How to Measure Legislative District Compactness If You Only Know it When You See it[1]

Gary King[2]

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Harvard University

Hubert M. Blalock Memorial Lecture, University of Michigan, 7/12/2017

[1] Based on joint work with Aaron Kaufman and Mayya Komisarchik

[2] GaryKing.org

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A simple single compactness dimension that you know when you see

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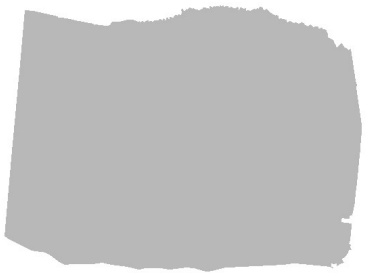

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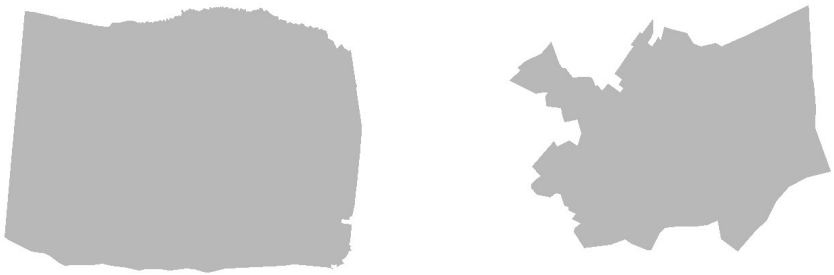

More Compact ← → Less Compact

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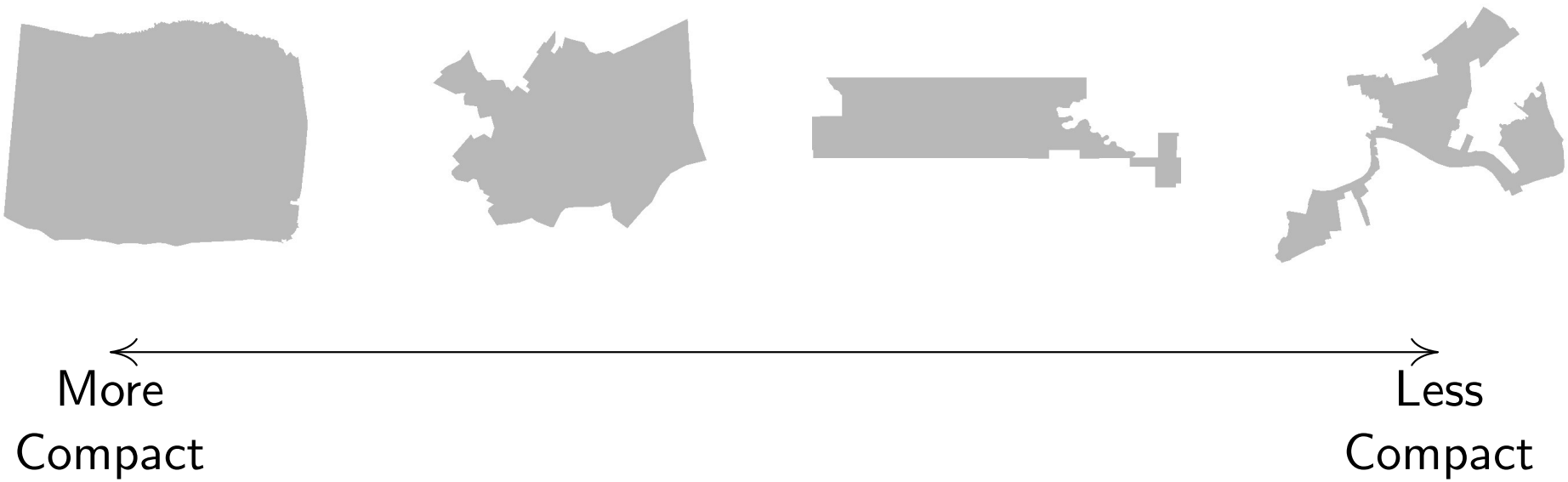

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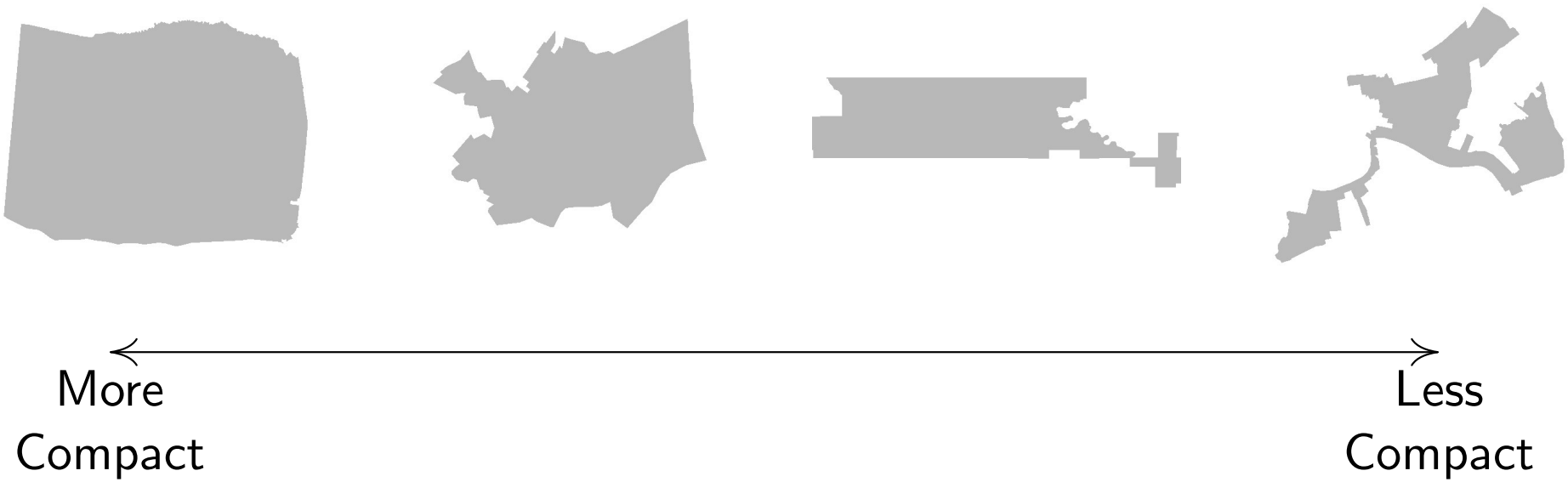

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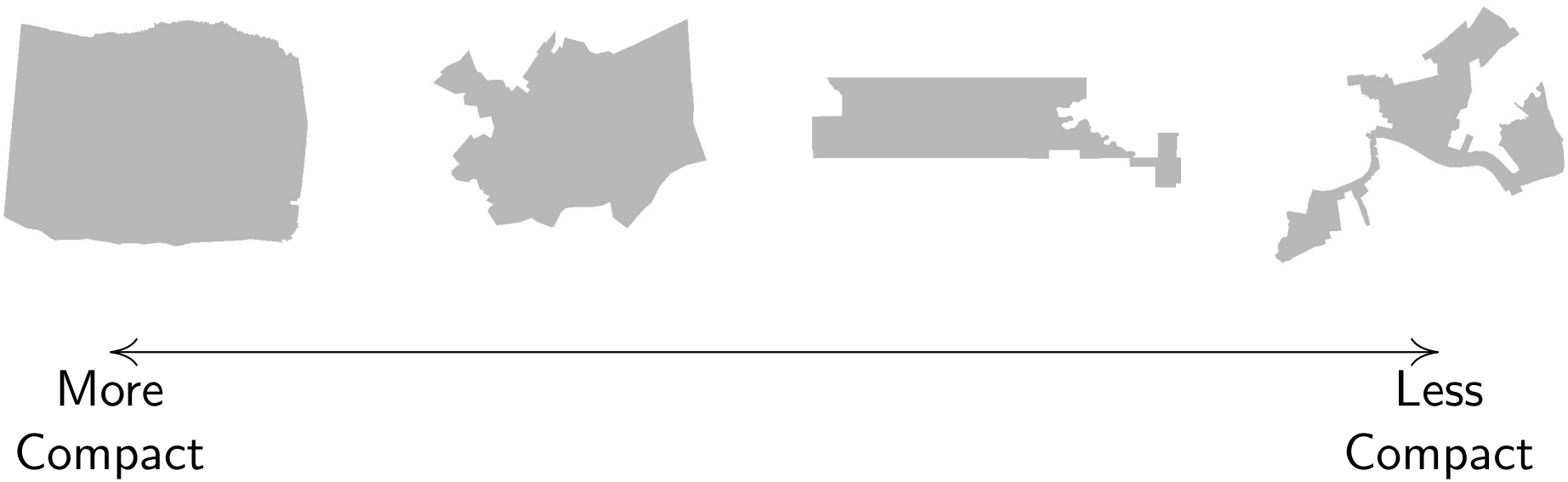

- The dimension is intuitive
- How to estimate where a new district shape falls on this dimension?

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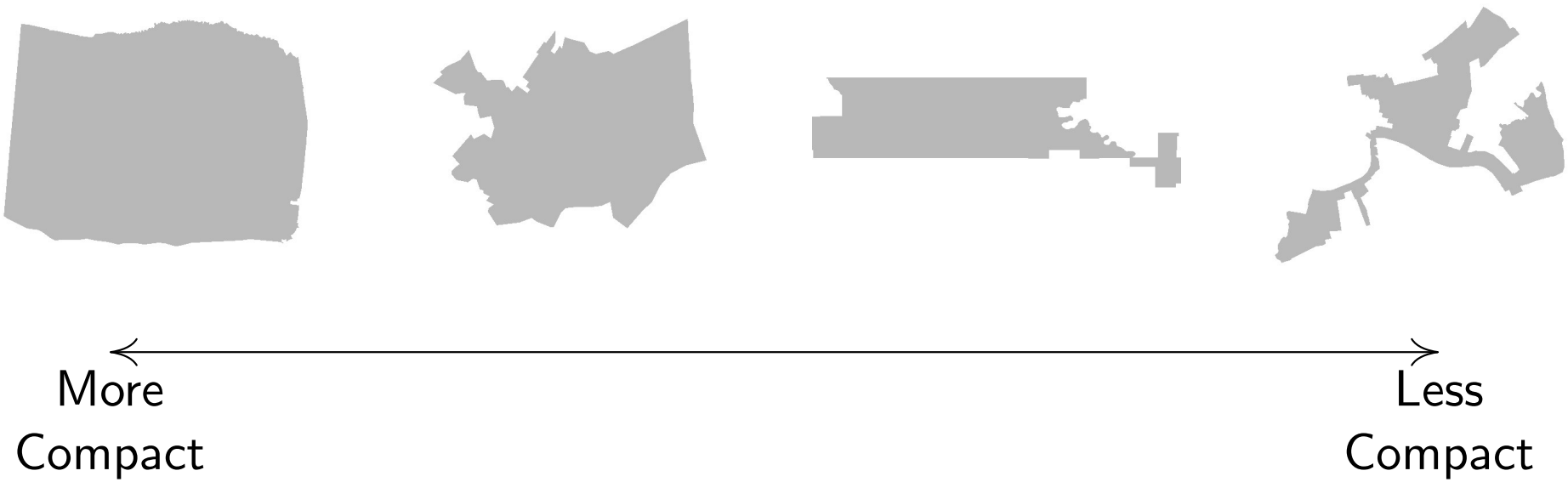

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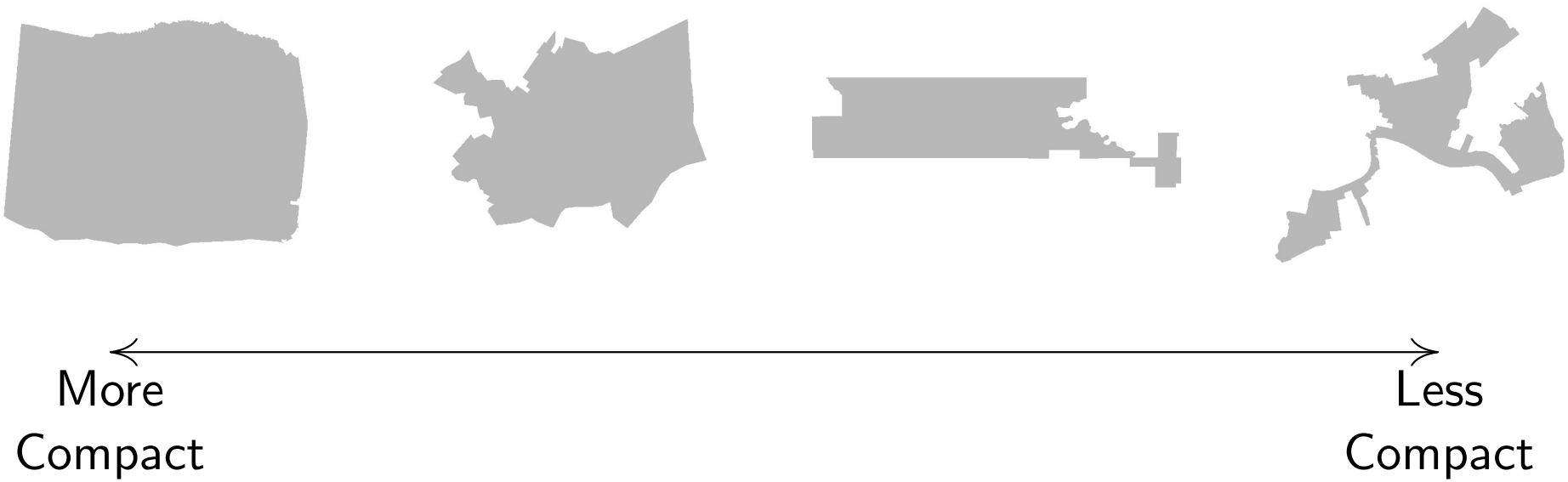

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- $\rightsquigarrow$ Let's start with existing measures by social scientists

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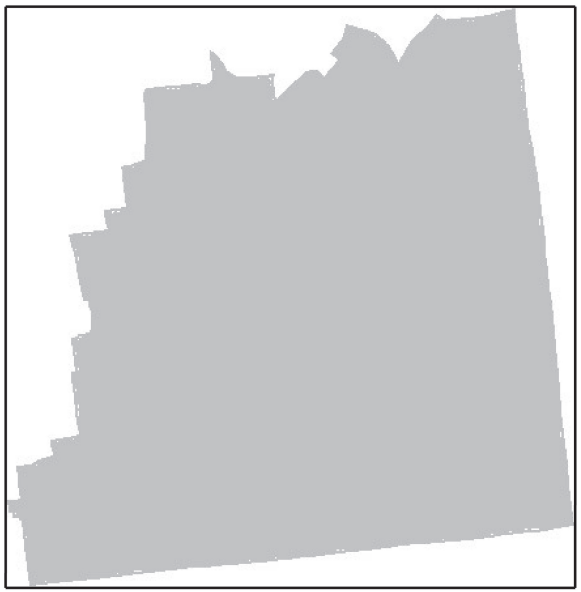

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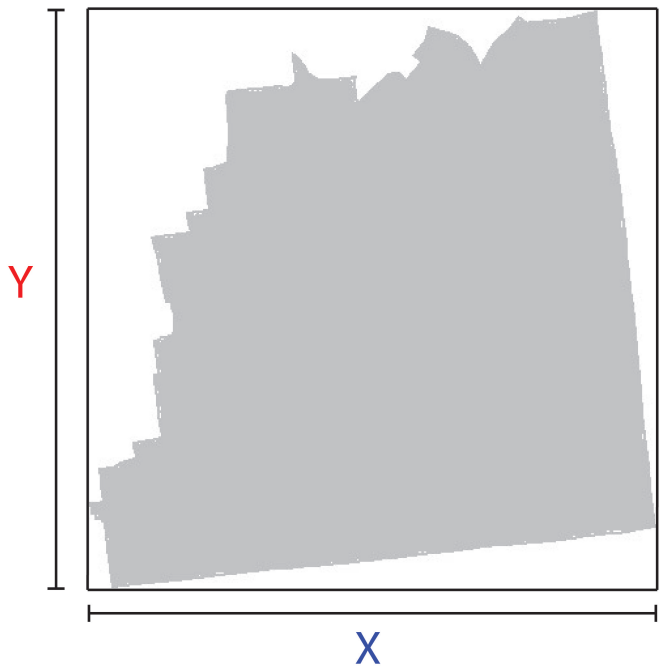

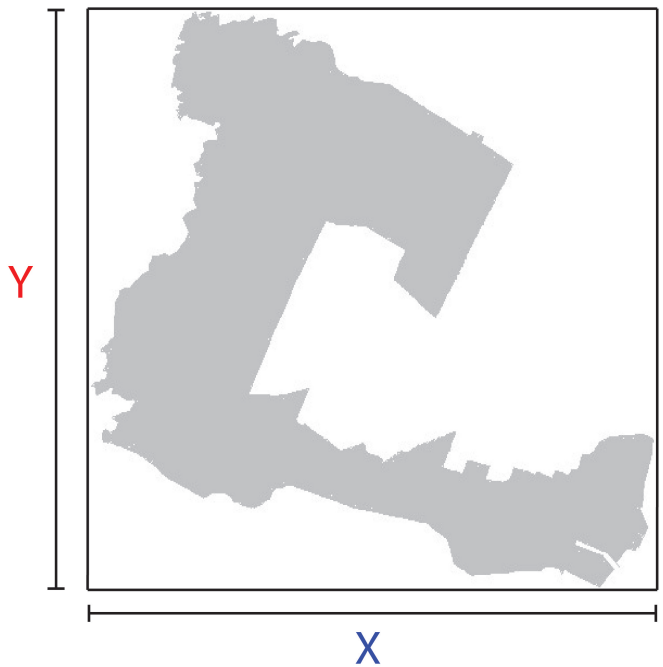

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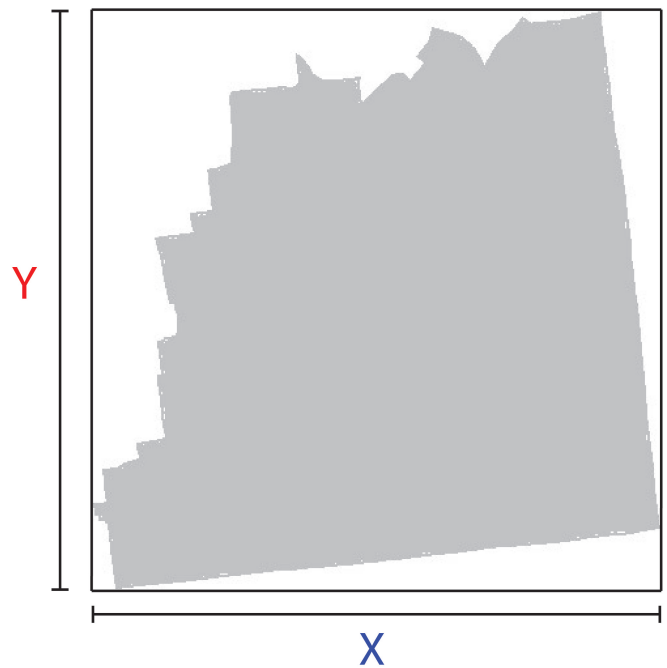

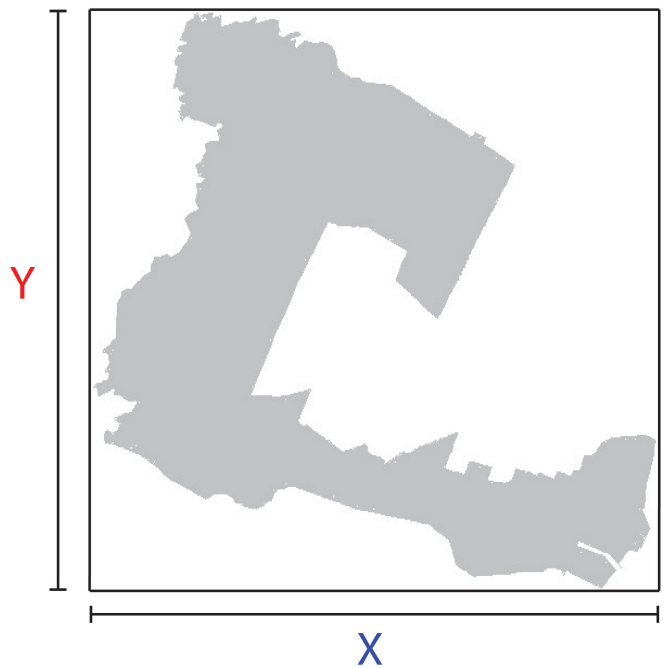

In both districts: $X/Y \approx 1.30$

Measure 2: Reock, District / Bounding Circle Areas

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Circular districts are most compact

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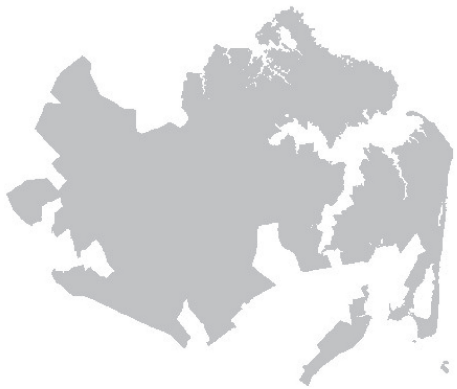

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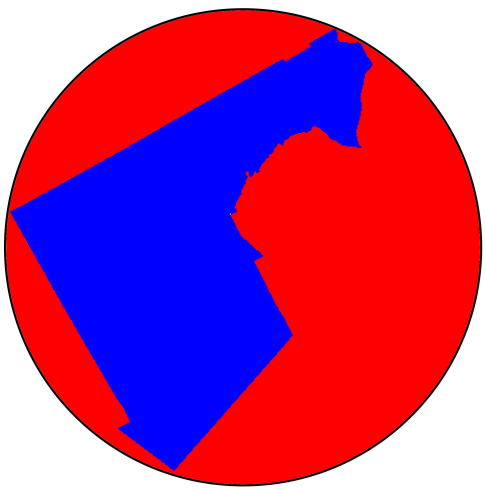

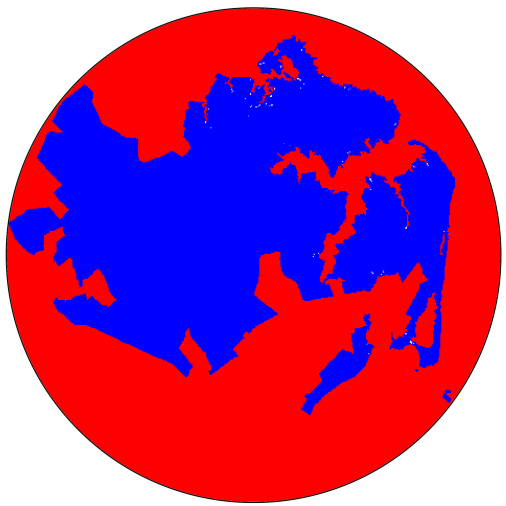

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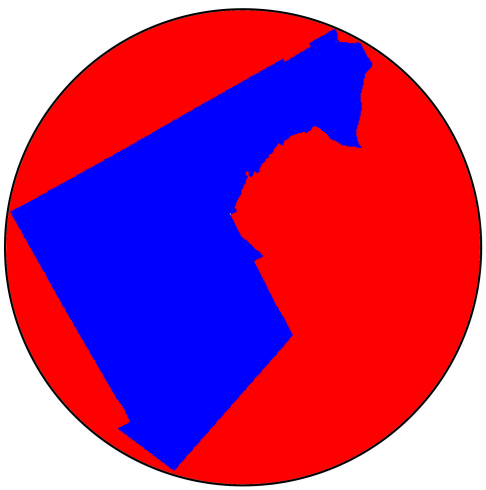

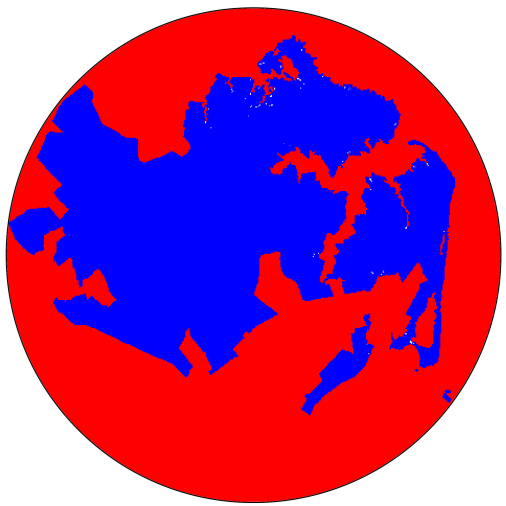

In both cases, $X/(Y + X) \approx 0.37$

Measure 3: Boyce-Clark, Variation in Centroid Deviations

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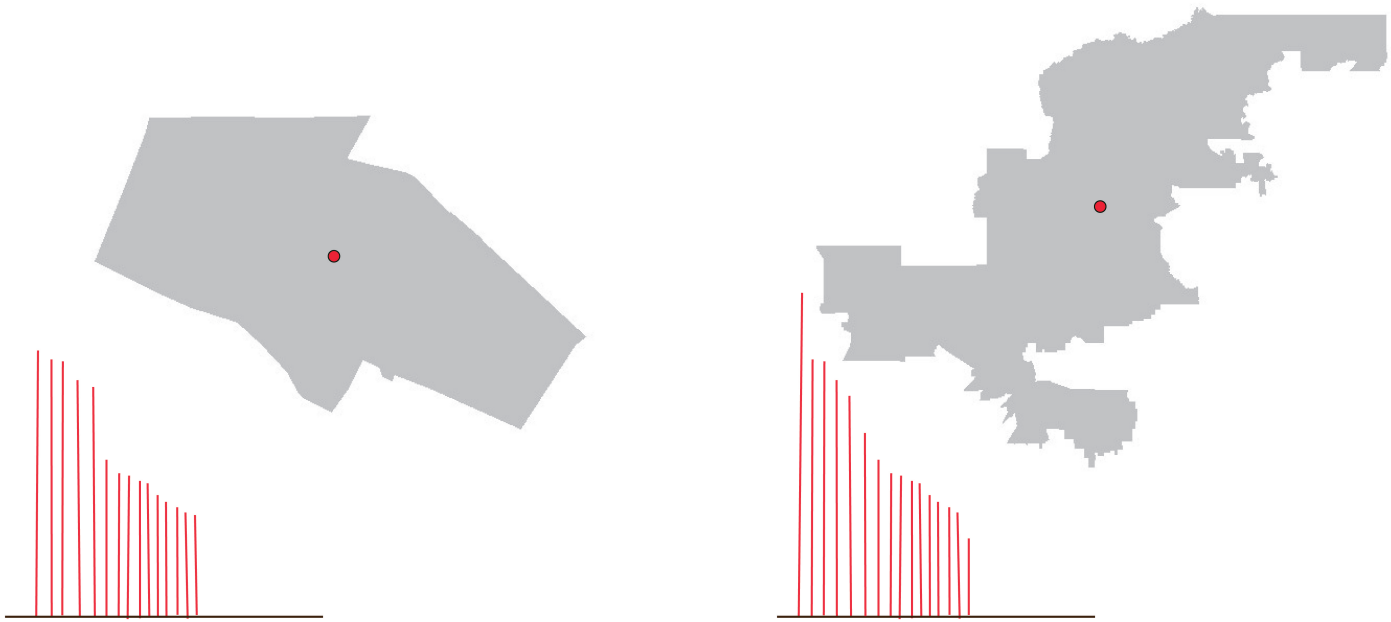

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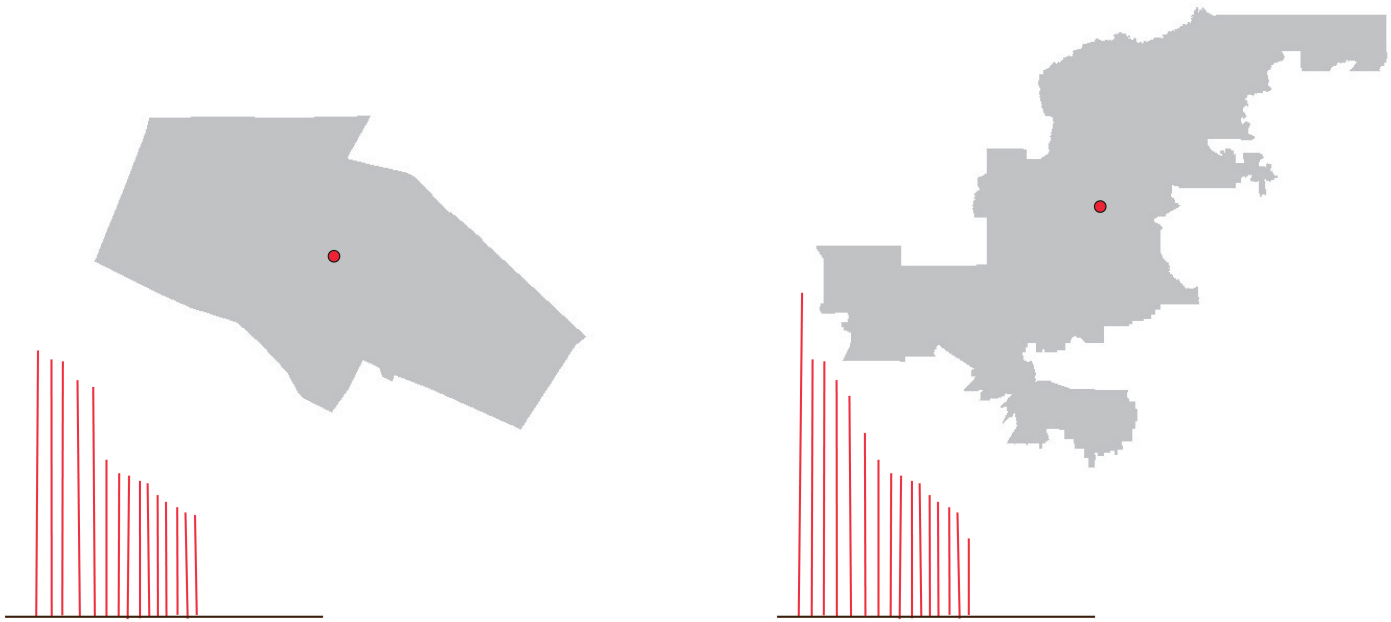

In both cases, MAD(r) ≈ 0.31

A Brief Rotational Invariance Interlude:

A Brief Rotational Invariance Interlude: Can you Name this Celebrity?

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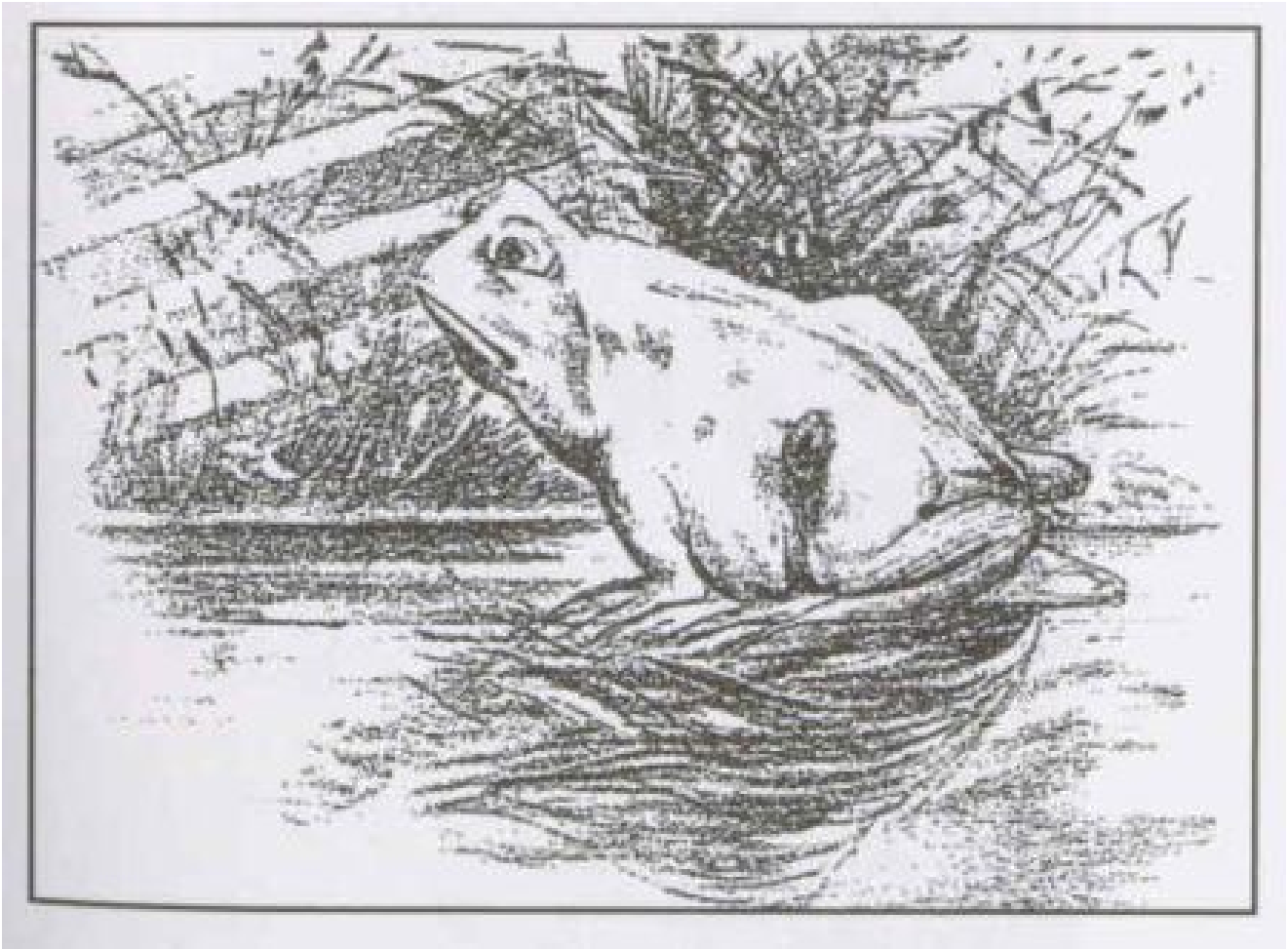
A Brief Rotational Invariance Interlude: See the Frog?



A Brief Rotational Invariance Interlude: See the ~~Frog~~ Horse?

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- $\rightsquigarrow$ Measuring "you know it when you see it": No rotational invariance

New Measure: Y-Symmetry, area of symmetric reflection

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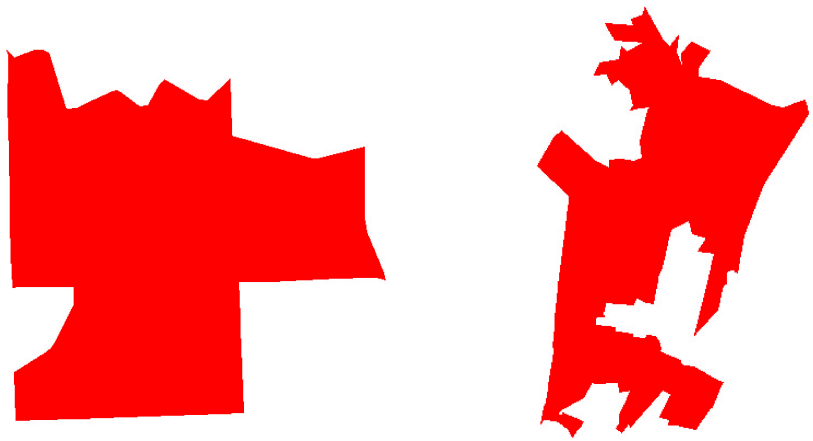

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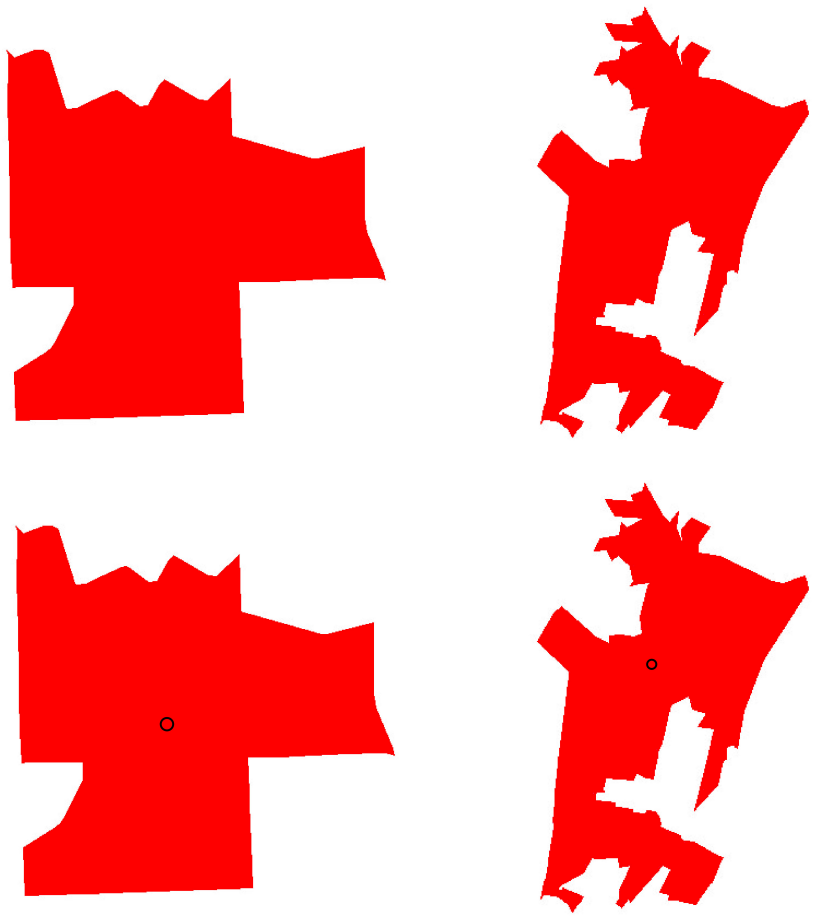

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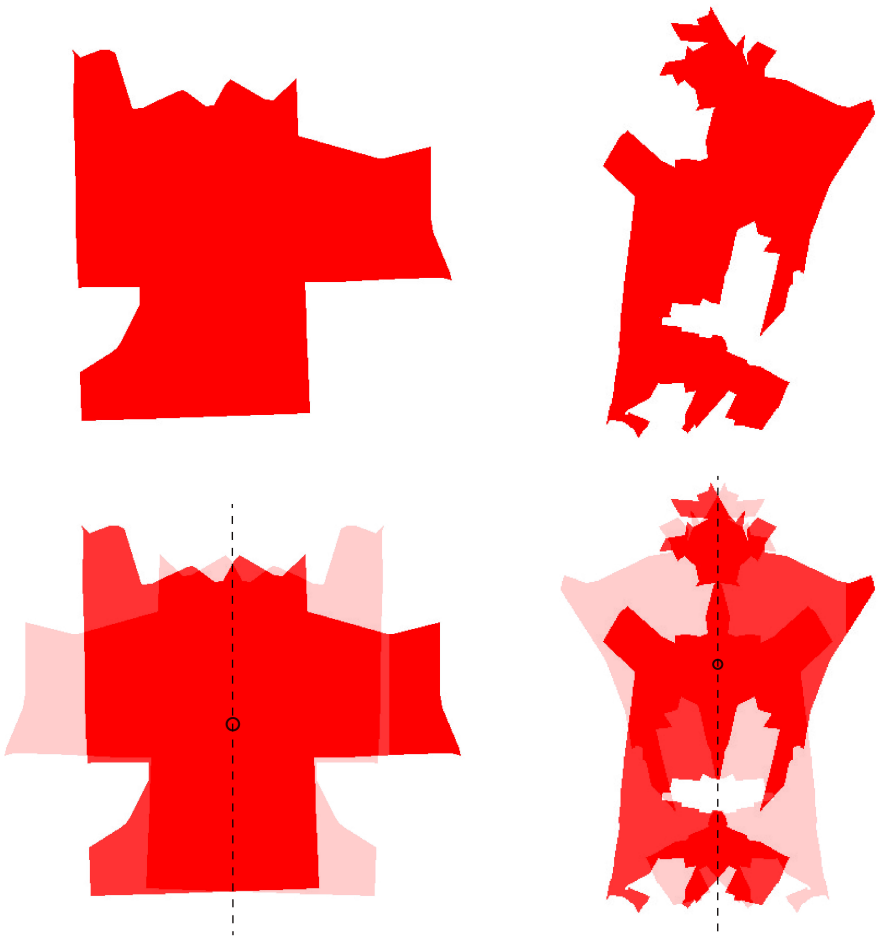

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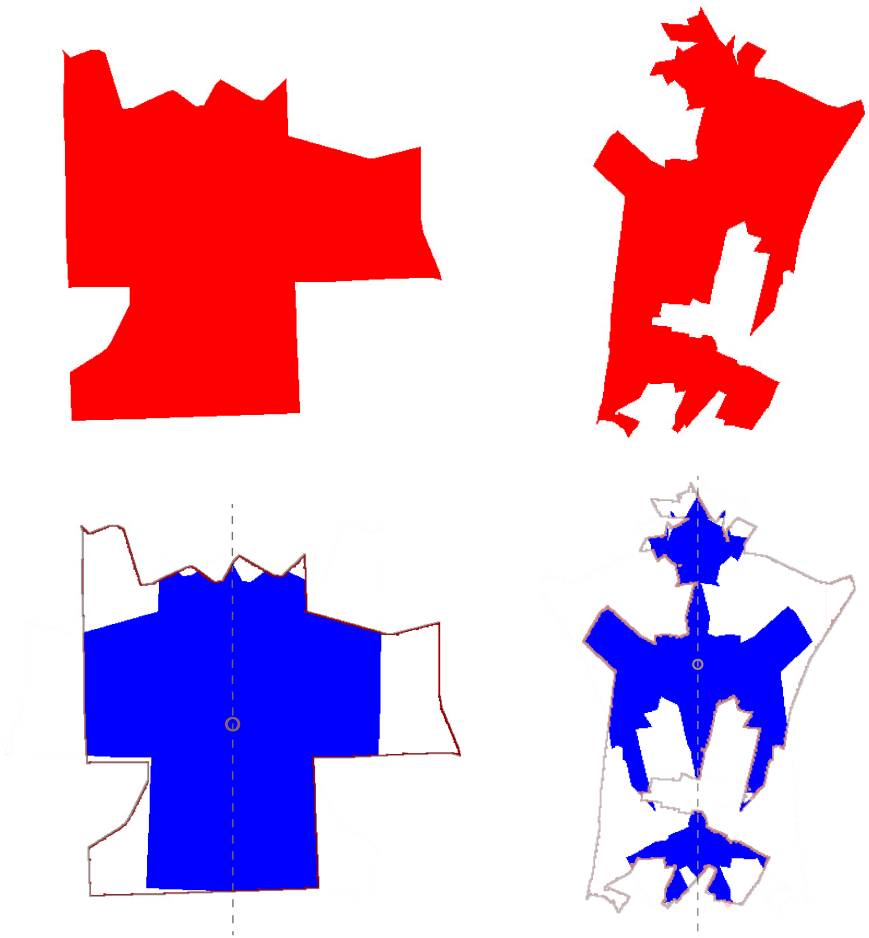

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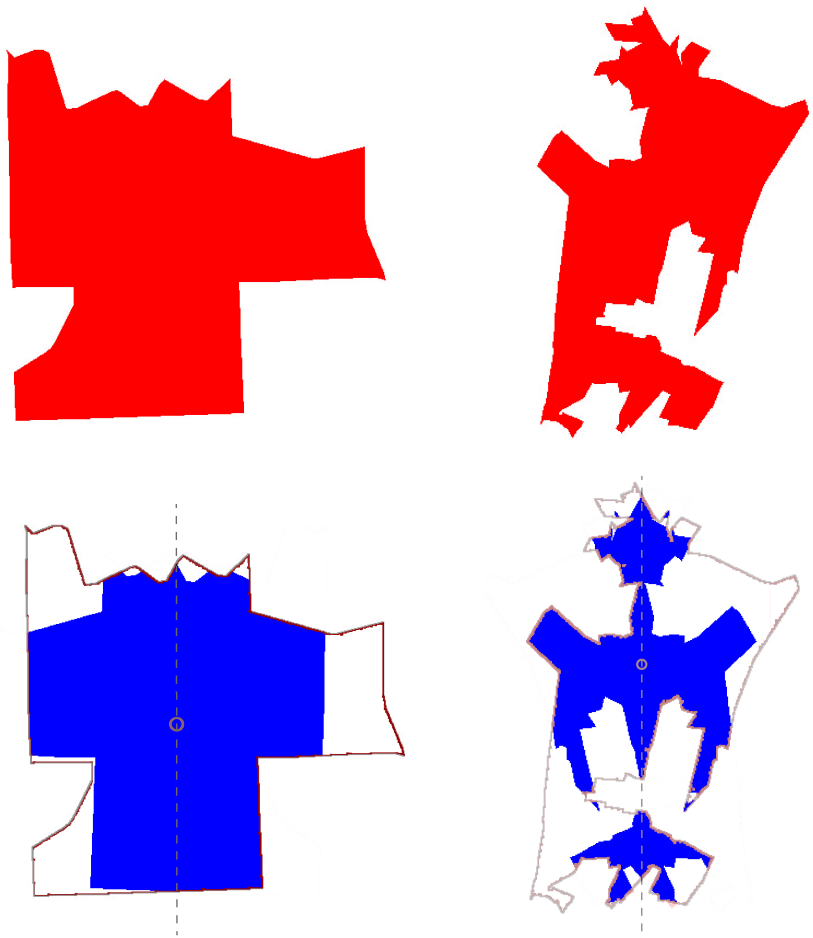

In both cases, Overlap/Original Area ≈ 0.34

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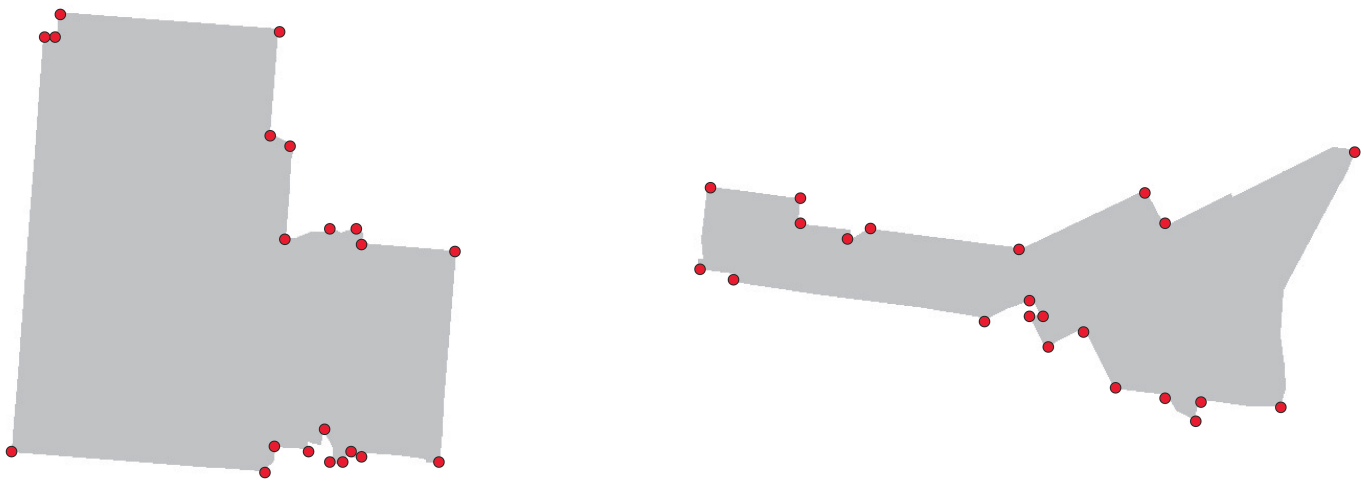

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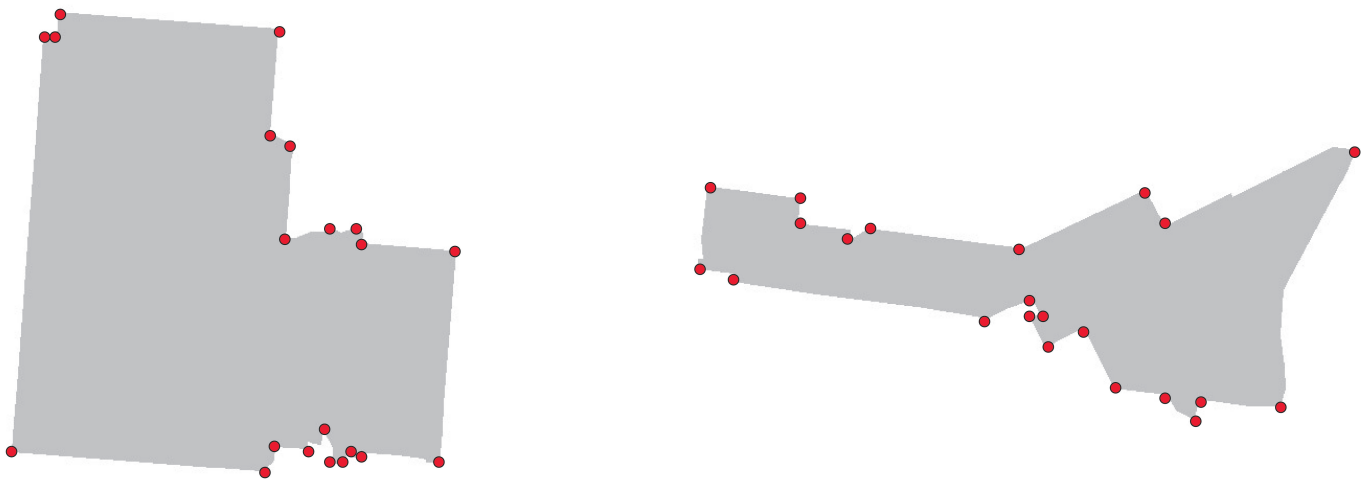
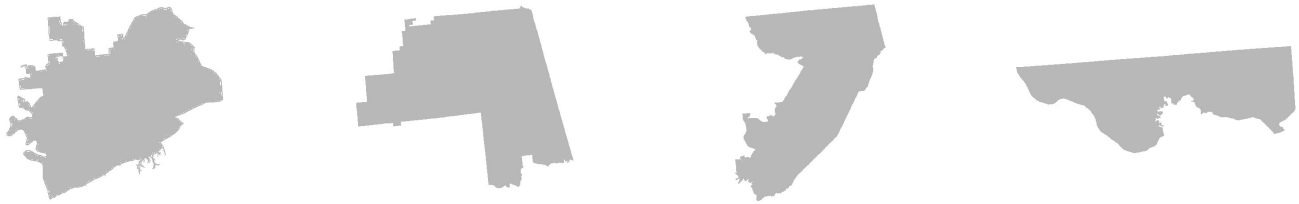


Both districts have 21 significant corners

Which is more compact?



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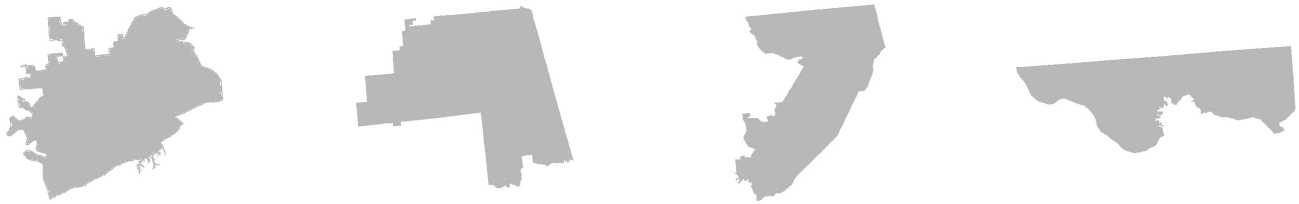

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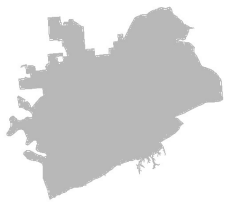
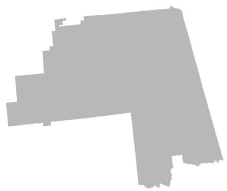

Reock	1	2	3	4

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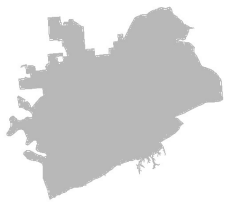
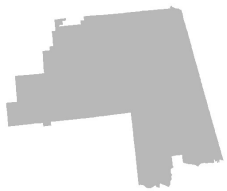

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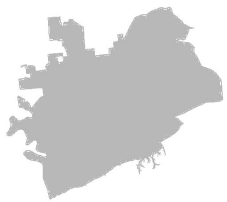
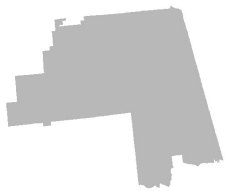

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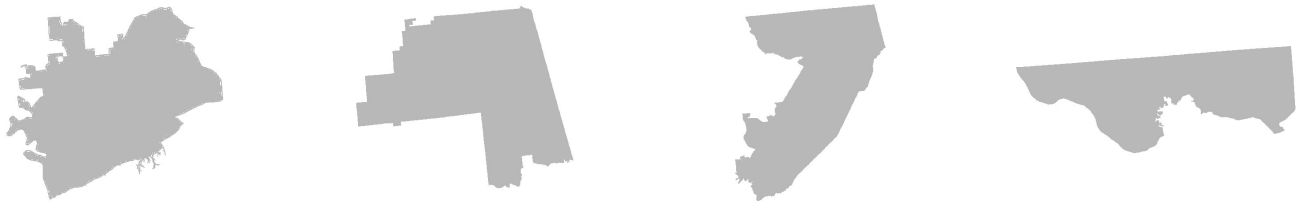

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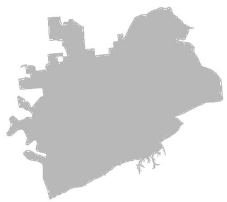
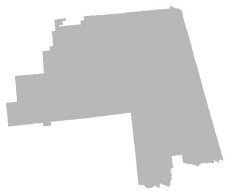

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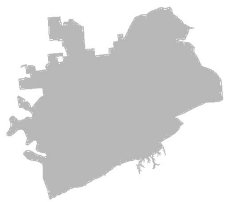
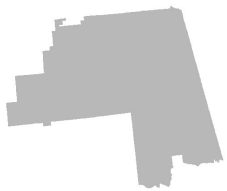

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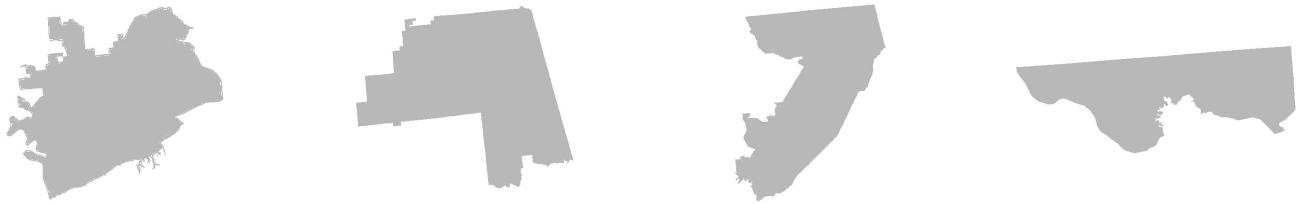

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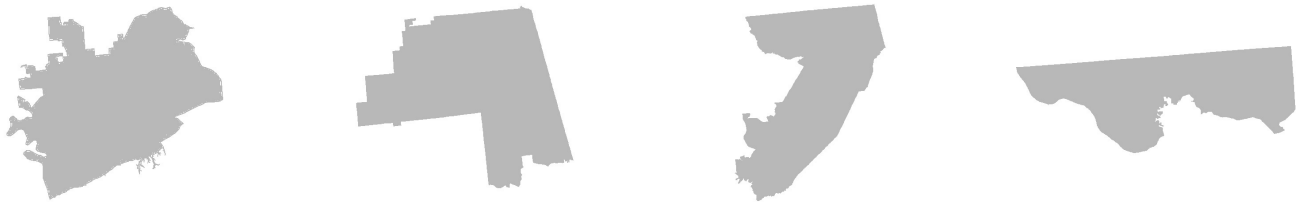

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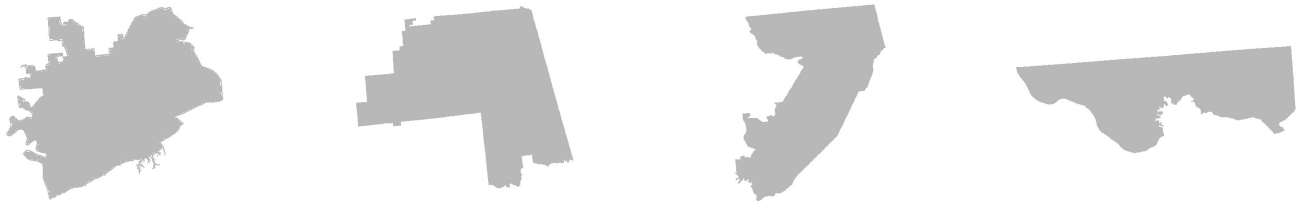

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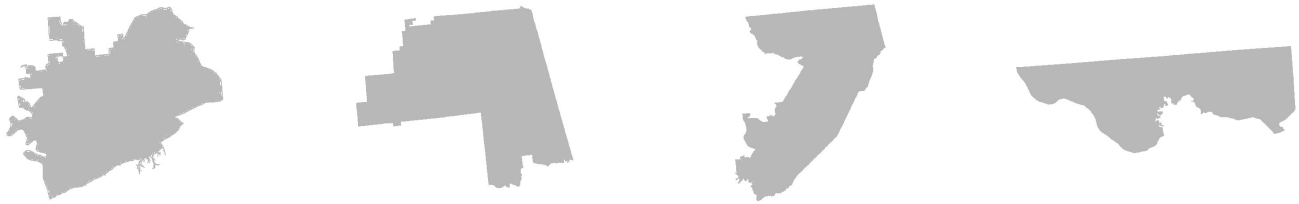

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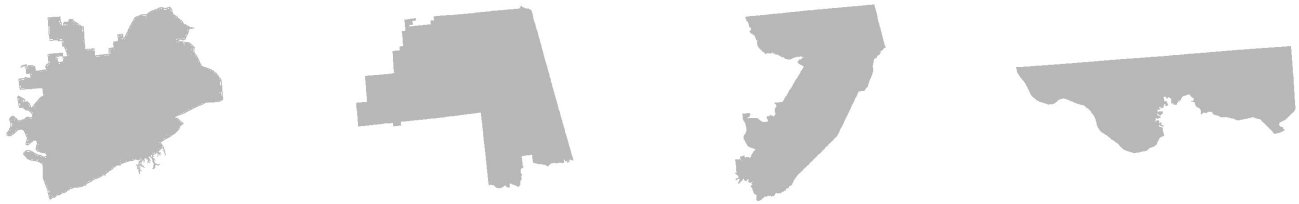

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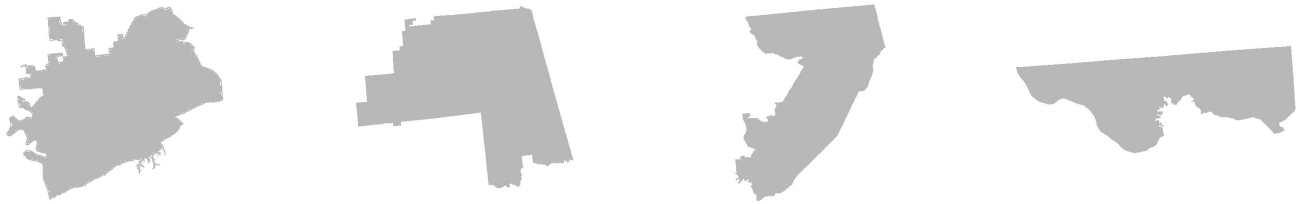

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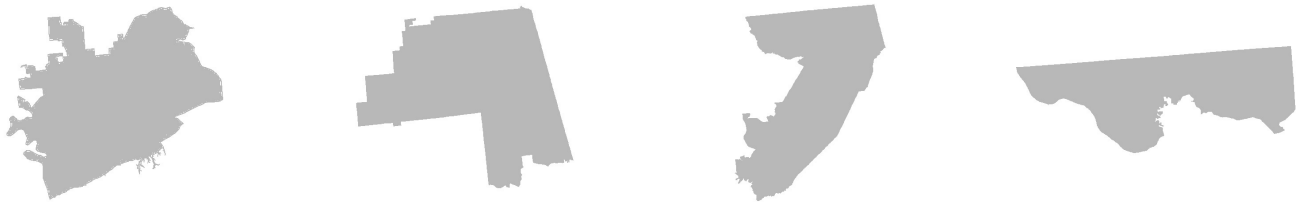

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Full Ranking

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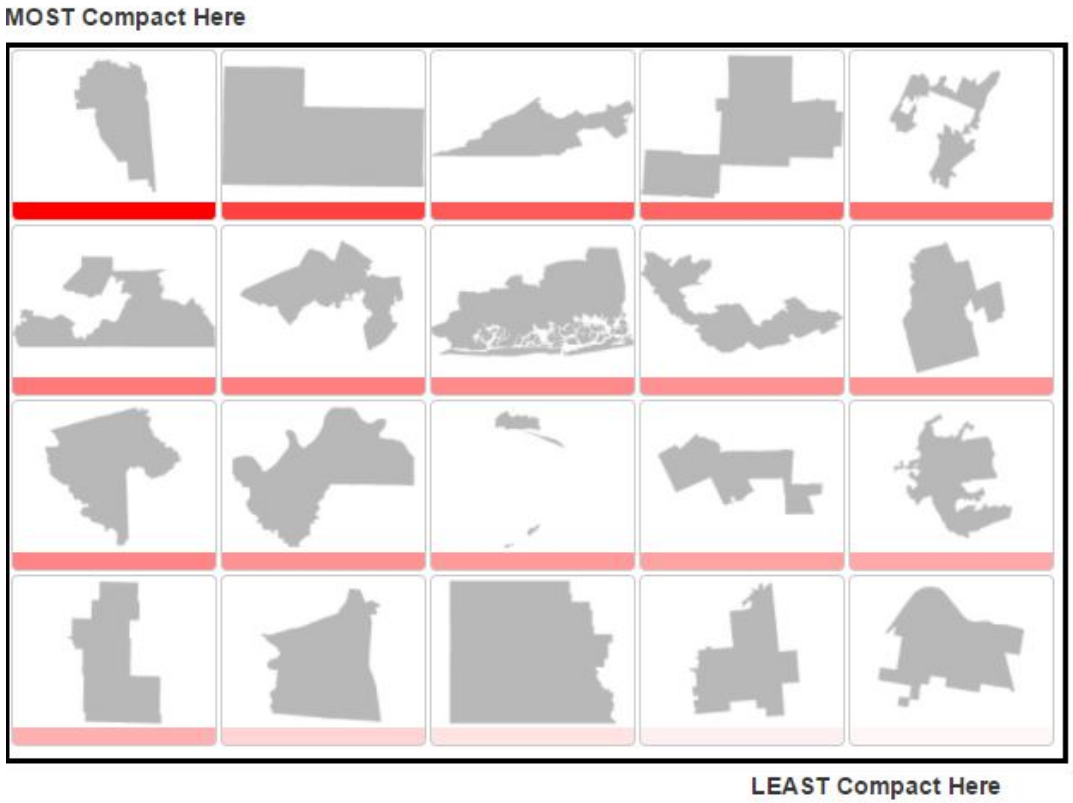
Full Ranking — on line



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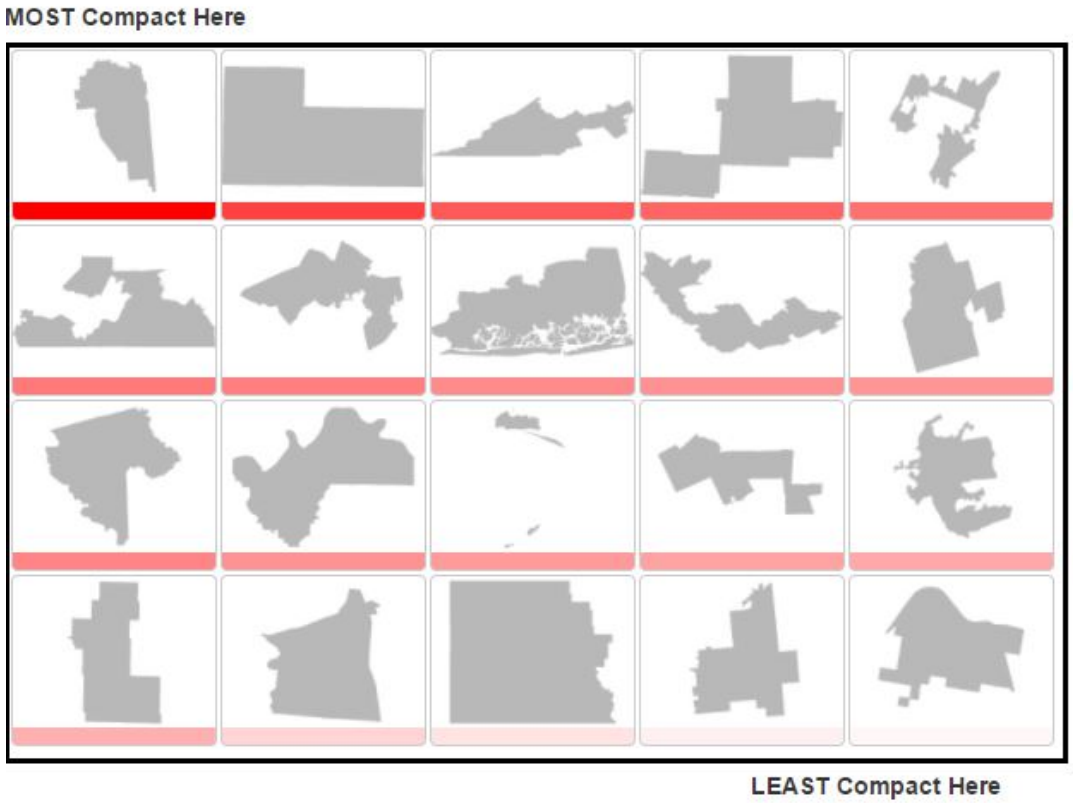

We show: very high reliability

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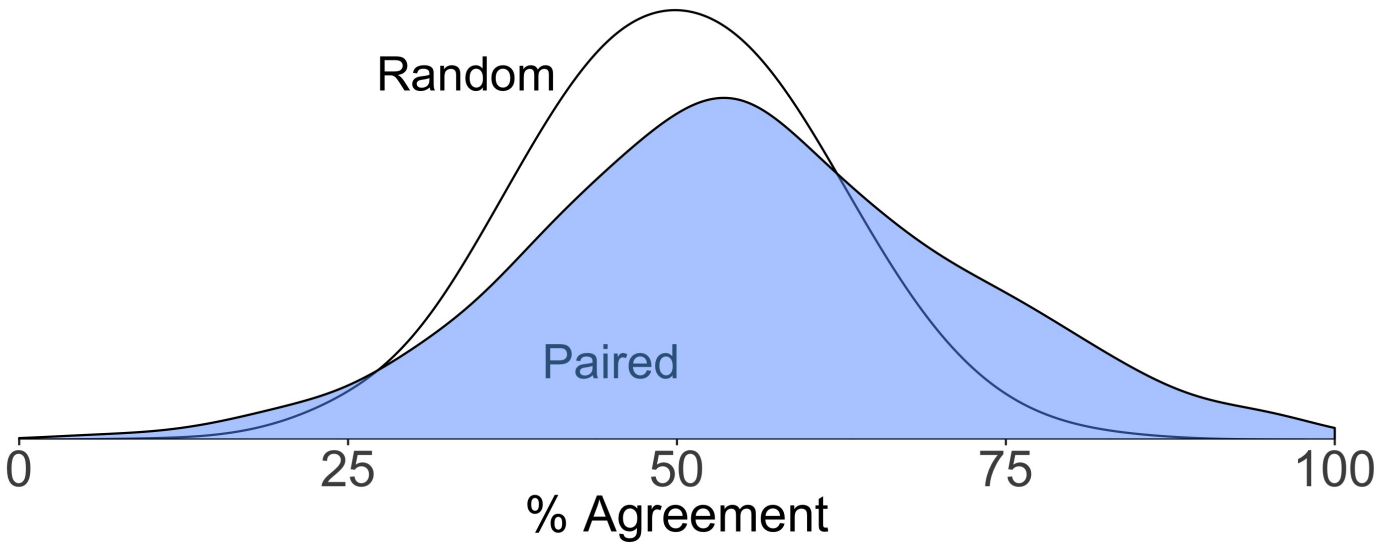

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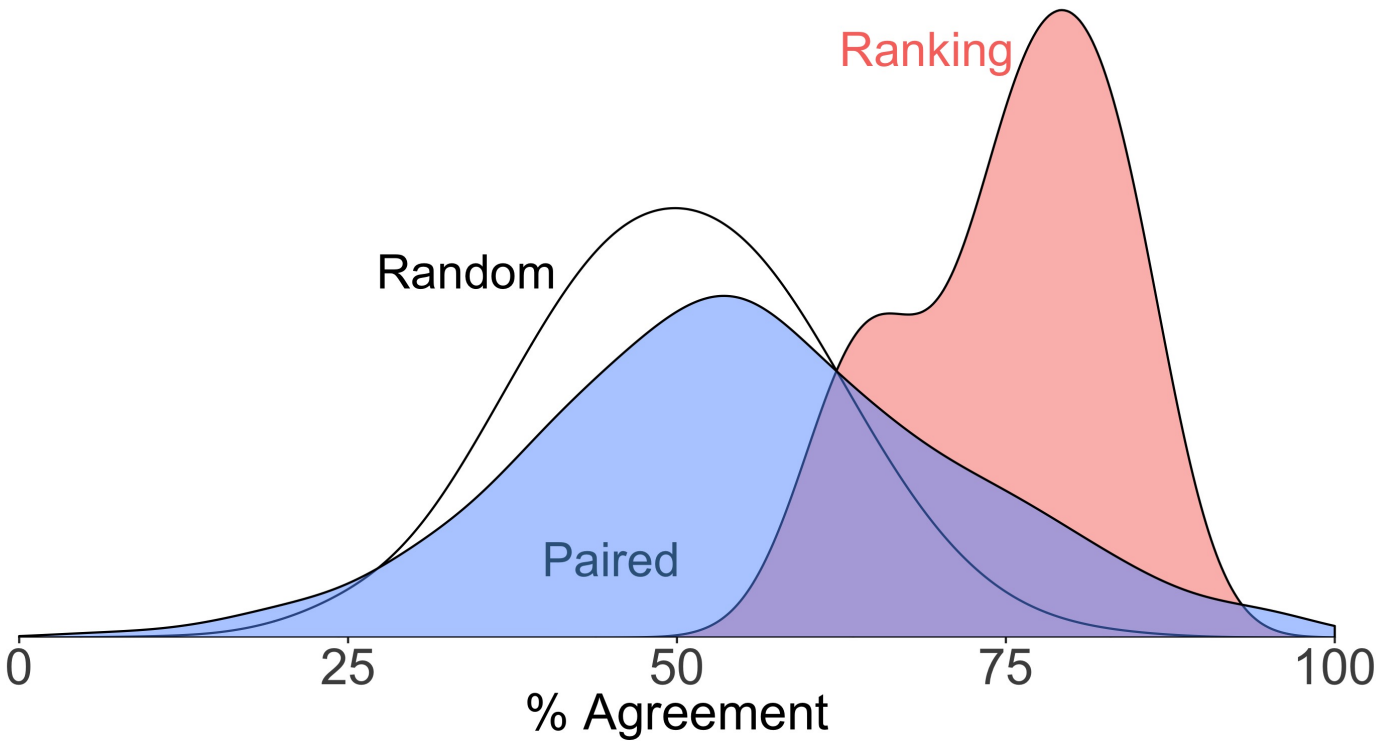

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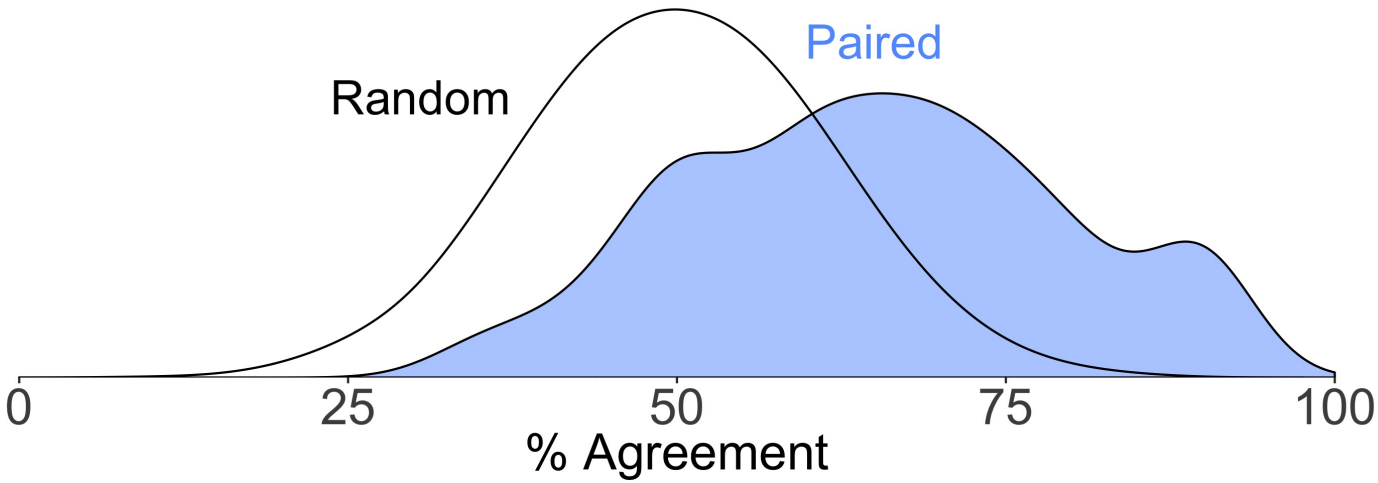

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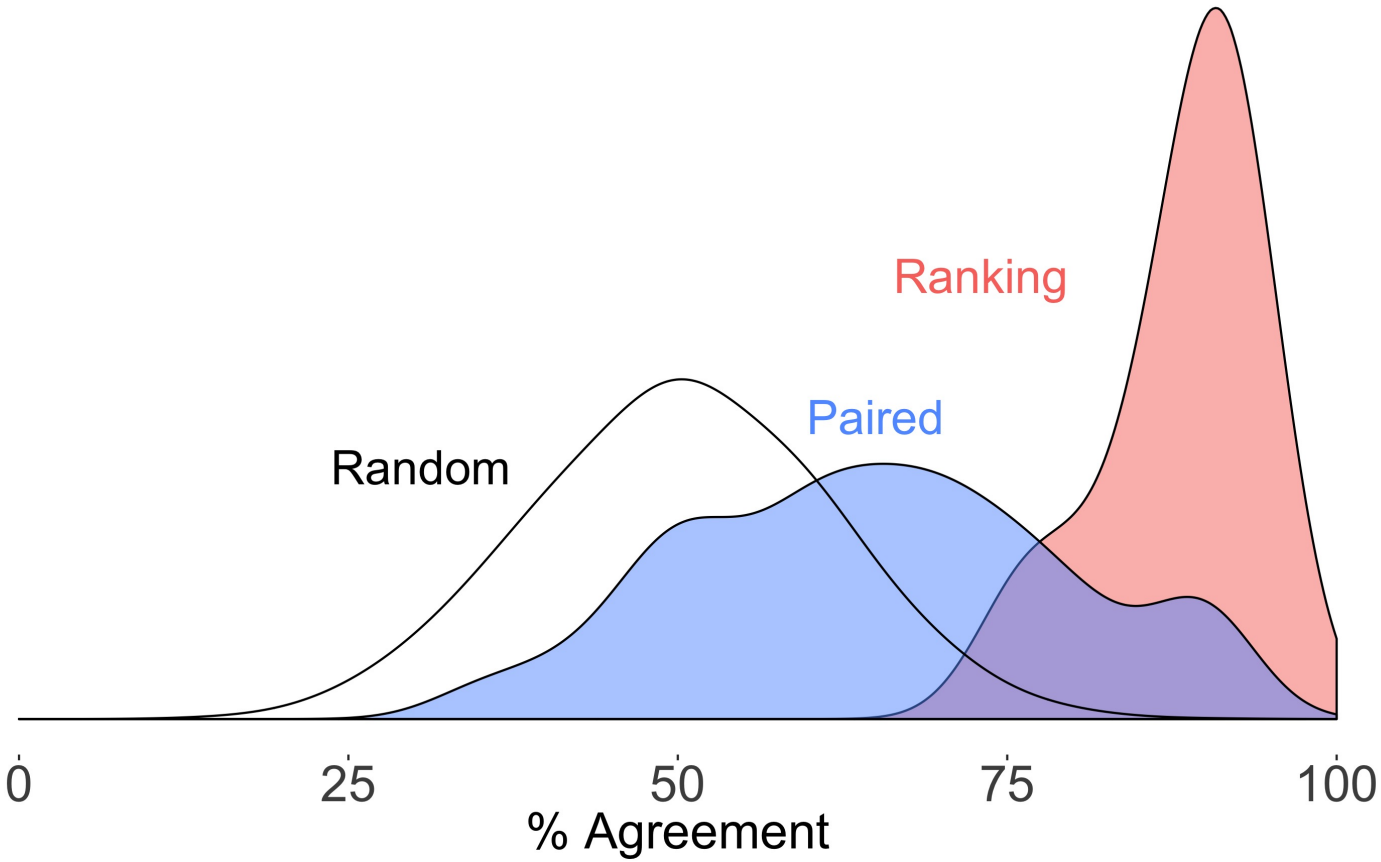

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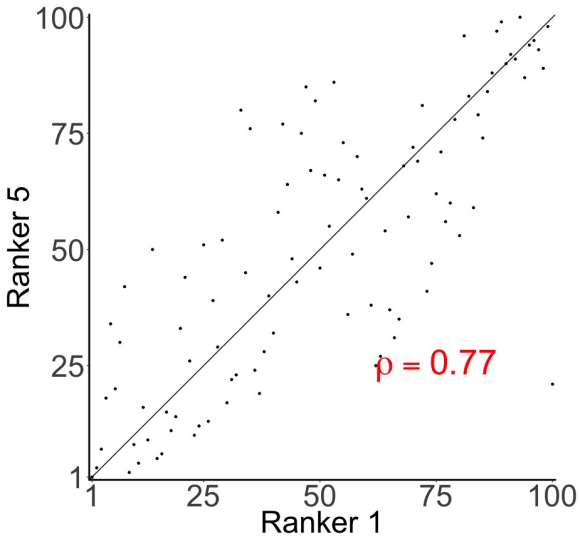

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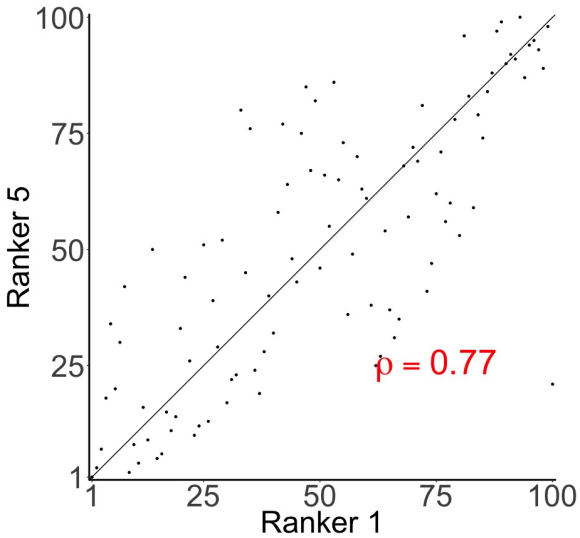

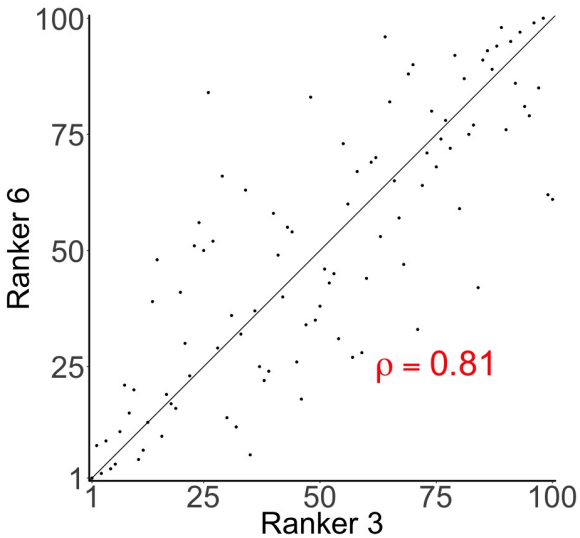

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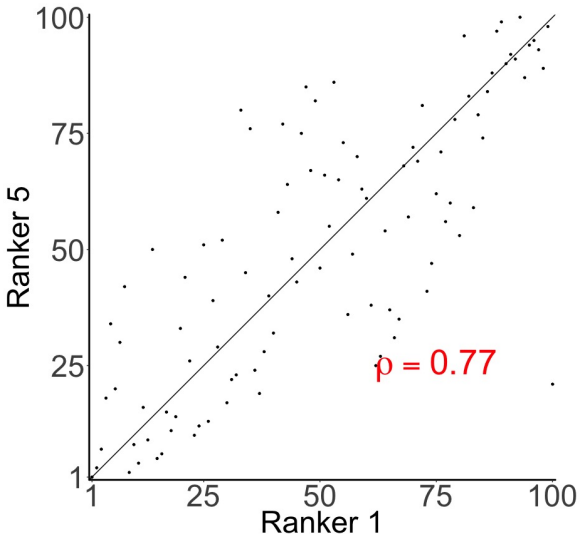

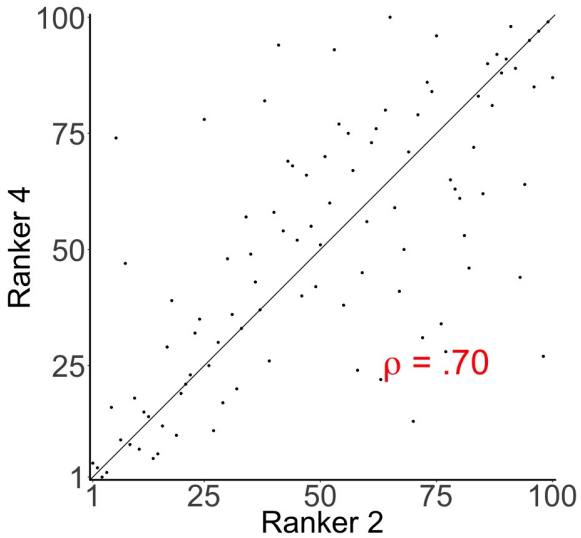

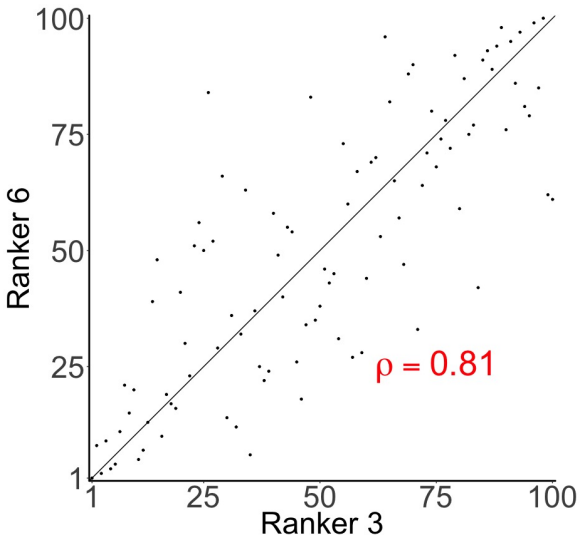

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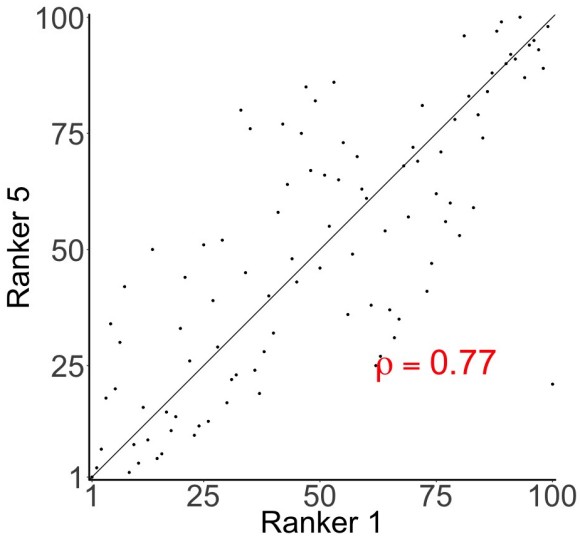

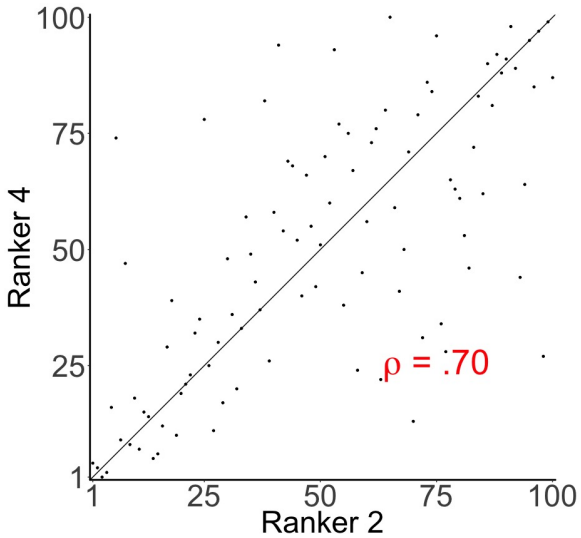

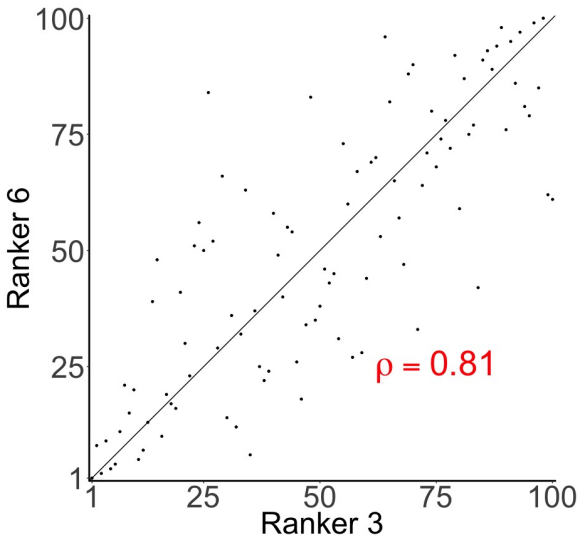

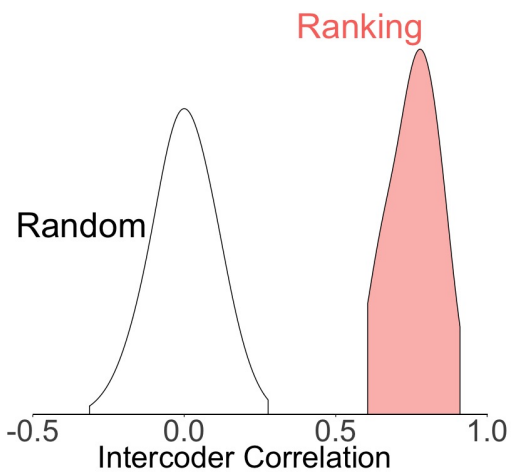

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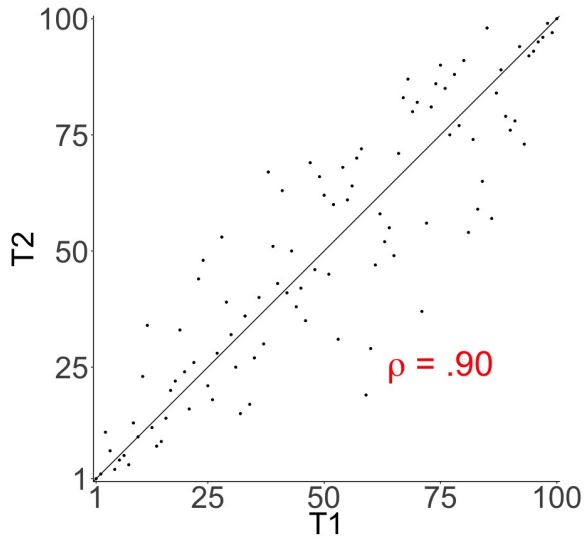

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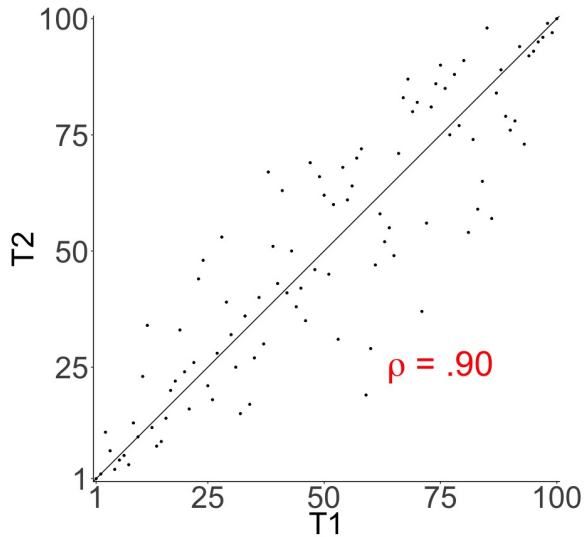

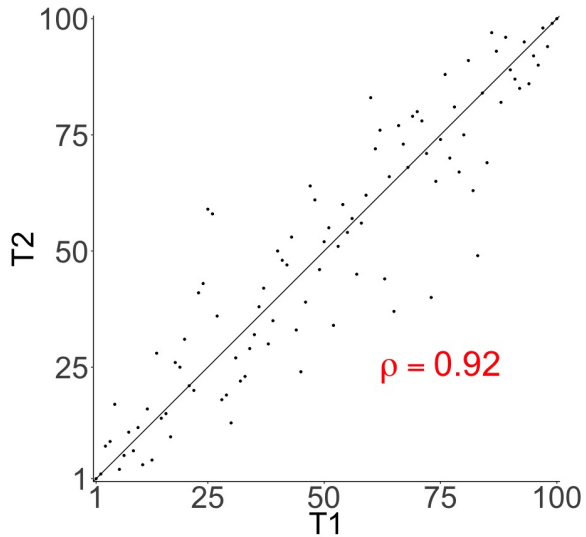

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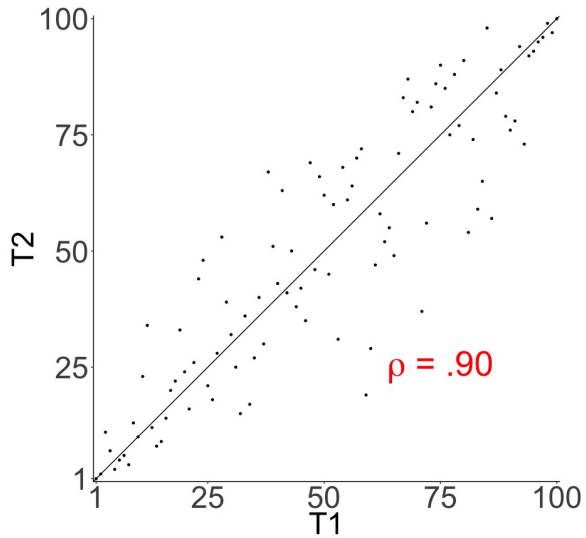

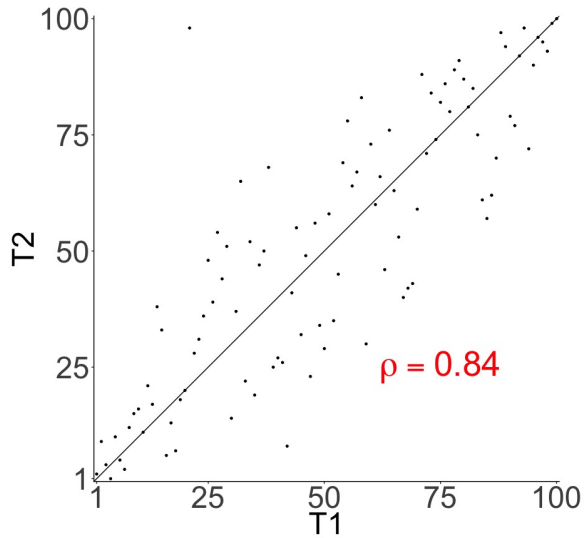

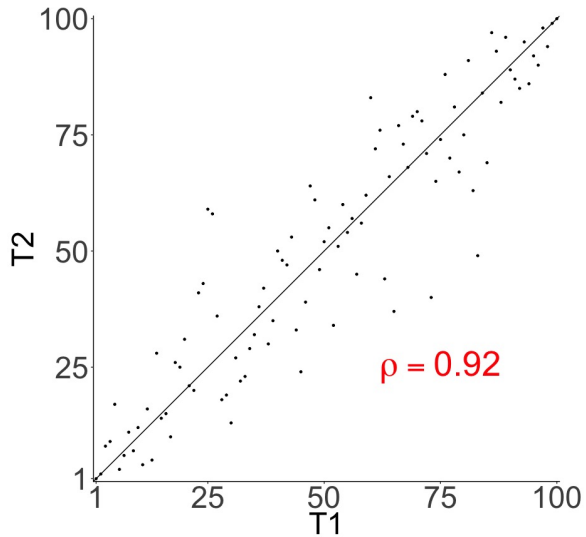

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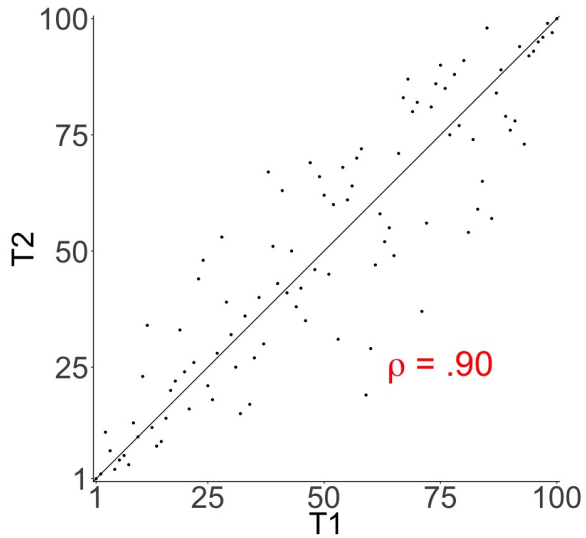

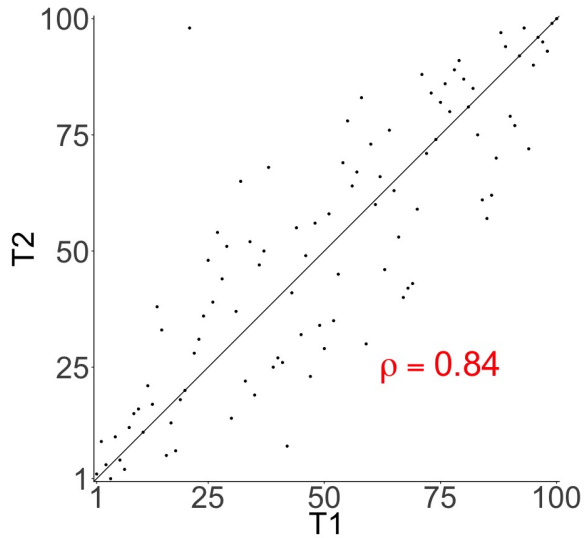

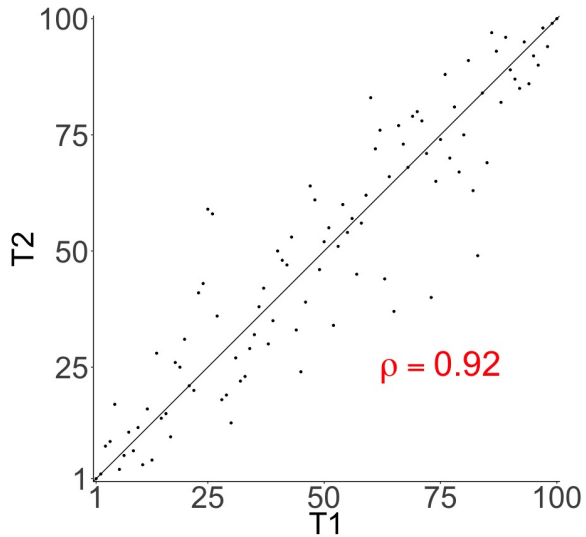

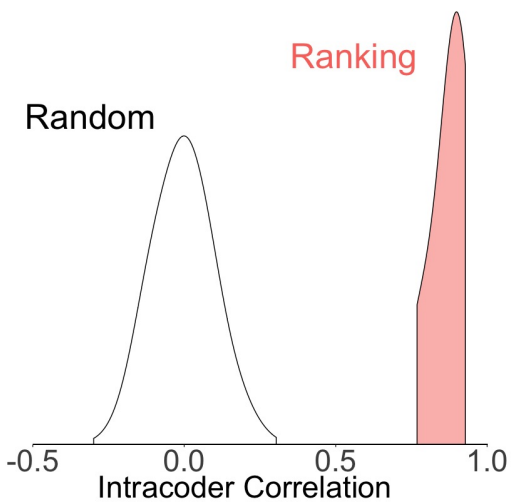

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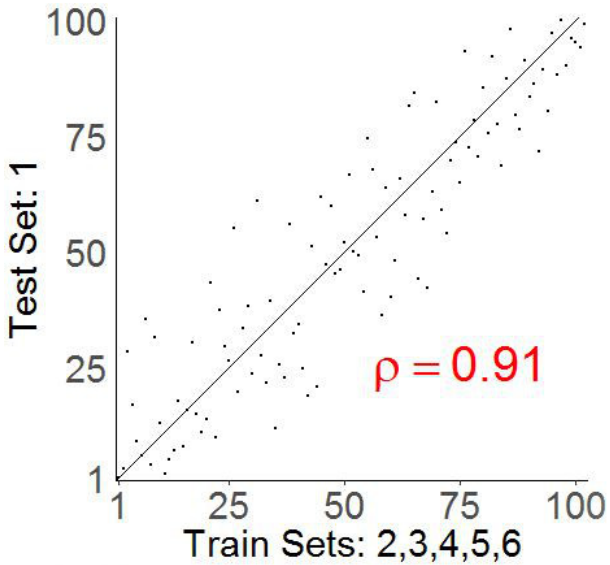

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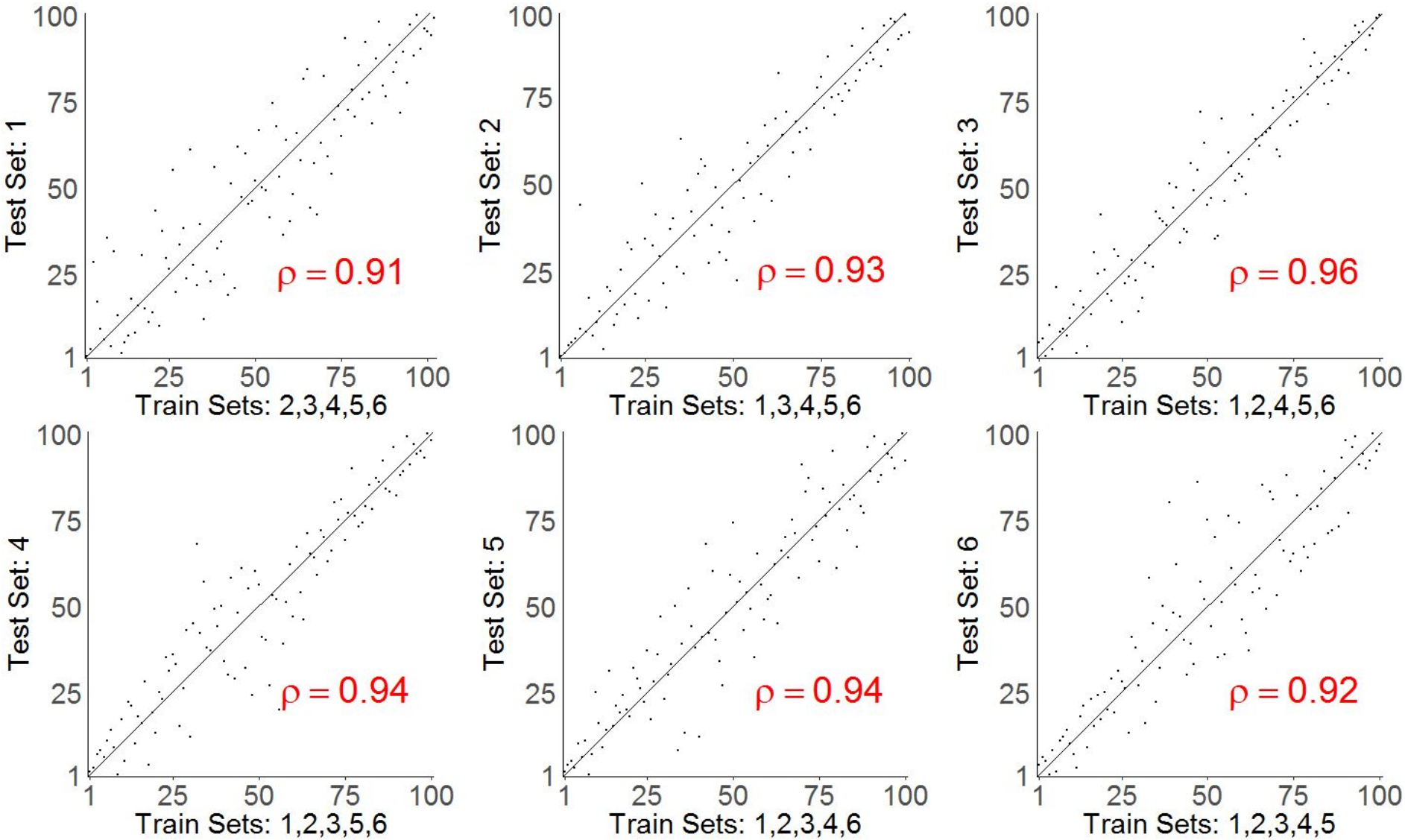

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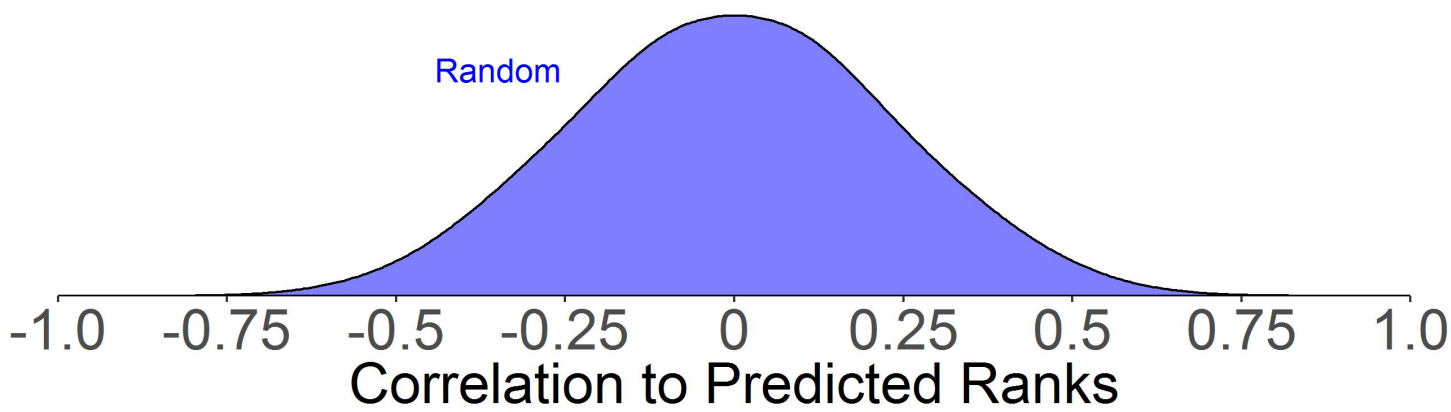

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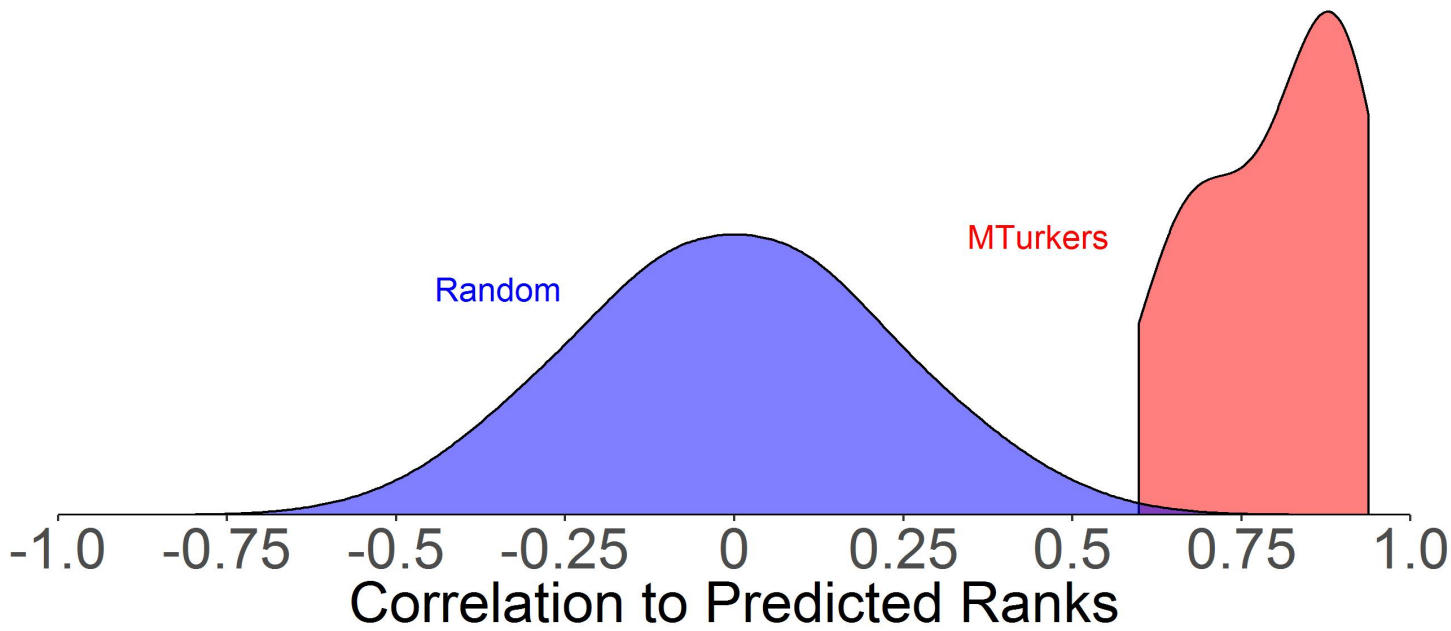

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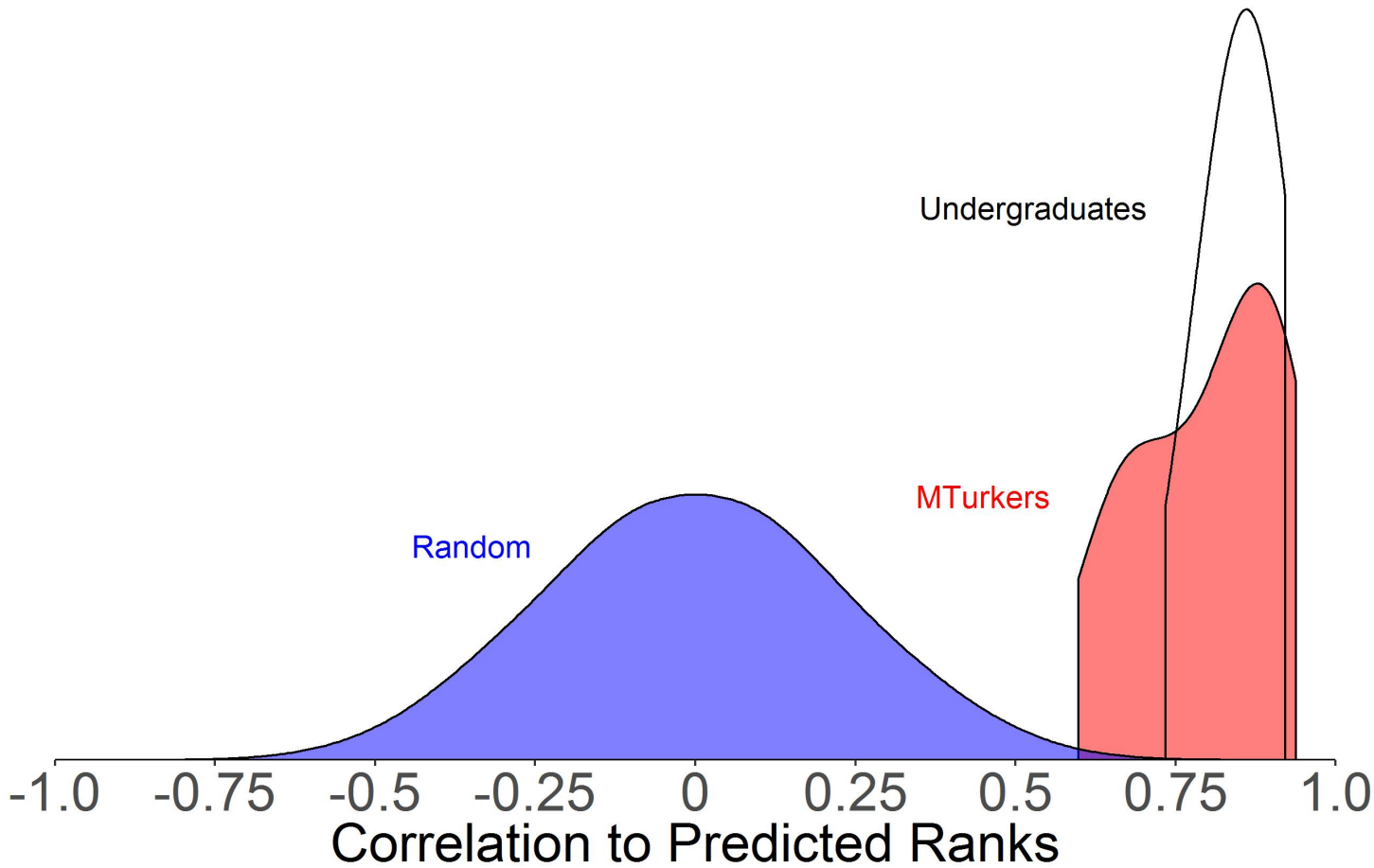

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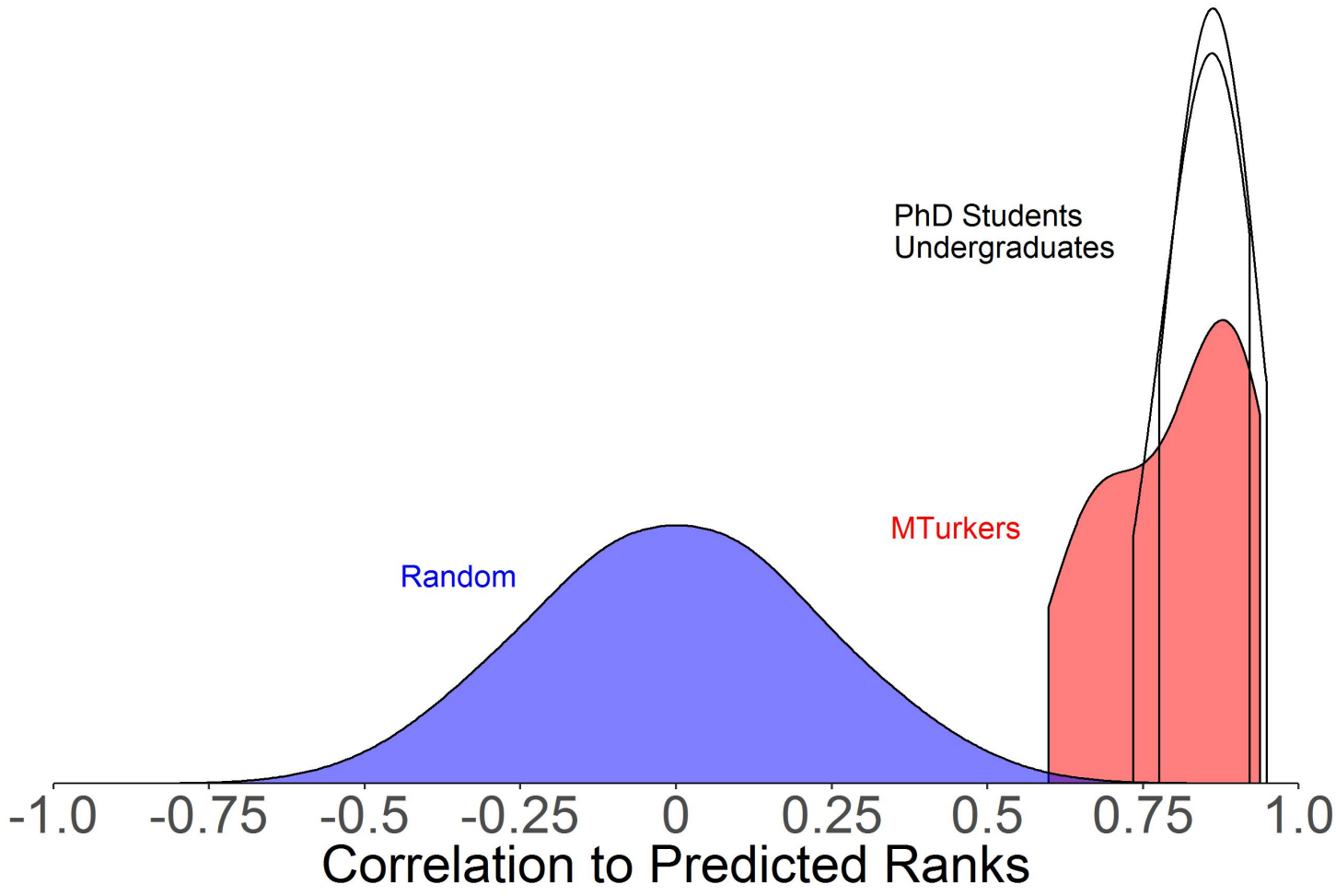

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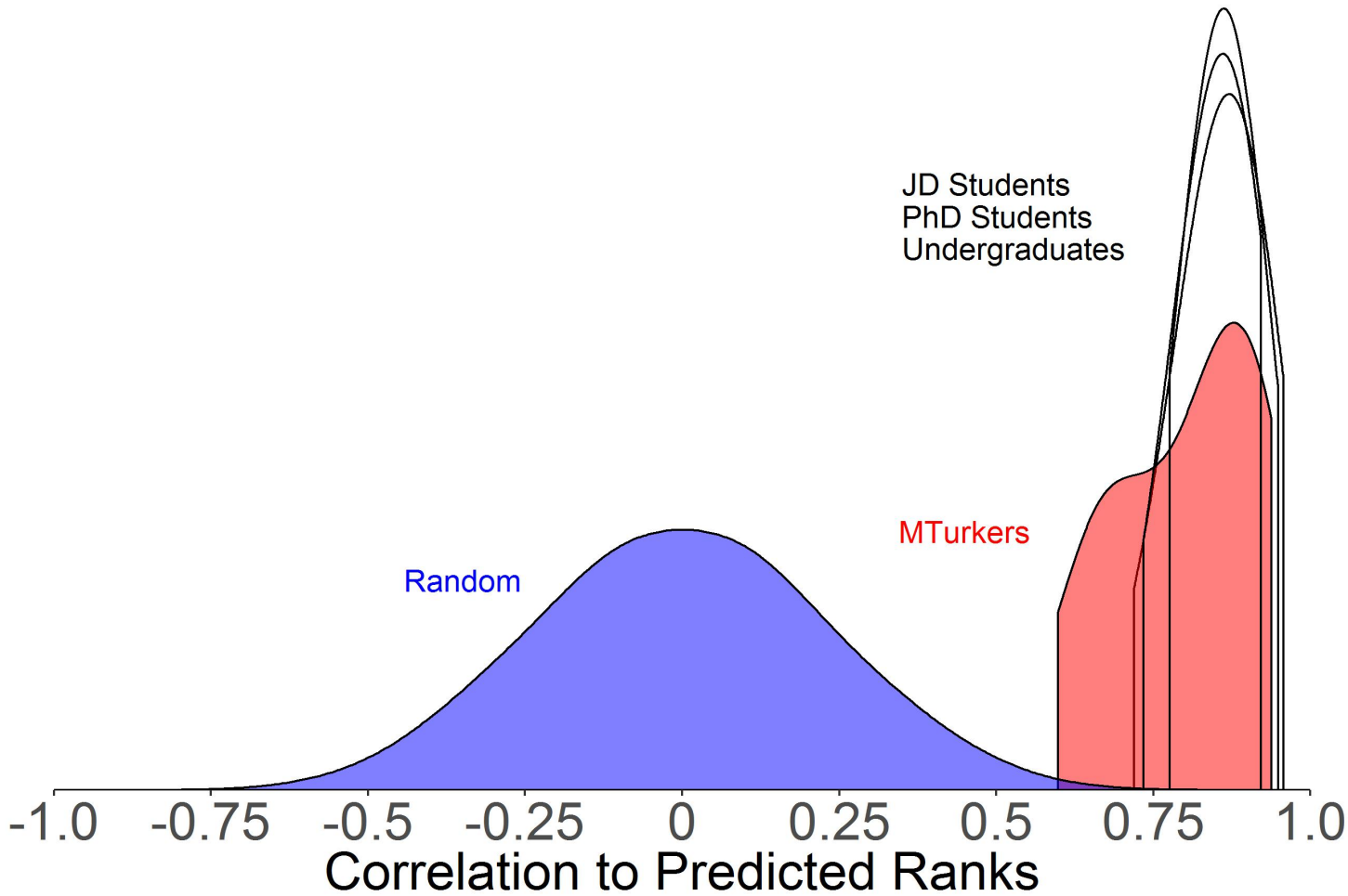

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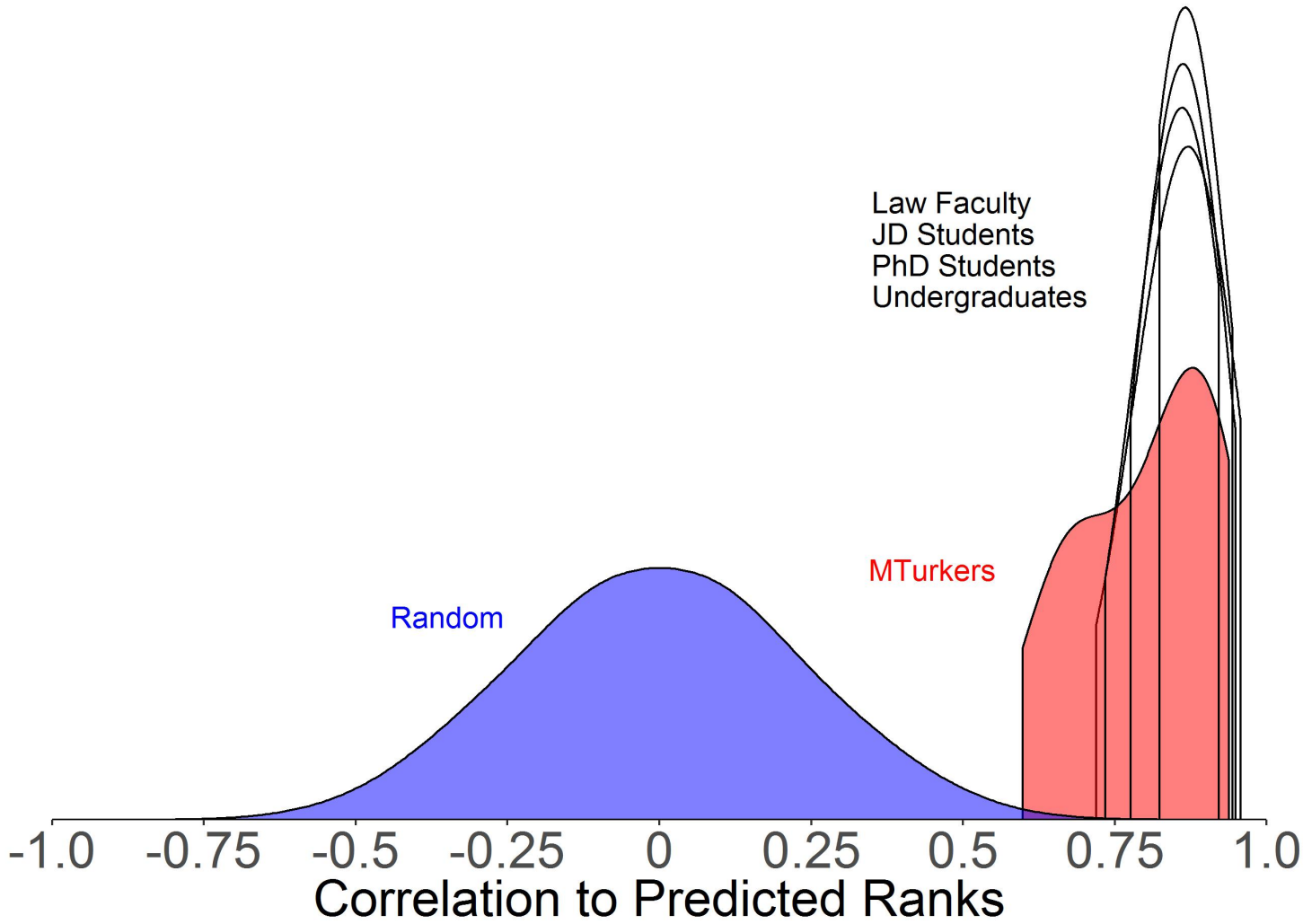

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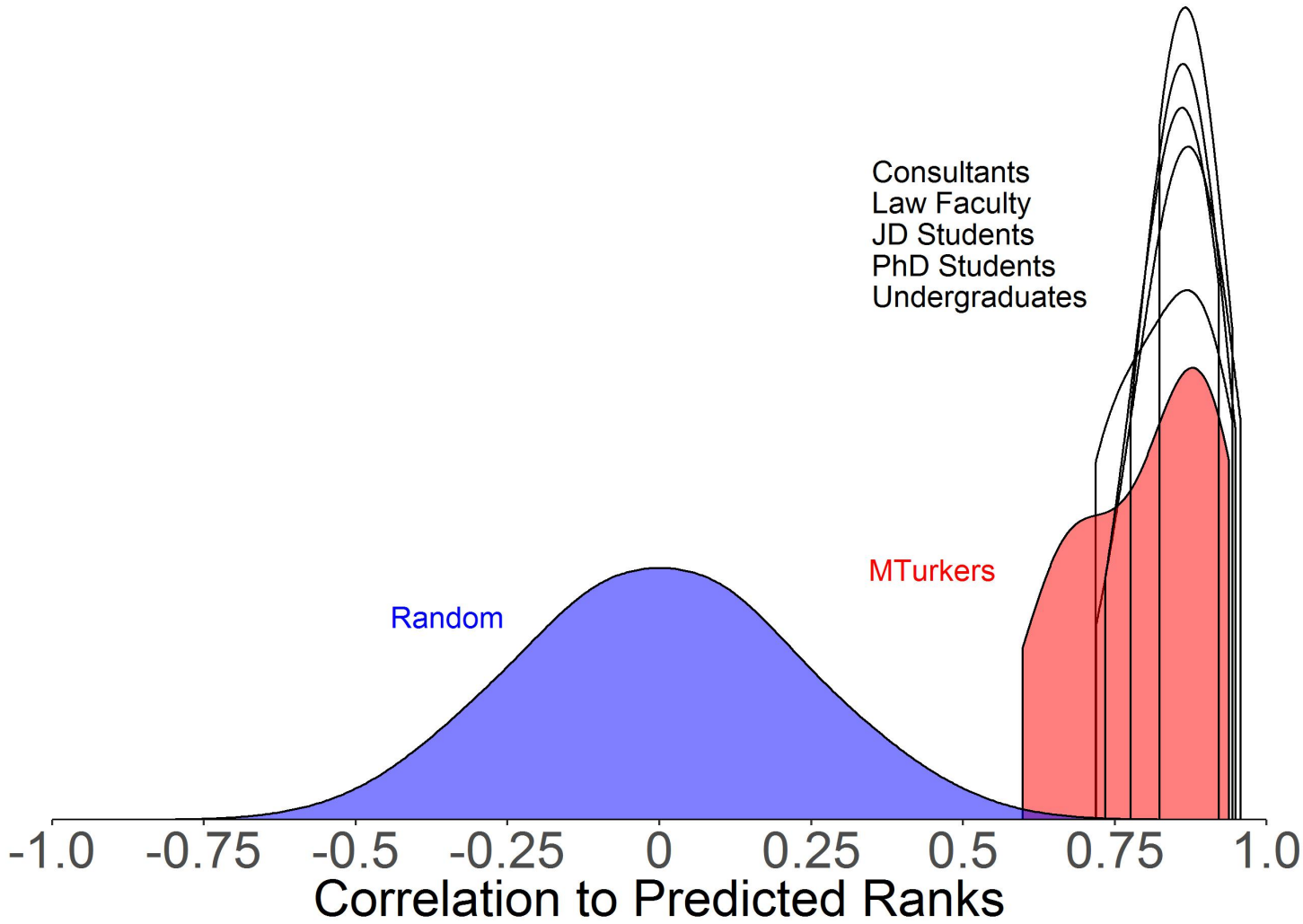

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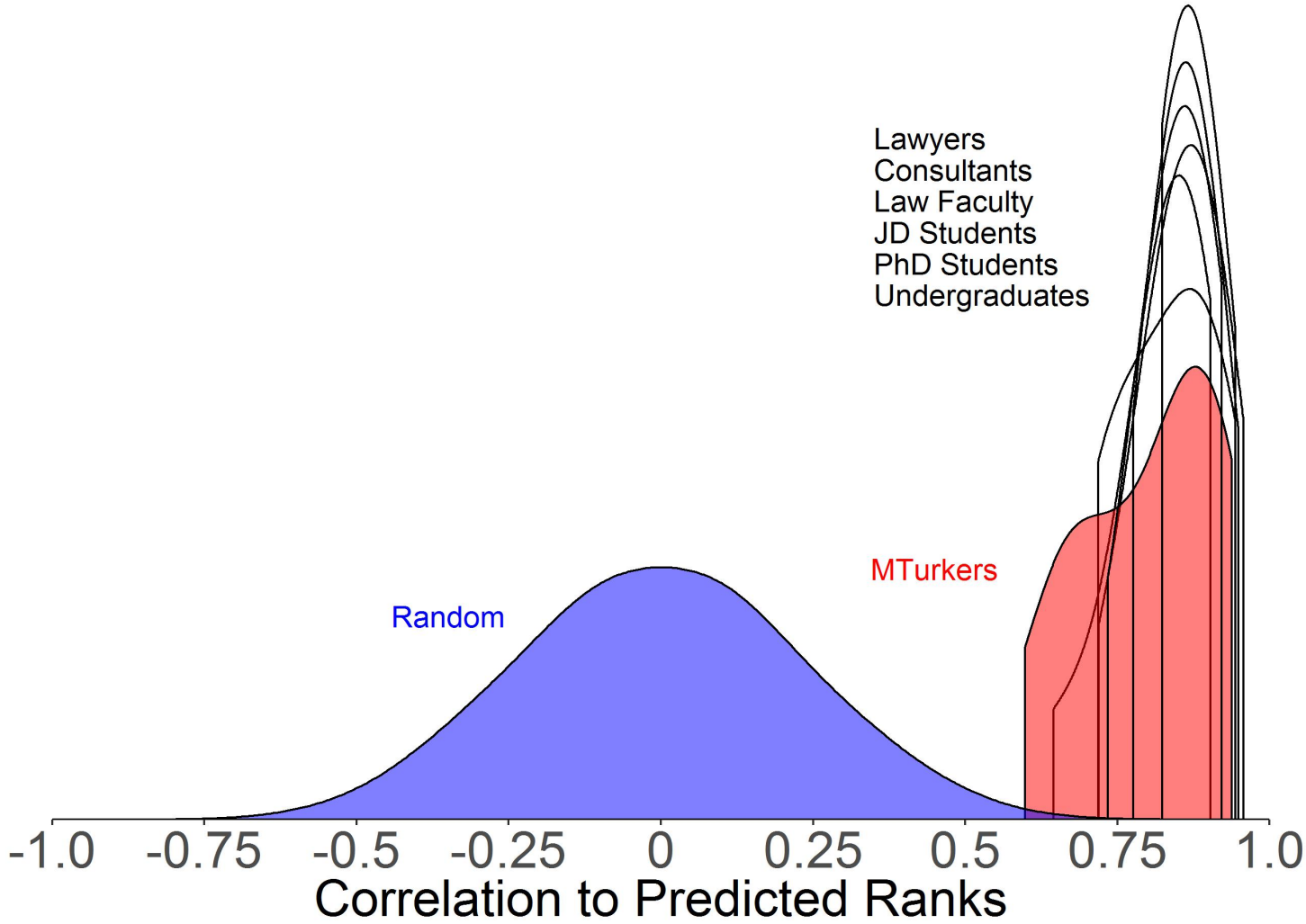

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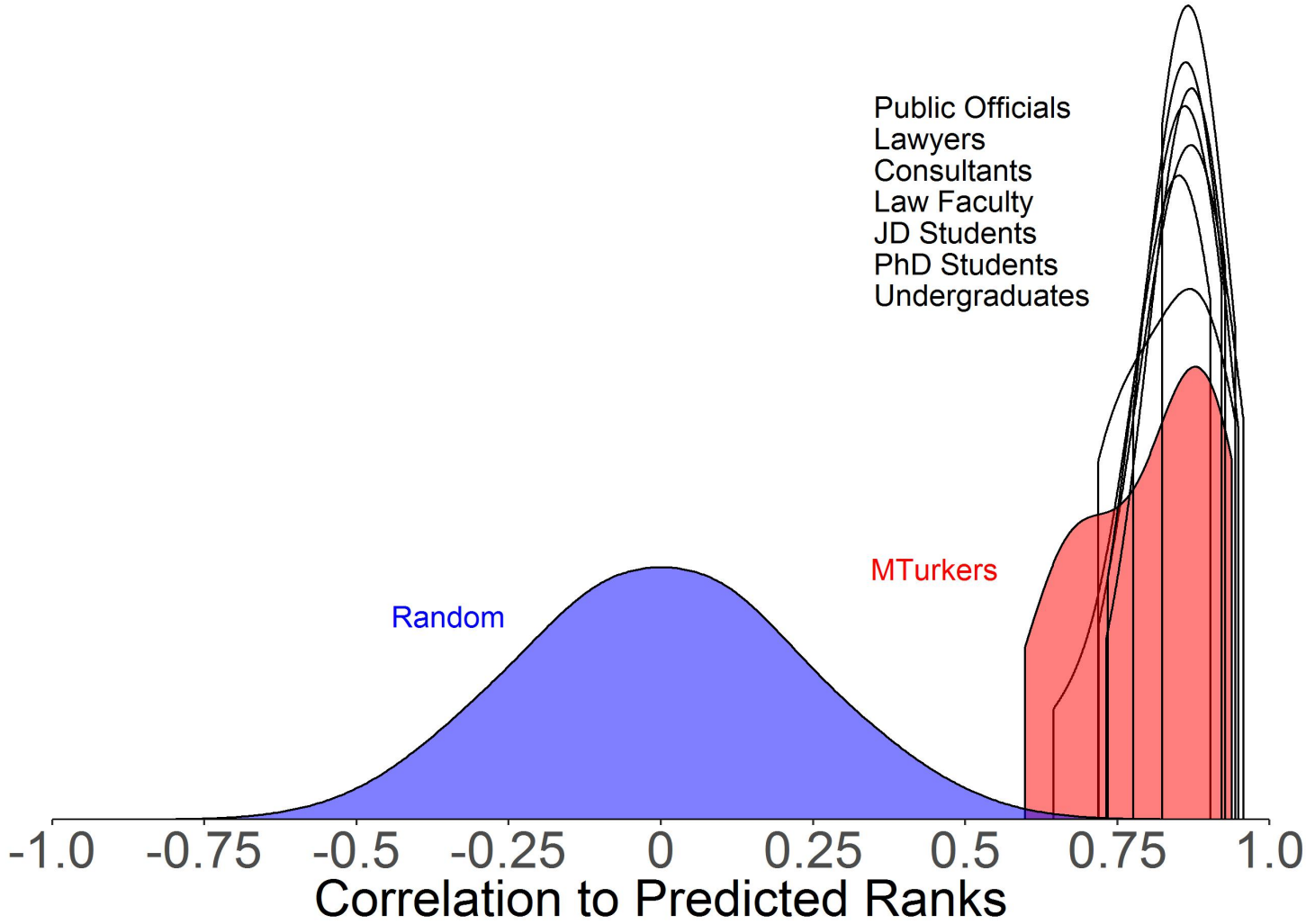

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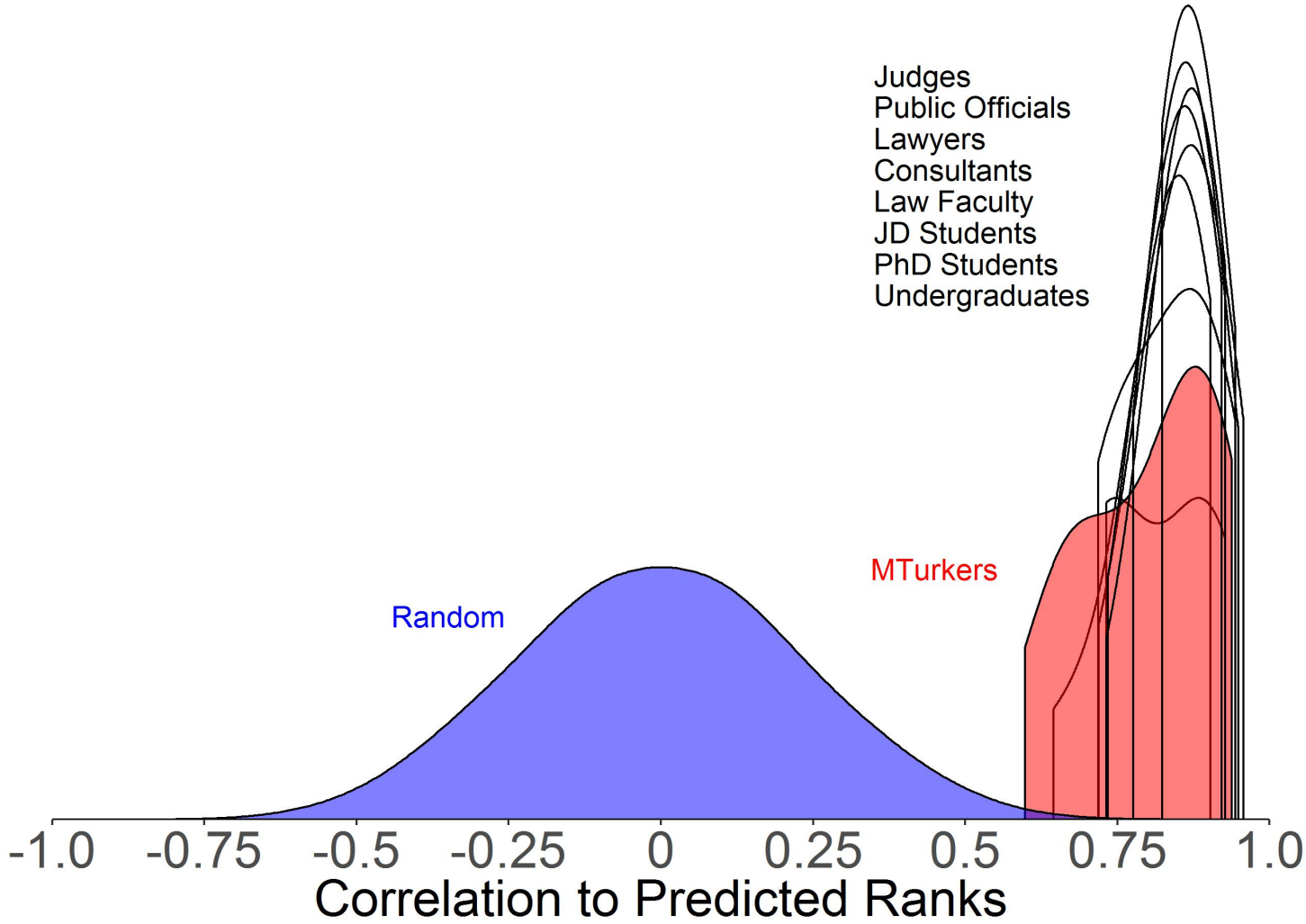

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  - Measures: for 18,215 Congressional & State Legislative districts
  - Software to calculate compactness from any district shape
- Along the way:
  - New perspective on $>$ 150 year consensus of ranking v paired comparisons

Concluding Remarks

- We address: Disconnect between political science & the real world
  - The Theoretical Concept: multidimensional and complex
  - The Legal Concept: one dimensional and simple
- A proposed resolution: measure the one dimension everyone sees
  - Calculated solely from district geometry
  - Very high intercoder & intracoder reliability
  - Very high predictive validity
  - Diverse people see it the same way
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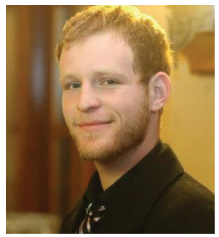
  - New directions for two venerable literatures

For more information



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Paper, data, software, slides: j.mp/Compactness