Anchoring Vignettes for Interpersonally Incomparable Survey Responses

Gary King


January 18, 2007
Readings


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- Papers, FAQ, examples, software, conferences, videos: http://GKing.Harvard.edu/vign
Two Problems in Survey Research

1. How to measure "big" concepts we can define only by example. E.g., freedom, political efficacy, pornography, health, etc.

The usual advice: You do not have a methodological problem. Get a theory and it will produce a more concrete question.

2. How to ensure interpersonal and cross-population comparability.

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Anchoring Vignettes & Self-Assessments:
Political Efficacy (about voting)

How much say [does ‘name’ / do you] have in getting the government to address issues that interest [him / her / you]?
(a) Unlimited say, (b) A lot of say, (c) Some say, (d) Little say, (e) No say at all
“[Alison] lacks clean drinking water. She and her neighbors are supporting an opposition candidate in the forthcoming elections that has promised to address the issue. It appears that so many people in her area feel the same way that the opposition candidate will defeat the incumbent representative.”

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“[Jane] lacks clean drinking water because the government is pursuing an industrial development plan. In the campaign for an upcoming election, an opposition party has promised to address the issue, but she feels it would be futile to vote for the opposition since the government is certain to win.”

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- “[Jane] lacks clean drinking water because the government is pursuing an industrial development plan. In the campaign for an upcoming election, an opposition party has promised to address the issue, but she feels it would be futile to vote for the opposition since the government is certain to win.”

- “[Moses] lacks clean drinking water. He would like to change this, but he can’t vote, and feels that no one in the government cares about this issue. So he suffers in silence, hoping something will be done in the future.”

How much say [does ‘name’ / do you] have in getting the government to address issues that interest [him / her / you]?
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Does $R_1$ or $R_2$ have More Political Efficacy?

The only reason for vignette assessments to change over respondents is DIF.

Assumption holds because investigator creates the anchors (Alison, Jane, Moses).

Our simple (nonparametric) method works this way.
A Simple, Nonparametric Method

Define self-assessment answers relative to vignettes answers. For respondents who rank vignettes, $z_1 < z_2 < \cdots < z_J$.

$$C_i = \begin{cases} 
1 & \text{if } y_i < z_{i1} \\
2 & \text{if } y_i = z_{i1} \\
3 & \text{if } z_{i1} < y_i < z_{i2} \\
\vdots & \\
2J+1 & \text{if } y_i > z_{iJ} 
\end{cases}$$

Apportion $C_i$ equally among tied vignette categories (This is wrong, but simple; we will improve shortly)

Treat vignette ranking inconsistencies as ties

Requires vignettes and self-assessments asked of all respondents (Our parametric method doesn’t)
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Comparing China and Mexico
Opposition leader Vicente Fox elected President.
71-year rule of PRI party ends.
Peaceful transition of power begins.

Plenty of political efficacy
China: How much say do you have in getting the government to address issues that interest you?
The left graph is a histogram of the observed categorical self-assessments.

The right graph is a histogram of $C$, our nonparametric DIF-corrected estimate of the same distribution.
Key Measurement Assumptions

1. **Response Consistency**: Each respondent uses the self-assessment and vignette categories in approximately the same way across questions. (DIF occurs across respondents, not across questions for any one respondent.)

2. **Vignette Equivalence**:
   - (a) The actual level for any vignette is the same for all respondents.
   - (b) The quantity being estimated exists.
   - (c) The scale being tapped is perceived as unidimensional.

In other words: we allow response-category DIF but assume stem question equivalence.
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Inconsistencies: $y < z_2 < z_1$; $y = z_2 < z_1$; $z_2 < y < z_1$; $z_2 < y = z_1$; $z_2 < z_1 < y$.
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**Ties:**

- $y < z_1$ $z_1 < y < z_2$ $z_1 < z_2 < y$
- $y = z_2$ $y = z_2$ $y = z_2$
- $y > z_2$ $y > z_2$ $y > z_2$
Ties and Inconsistencies Produce *Ranges*

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**Ties:**

| 6                 | $y < z_1 = z_2$ | $T$       | {1} |

**Inconsistencies:**

| 7                 | $y < z_1 = z_2$ | $T$       | {1} |
| 8                 | $z_1 = y = z_2$ | $T$       | {2,3,4} |
| 9                 | $z_1 < y < z_2$ | $T$       | {1} |
| 10                | $z_1 < y < z_2$ | $T$       | {2,3,4,5} |
| 11                | $z_1 < z_2 < y$ | $T$       | {2,3,4,5} |
| 12                | $z_1 < z_2 < y$ | $T$       | {5} |
Ties and Inconsistencies Produce \textit{Ranges}

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**Ties:**

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| 7       | $y = z_1 = z_2$  | T            |              | T                 |              |              | {2,3,4} |
| 8       | $z_1 = z_2 < y$  |              |              | T                 |              | T            | {5} |

**Inconsistencies:**

### Ties and Inconsistencies Produce Ranges

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#### Ties:

| 6       | $y < z_1 = z_2$ | T            |              |                   |              |              | \{1\} |
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#### Inconsistencies:

| 9       | $y < z_2 < z_1$ | T            |              |                   |              |              | \{1\} |
| 10      | $y = z_2 < z_1$ | T            |              | T                 |              |              | \{1,2,3,4\} |
| 11      | $z_2 < y < z_1$ | T            |              | T                 |              |              | \{1,2,3,4,5\} |
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<tr>
<td>4</td>
<td>$z_1 &lt; y = z_2$</td>
<td></td>
<td></td>
<td></td>
<td>T</td>
<td></td>
<td>{4}</td>
</tr>
<tr>
<td>5</td>
<td>$z_1 &lt; z_2 &lt; y$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>T</td>
<td>{5}</td>
</tr>
</tbody>
</table>

Ties:

| 6       | $y < z_1 = z_2$ | T          |            |                |            |            | {1} |
| 7       | $y = z_1 = z_2$ |            | T          |                | T          |            | {2,3,4} |
| 8       | $z_1 = z_2 < y$ |            |            |                |            | T          | {5} |

Inconsistencies:

| 9       | $y < z_2 < z_1$ | T          |            |                |            |            | {1} |
| 10      | $y = z_2 < z_1$ | T          |            |                | T          |            | {1,2,3,4} |
| 11      | $z_2 < y < z_1$ | T          |            |                |            | T          | {1,2,3,4,5} |
| 12      | $z_2 < y = z_1$ |            |            | T              |            | T          | {2,3,4,5} |
| 13      | $z_2 < z_1 < y$ |            |            |                |            | T          | {5} |
How to analyze a variable with scalar and vector responses?

Define an unobserved variable:

\[ Y_i \sim \text{Normal}(x_i \beta, 1) \]

With observation mechanism, for scalar \( C \), the same as ordered probit:

\[ C_i = c \text{ if } \tau_{c-1} \leq Y_i < \tau_c \]

Probability of observing category \( c \), for \( X = x_0 \):

\[ \Pr(C_i = c | x_0) = \int_{\tau_{c-1}}^{\tau_c} \text{Normal}(y | x_0 \beta, 1) \, dy \]

Observation mechanism for vector valued \( C \):

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Analyzing the DIF-Free Variable: More Efficiencies

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Robust Analysis via Conditional Model

Condition on observed value of $c_i$:

$$\Pr(C = c | x_0, c_i) = \begin{cases} 
\Pr(C = c | x_0) & \text{for } c \in c_i \\
0 & \text{otherwise}
\end{cases}$$

Advantages compared to unconditional probabilities:

Conditions on $c_i$ by normalizing the probability to sum to one within the set $c_i$ and zero outside that set. For scalar values of $c_i$, this expression simply returns the observed category: $\Pr(C = c | x_i, c_i) = 1$ for category $c$ and 0 otherwise. For vector valued $c_i$, it puts probability density over categories within $c_i$, which in total sum to one. Probabilities can be interpreted for causal effects or summed to produce a histogram.

Result: highly robust to model mispecification, extracts considerably more information from anchoring vignette data.
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Improved Efficiency in Practice

Uniform

Unconditional

Conditional
Ultimate Goal:
Learn about a continuous unobserved variable (health, efficacy).

Observed:
Proportions of the mass of the continuous variable (and hence observations) falling in each discrete category defined by the vignettes

Worst choice:
All in one category; i.e., information = discriminatory power (E.g., “Bob ran two marathons last week. . . ” does not discriminate among respondents)

Best choice:
Largest number of categories, with mass of the unobserved variable spread uniformly over categories

Immediate Goal:
Measure information in a categorization scheme (defined by the choice of vignettes)

Formalization of the goal:
Define a function $H(C)$ measuring information.

Operational use:
Run a pretest with lots of vignettes
Compute $C$ and $H(C)$ for each possible subset,
Choose a subset for the main survey based on values of $H$ and cost of survey questions.
Ultimate Goal: Learn about a continuous unobserved variable (health, efficacy).
Optimally Choosing Vignettes

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Step 1: Criteria for Defining $H(C)$ for scalar $C$

Summarize $C$ with a histogram, so $H(C) = H(p_1, \ldots, p_{2J+1})$.

1. $H(0, 1, 0, 0, 0) = 0$, i.e., when all mass is in (any) one category and at a maximum when $p_1 = p_2 = \cdots = p_{2J+1}$.

2. $H$ is a monotonically increasing function of the number of vignettes (and hence $2J + 1$, the number of categories of $C$).

3. Assume consistent decomposition:
   - With one vignette, $C$ has 3 categories (below, equal to, or above the vignette) and proportions $p_1 + p_2 + q = 1$.
   - Add a new vignette and we can decompose the "above" category (into between the two vignettes, equal to the second, or above the second).
   - We now have 5 categories, with proportions $p_1 + p_2 + p_3 + p_4 + p_5 = 1$.
   - The information in the union of the smaller bins ($3, 4, 5$) should equal that in the original undecomposed bin since $q = p_3 + p_4 + p_5$.
   - The information in the unaffected bins (1, 2) should remain the same with the addition of the new vignette.

   More formally:
   $$ H(p_1, p_2, p_3, p_4, p_5) = H(p_1, p_2, q) + qH(p_3, p_4, p_5) $$
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Step 1: Criteria for Defining $H(C)$ for scalar $C$

Summarize $C$ with a histogram, so $H(C) = H(p_1, \ldots, p_{2J+1})$. Add 3 criteria:

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Anchoring Vignettes
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Lots of candidates exist: Gini index, variance, absolute deviations etc. Only one measure satisfies all three criteria: entropy. Thus, formally, we set:

$$H(p_1, \ldots, p_{J+1}) = -\sum_{j=1}^{J+1} p_j \ln(p_j)$$

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Without ties or inconsistencies, we simply compute entropy.

With ties and inconsistencies, we somehow estimate $p$'s and then compute entropy, $H$.

Rules for estimating the $p$'s, and thus types of entropy:

- **Estimated entropy**: using the multiple response ordered probit model.
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Estimated Entropy

Measures the informativeness of the vignettes, as supplemented by the predictive information in the covariates. A reasonable approach, uses a modification of a standard statistical model, and robust to misspecification. But it assumes the probit specification is correct. Normally this is ok, but decisions here are more consequential since they affect data collection decisions and thus can preclude asking some questions. Thus, we also want "known entropy."
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Computing Known Entropy (no assumptions required)

Scalar-valued $C_i$ observations are set to observed values.

Vector-valued $C_i$: Elements of all possible vector responses are parameterized: (e.g., $p_1, p_2, p_3$ for $C_i = \{2, 3, 4\}$)

All mass is restricted to within the vector (e.g., $p_1 + p_2 + p_3 = 1$)

Choose all $p$'s to minimize entropy (i.e., adjust the $p$'s to see how spiky the distribution can become)

Some tricks make this easy with a genetic optimizer.

Then form the histogram (summing the $p$'s) and compute entropy.

We now compute estimated entropy and known entropy for all possible subsets of vignettes.
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We now compute *estimated entropy* and *known entropy* for all possible subsets of vignettes.
One vignette can be better than three: Sleep (China)
Some vignette sets are uninformative: Self-Care (China)
Some covariates are unhelpful: Pain (China)
If thresholds vary, categorical answers are meaningless.

Our parametric model works by estimating the thresholds.

Vignettes provide identifying information for the $\tau$’s.
Model Summary

An ordinal probit model.

Actual:

Perceived:

Reported:

Self-Assessment
An ordinal probit model.
with varying thresholds,

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optional multiple self-assessment questions,
An ordinal probit model, with varying thresholds, a vignette for identification, more vignettes for better discrimination, optional multiple self-assessment questions, and an optional random effect.
Self-Assessment:
In the last 30 days, how much difficulty did [you/name] have in seeing and recognizing a person you know across the road (i.e. from a distance of about 20 meters)?

(A) none, (B) mild, (C) moderate, (D) severe, (E) extreme/cannot do
Self-Assessments v. Medical Tests

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The Snellen Eye Chart Test:
Fixing DIF in Self-Assessments of Visual (Non)acuity

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<td>Mean</td>
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<td>μ</td>
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The medical test shows Slovakians see much better than the Chinese. Ordinal probit finds no difference. Chopit reproduces the same result as the medical test (though on different scale).
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Anchoring vignettes will not eliminate all DIF, but problems would have to occur at unrealistically extreme levels to make the unadjusted measures better than the adjusted ones.

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If you think you have DIF-free questions, you now have the first real opportunity to test that hypothesis.

Whether or not you have DIF, vignettes can help us follow the usual survey advice of making questions concrete. (Compare “say in government” with that question plus the vignettes)

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For More Information

http://GKing.Harvard.edu/vign

Includes:

- Academic papers
- Anchoring vignette examples by researchers in many fields,
- Frequently asked questions,
- Videos
- Conferences
- Statistical software
Define $\mu$ as the quantity of interest; $D$ as DIF.

1. If model assumptions hold:
   
   Self-assessments estimate: $\left( \mu + D \right)$.
   
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Since the same person generates both $D_s$ and $D_v$, (b) will usually be smaller.

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Anchoring Vignettes Measure DIF, not Vision: A Heuristic

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Self-Assessment Component: for $i = 1, \ldots, n$

Actual level:

$$\mu_i = X_i \beta + \eta_i,$$

with random effect $\eta_i \sim N(0, \omega^2)$

Perceived level:

$$Y^*_i \sim N(\mu_i, 1).$$

Reported Level:

$$y_{i1} = k \text{ if } \tau_{k-1} \leq Y^*_i < \tau_k \ldots y_{iS} = k \text{ if } \tau_{k-1} \leq Y^*_i < \tau_k.$$
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\tau_{is}^k &= \tau_{is}^{k-1} + e^{\gamma_k V_i} \quad (k = 2, \ldots, K_s) \end{align*}
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where
Vignette Component: for $\ell = 1, \ldots, N$

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$Z^*_{\ell 1} \sim N(\theta_1, \sigma^2)$

$Z^*_{\ell J} \sim N(\theta_J, \sigma^2)$

Reported Level:

$z_{\ell j} = k$ if $\tau_{k-1} < Z^*_{\ell j} < \tau_k$

where $\tau_{1s} = V_{\ell s}$

$\tau_{k} = \tau_{k-1} + e_{\gamma k}$

(k = 2, ..., K_s)

Gary King ()

Anchoring Vignettes
Vignette Component: for $\ell = 1, \ldots, N$

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($S$ ordered probits with varying thresholds).
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$$L_s(\beta, \omega^2, \gamma | y) \propto \prod_{i=1}^{n} \int_{-\infty}^{\infty} \prod_{s=1}^{S} \prod_{k=1}^{K_s} \left[ F(\tau_{is}^k | X_i \beta + \eta, 1) - F(\tau_{is}^{k-1} | X_i \beta + \eta, 1) \right]^{1(y_{is}=k)} N(\eta | 0, \omega^2) d\eta$$
The Likelihood Function: Self-Assessment Component

If $\eta_i$ were observed:

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($S$ ordered logits with varying thresholds). Since $\eta_i$ is unobserved,

$$L_s(\beta, \omega^2, \gamma|y) \propto \prod_{i=1}^n \int_{-\infty}^{\infty} \prod_{s=1}^S \prod_{k=1}^{K_s} \left[ F(\tau_{is}^k|X_i\beta + \eta, 1) - F(\tau_{is}^{k-1}|X_i\beta + \eta, 1) \right]^{1(y_{is}=k)} N(\eta|0, \omega^2) d\eta$$

In the special case where $S = 1$, this simplifies to

$$L_s(\beta, \omega^2, \gamma|y) = \prod_{i=1}^n \prod_{k=1}^{K_1} \left[ F(\tau_{i1}^k|X_i\beta, 1 + \omega^2) - F(\tau_{i1}^{k-1}|X_i\beta, 1 + \omega^2) \right]^{1(y_{i1}=k)}$$
The vignette component is a \( J \)-variate ordinal probit with varying thresholds:

\[
L_v(\theta, \sigma^2, \gamma | z) \propto \prod_{\ell=1}^N \prod_{j=1}^J \prod_{k=1}^{K_1} \left[ F(\tau_{\ell 1}^k | \theta_j, 1) - F(\tau_{\ell 1}^{k-1} | \theta_j, \sigma^2) \right] 1(z_{\ell j} = k)
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\]

The joint likelihood shares parameter \( \gamma \):

\[
L(\beta, \sigma^2, \omega^2, \theta, \gamma | y, z) = L_s(\beta, \sigma^2, \omega^2, \gamma | y) \times L_v(\theta, \gamma | z).
\]

and nests the ordinal probit model as a special case.
Fixing DIF in China and Mexico

<table>
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<th>Eqn.</th>
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<td>Coeff. (s.e.)</td>
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<td>ln $\sigma$</td>
<td>$- .238$ (.042)</td>
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The Source of DIF in China and Mexico: Threshold Variation

![Diagram showing the relationship between political efficacy and density in China and Mexico.](image)
Computing Quantities of Interest

1. Effect Parameters

The effect parameters $\beta$ are interpreted as in a linear regression of actual levels $\mu_i$ on $X_i$ and $\eta_i$.

2. Actual Levels, without a Self-Assessment

Choose hypothetical values of the explanatory variables, $X_c$. The posterior density of $\mu_c$ is similar to regression:

$$P(\mu_c | y) = N(\mu_c | X_c \hat{\beta}, X_c' \hat{V}(\hat{\beta}) X_c + \hat{\omega}^2)$$

E.g., we can use the mean, $X_c \hat{\beta}$ as a point estimate of the actual level when $X = X_c$. 

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Anchoring Vignettes
1. If we know $y_i$, why not use it?

2. For example, suppose John and Esmeralda have the same $X$ values. By Method 1, they give the same inferences: $P(\mu_J | y) = P(\mu_E | y)$. Suppose John's $y_J$ value is near $\hat{\mu}_J$ and but Esmeralda's is far away. Under Method 1, nothing's new. Predictions are unchanged. Intuitively, John is average and Esmeralda is an outlier. We should adjust our prediction from $\hat{\mu}_E$ toward $y_E$. So the new method takes roughly the weighted average of the model prediction $\hat{\mu}_E$ and the observed $y_E$, with weights determined by the how good a prediction it is.
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More formally, we use Bayes theorem

\[ P(\mu_i | y, y_i) \propto P(y_i | \mu_i, y) P(\mu_i | y), \]
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$$\times N(\mu_i|X_i\hat{\beta}, X_i\hat{\nu}(\hat{\beta})X'_i + \hat{\omega}^2)$$
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Key Difference: 
- \( P(\mu_i | y) \) works for out-of-sample prediction
- \( P(\mu_i | y, y_i) \) works better when \( y_i \) is available
Unconditional posterior for a hypothetical 65-year-old respondent in country 1, based on one simulated data set.
Conditional posteriors for two different 21 year old respondents. Person 1 gave responses (1,1) on the two self-evaluation questions; Person 2 gave responses (4,3). The unconditional posterior, drawn with a dashed line, gives less specific predictions. Each curve was computed from one simulated data set.