

# How to Measure Legislative District Compactness If You Only Know it When You See it<sup>1</sup>

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Harvard University

Society for Political Methodology, University of Wisconsin, 7/14/2017

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<sup>1</sup>Paper available at [j.mp/Compactness](http://j.mp/Compactness)

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  - E.g., Population equality, partisan fairness, racial fairness, respect for municipal boundaries . . . **compactness**

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    - Required in many other jurisdictions

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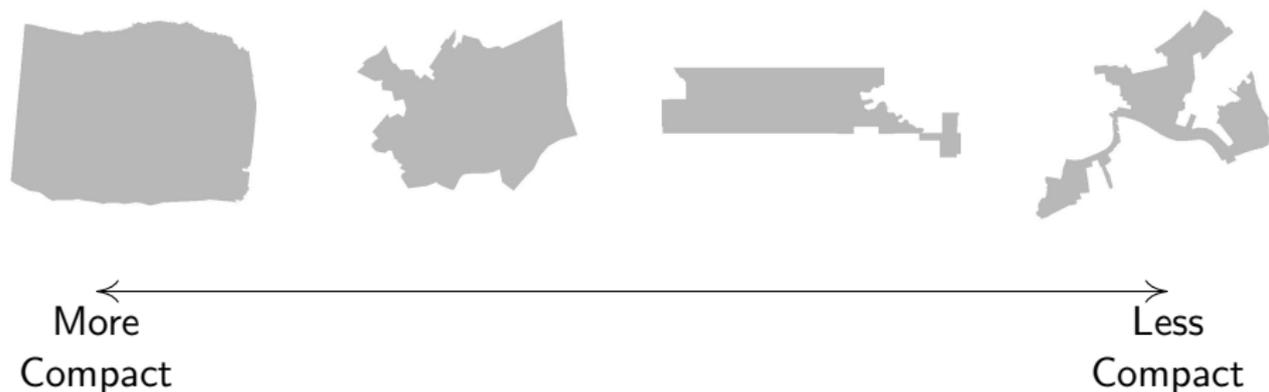


←  
More  
Compact

→  
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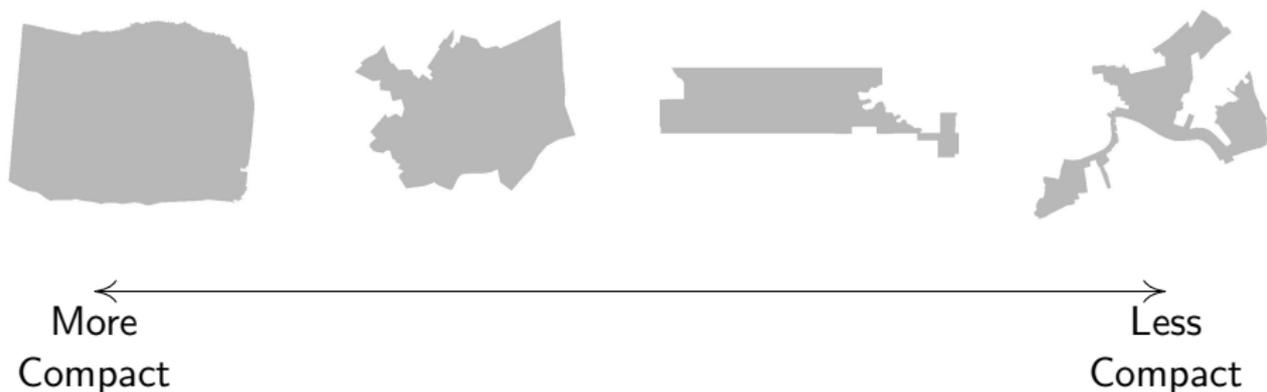
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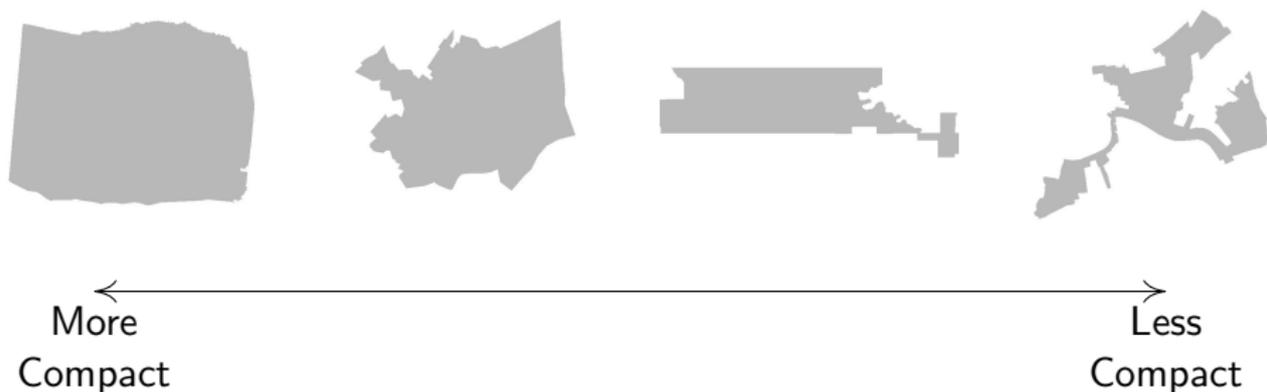
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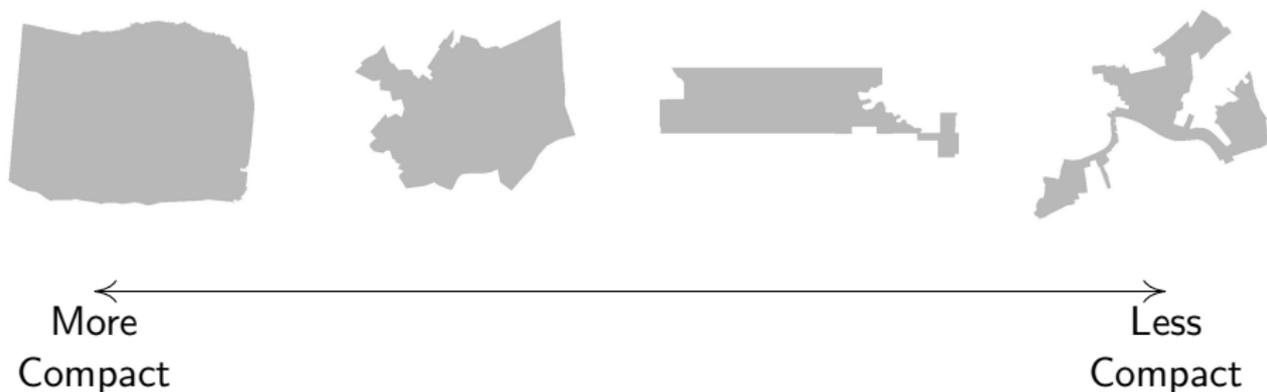
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- Only a **consensus measure** can constrain advocates
- **↪ Let's start with existing measures by social scientists**

## Measure 1: Length/Width Ratio of Min Bounding Box

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Squarish districts more compact than long thin ones

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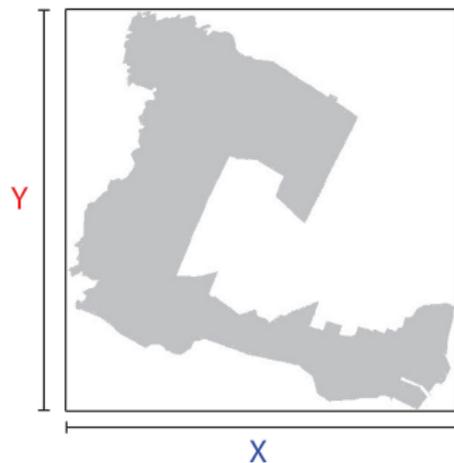
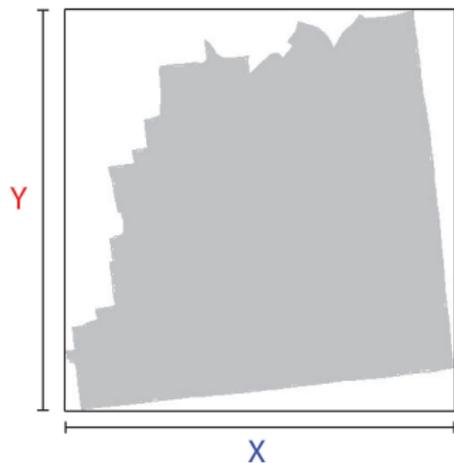
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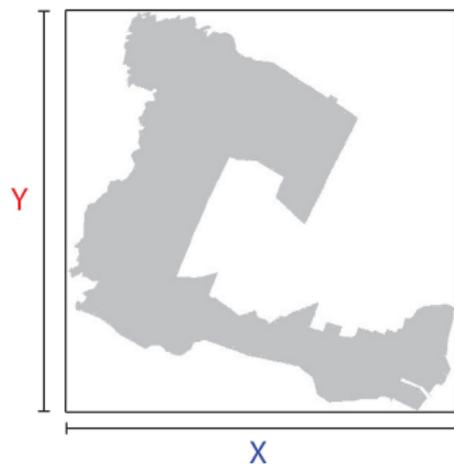
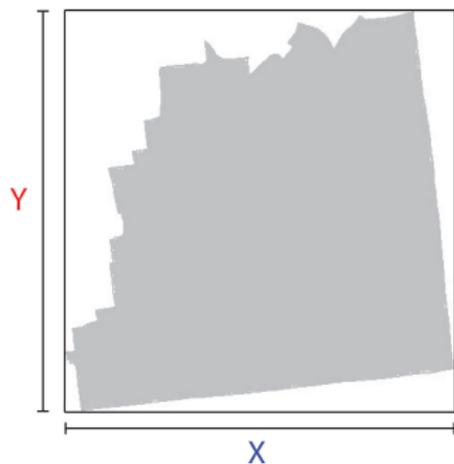
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In both districts:  $X/Y \approx 1.30$

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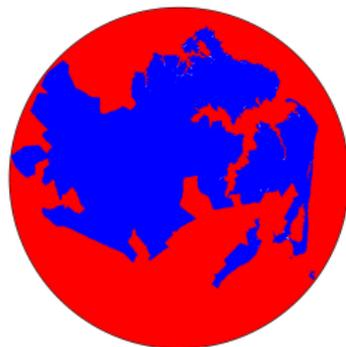
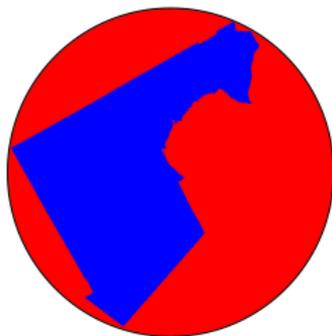
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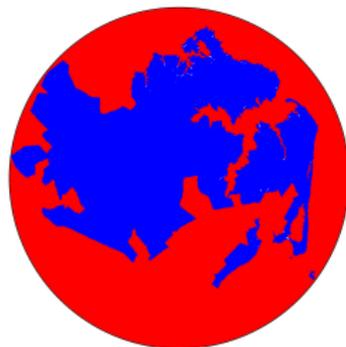
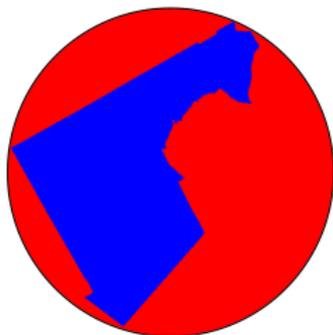
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In both cases,  $X/(Y + X) \approx 0.37$

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In both cases,  $MAD(r)/\bar{r} \approx 0.31$

## A Brief Interlude: Can you Name this Celebrity?

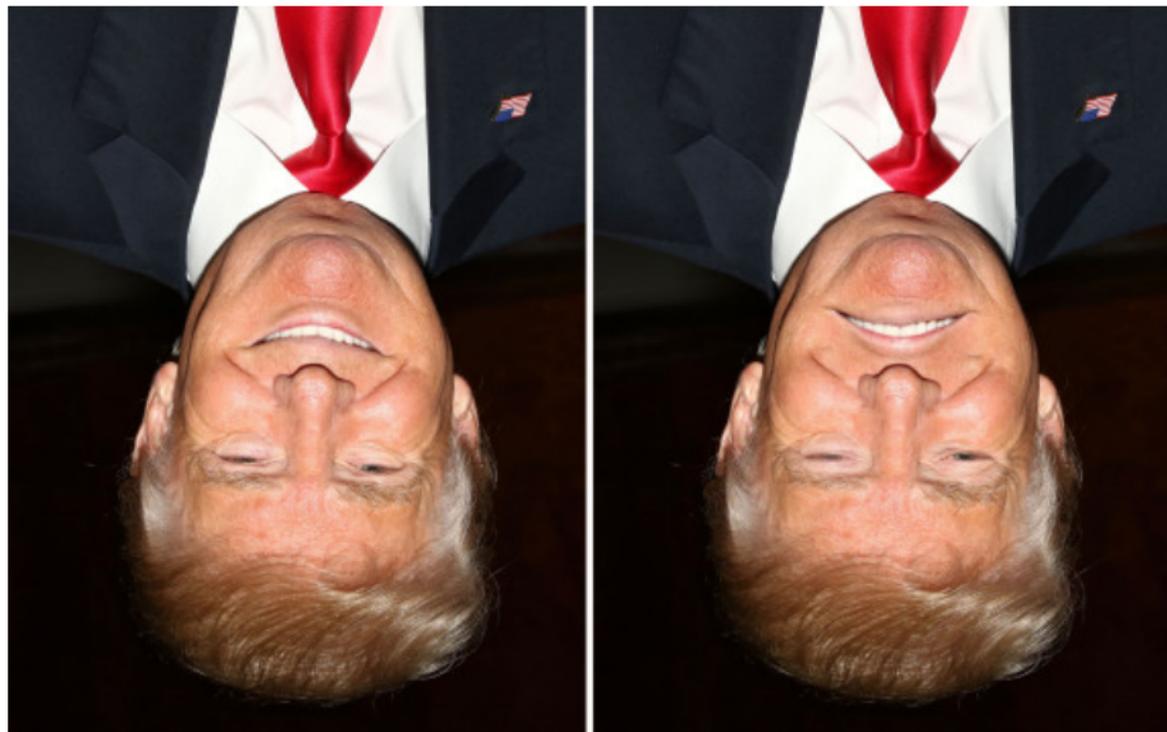
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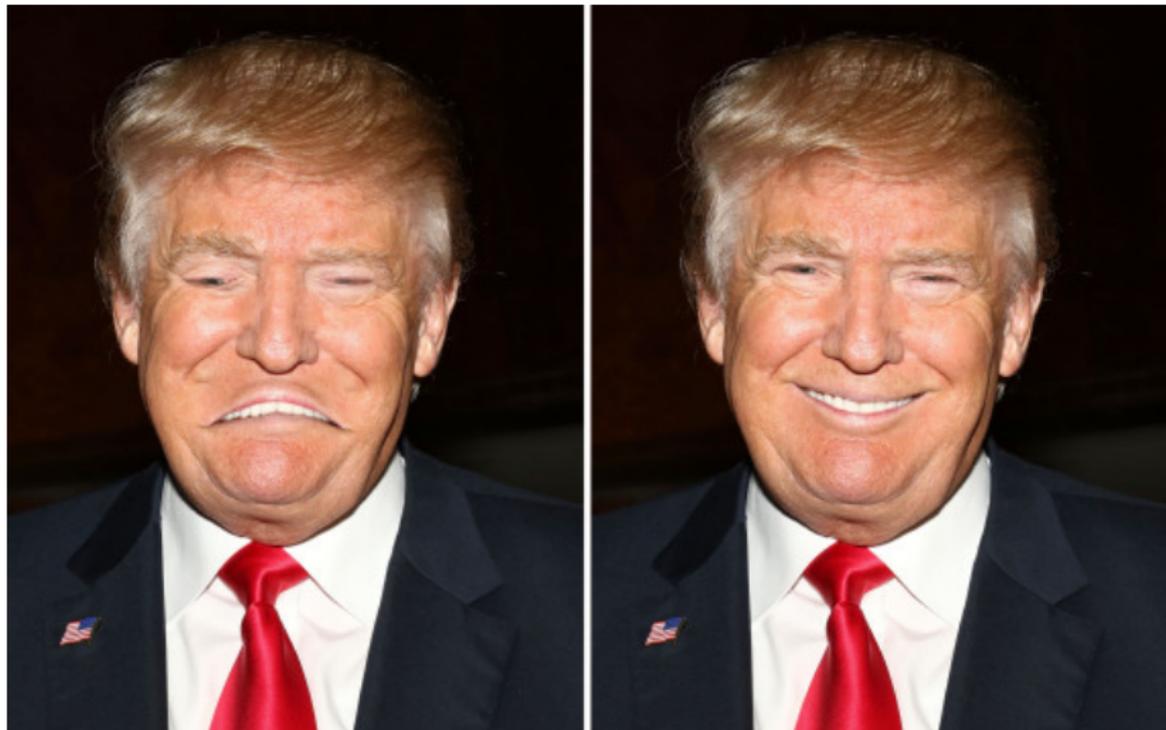
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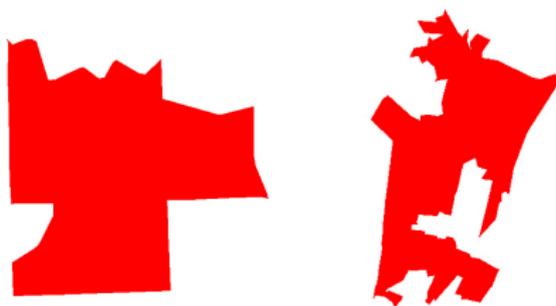
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Symmetric figures (circles, squares) are more compact

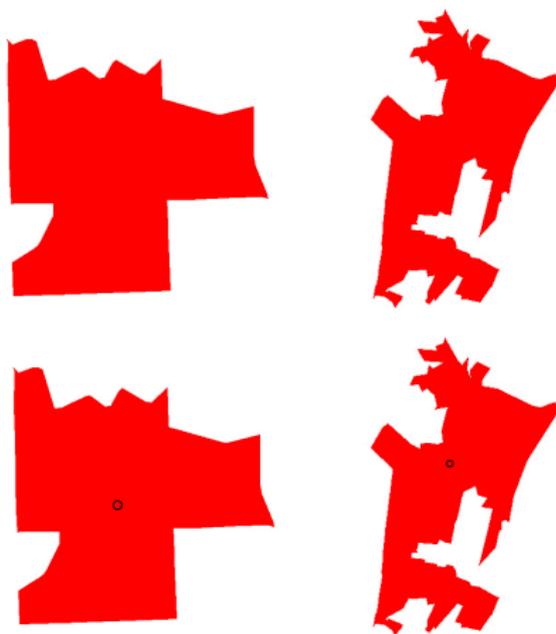
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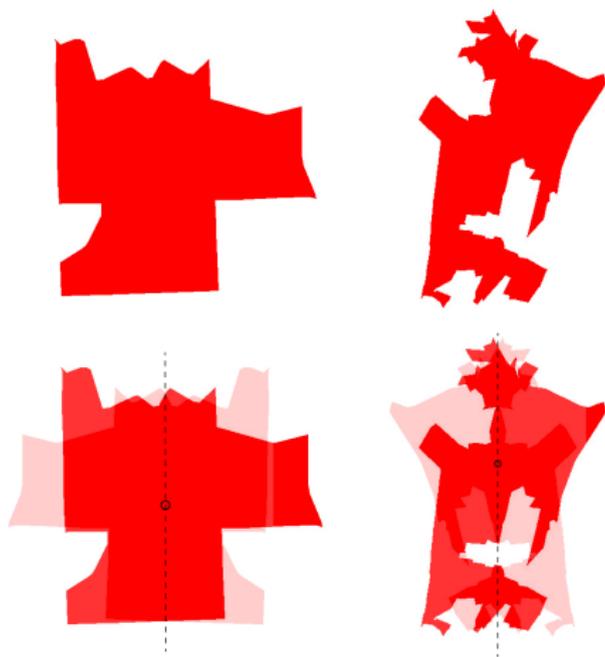
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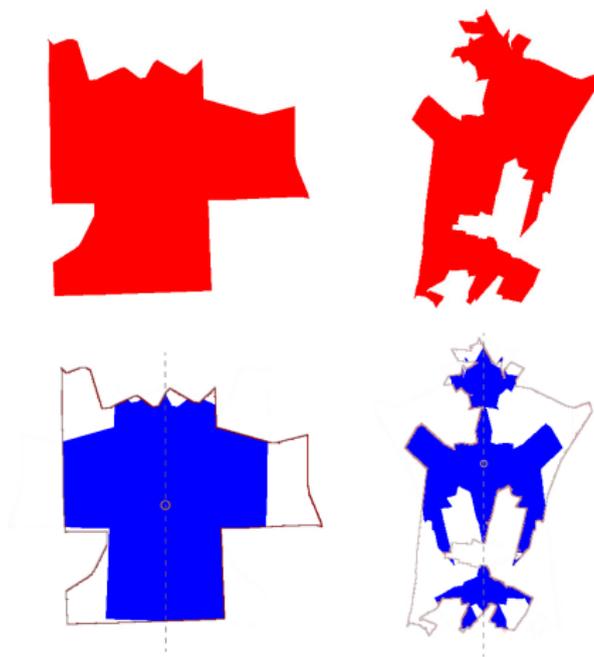
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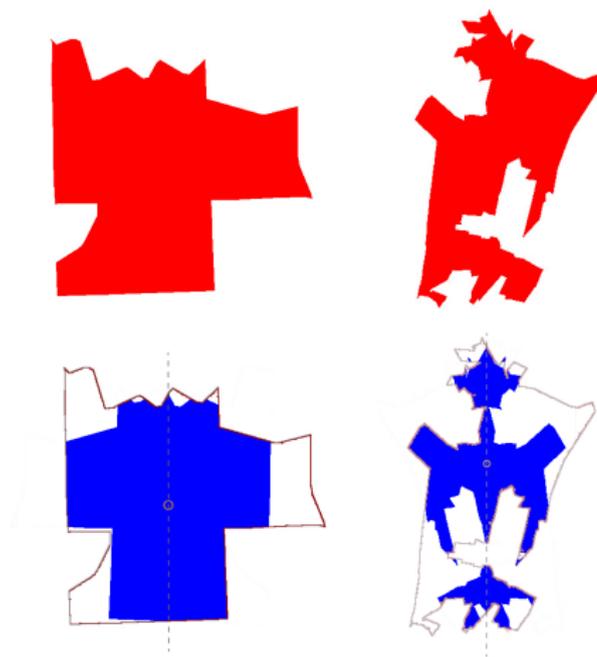
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In both cases,  $\text{Overlap} / \text{Original Area} \approx 0.34$

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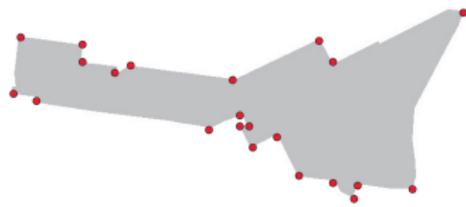
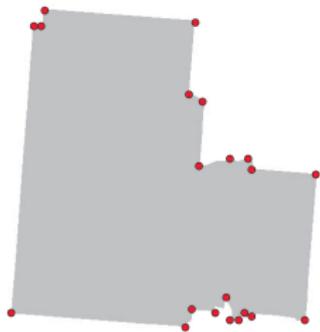
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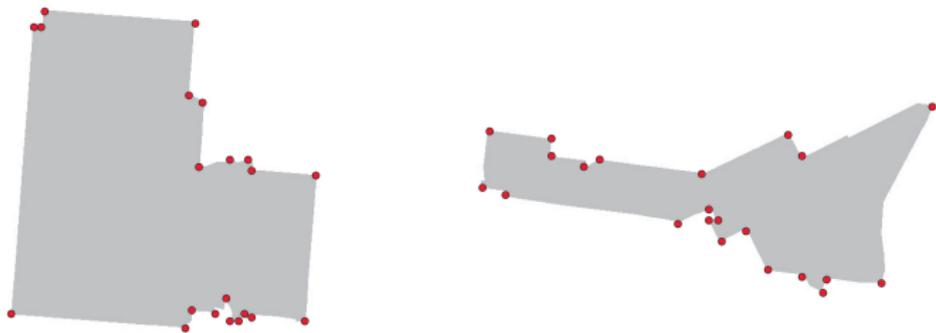
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Both districts have 21 significant corners

Which is more compact?



Which is more compact? Depends on the standard!



Which is more compact? Depends on the standard!



Convex Hull

4

3

2

1

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3

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Convex Hull	4	3	2	1
Reock	1	2	3	4
Polsby-Popper	4	1	2	3

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Reock	1	2	3	4
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Boyce-Clark	2	3	1	4
Length/Width	3	2	1	4

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X-Axis Symmetry	1	4	3	2

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Significant Corners	4	1	3	2

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Polsby-Popper	4	1	2	3
Boyce-Clark	2	3	1	4
Length/Width	3	2	1	4
X-Axis Symmetry	1	4	3	2
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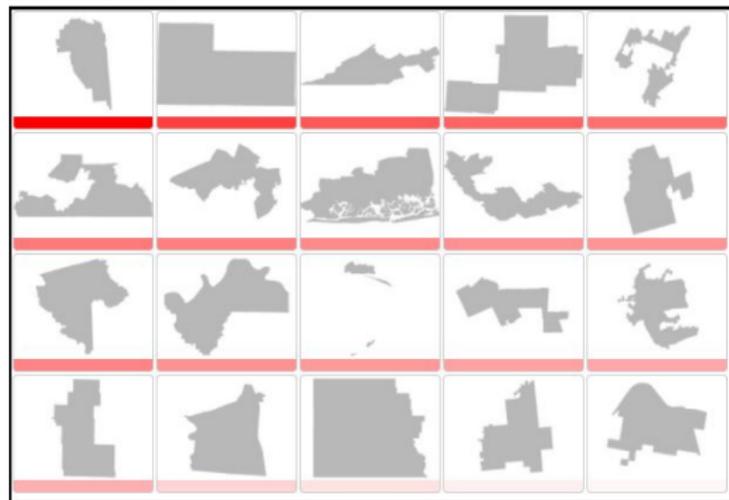


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MOST Compact Here



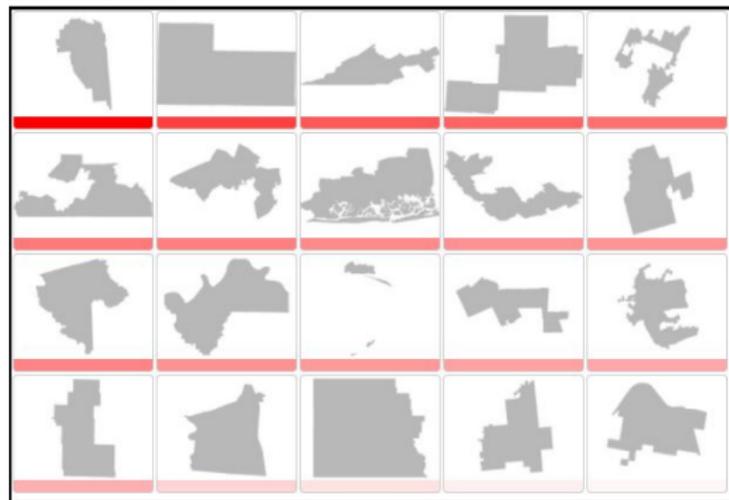
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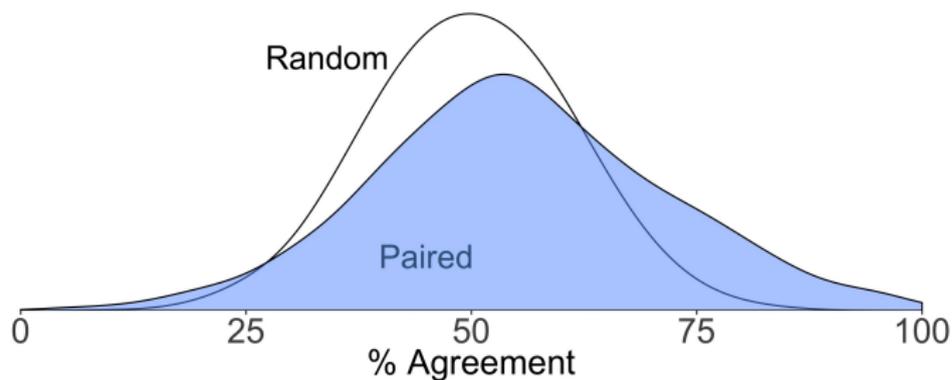
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Ranking: all evaluations on **one dimension** of user's choice

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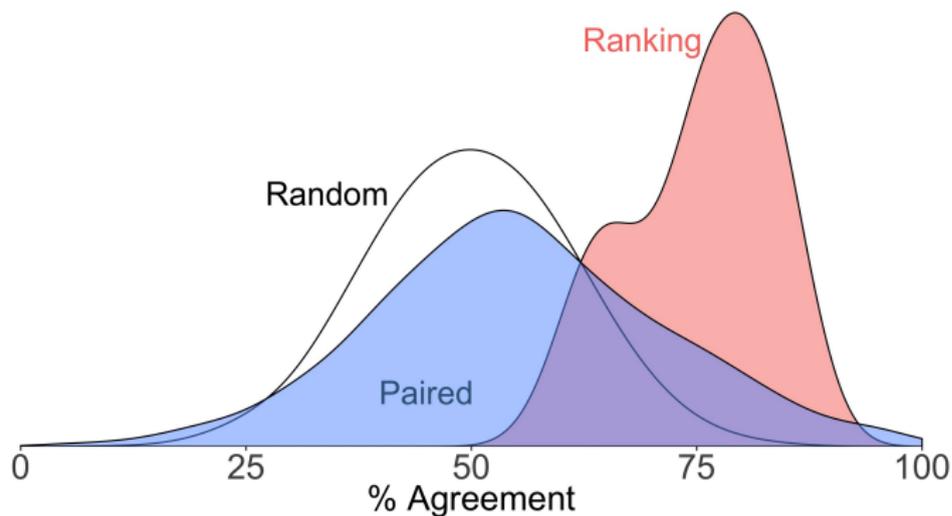
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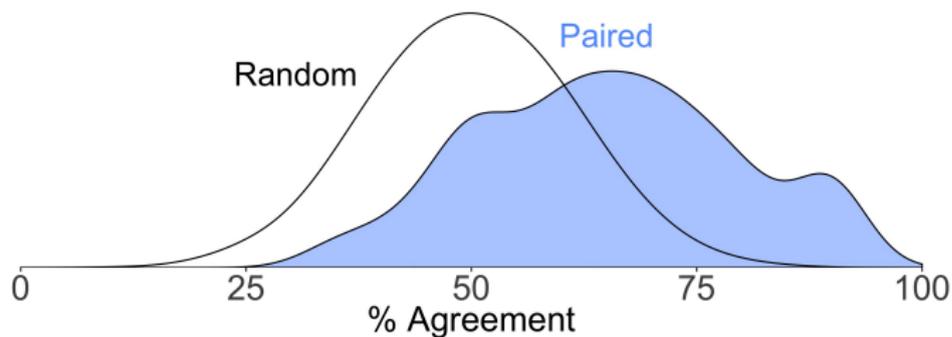
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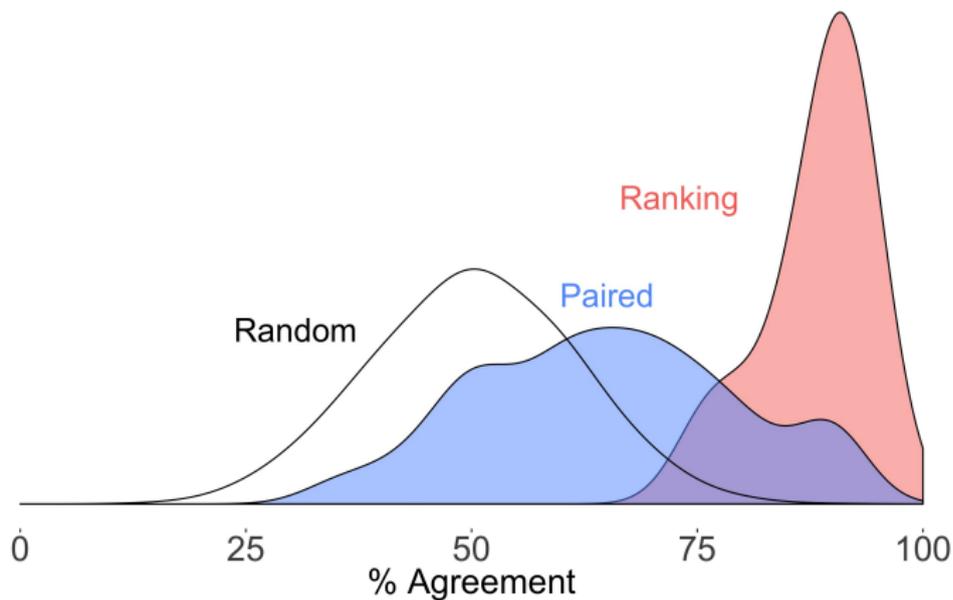
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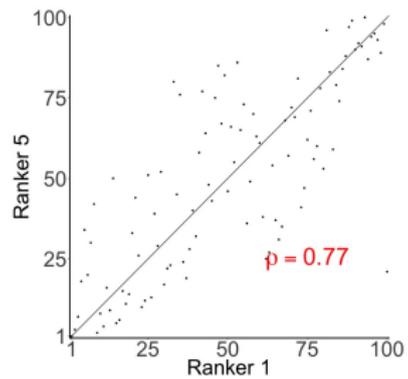


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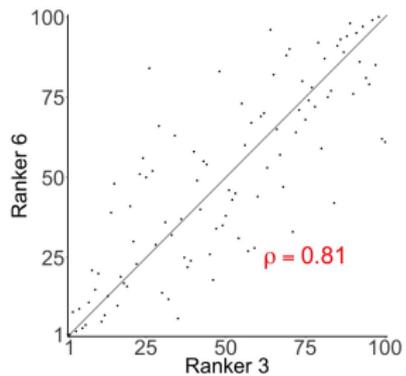
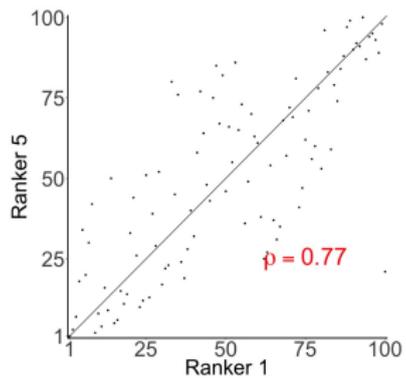
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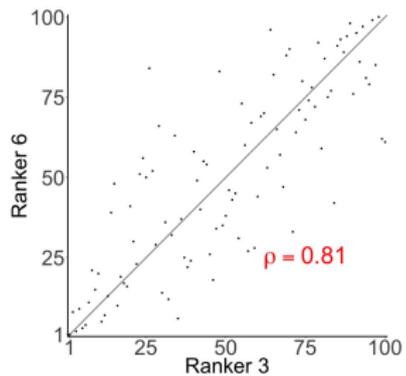
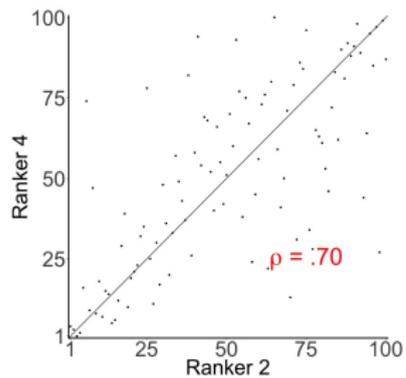
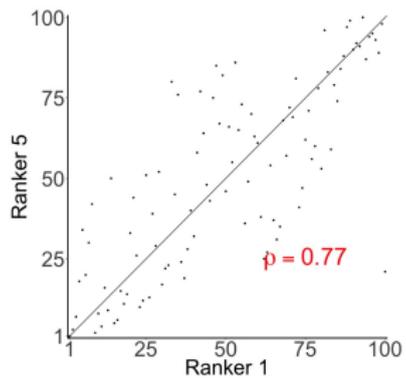
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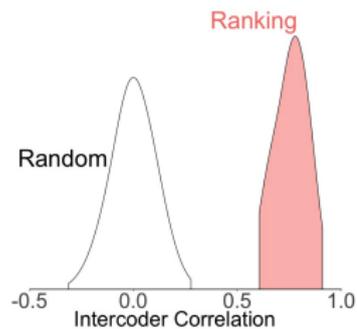
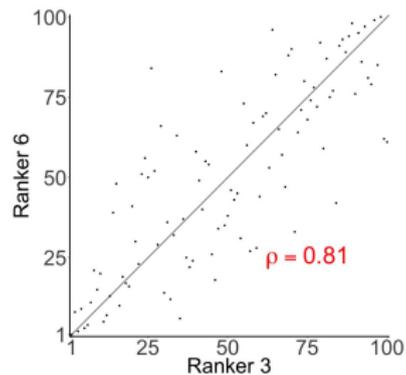
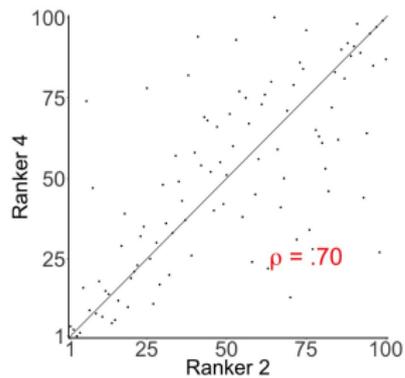
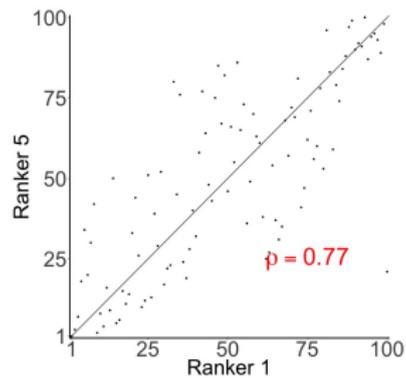
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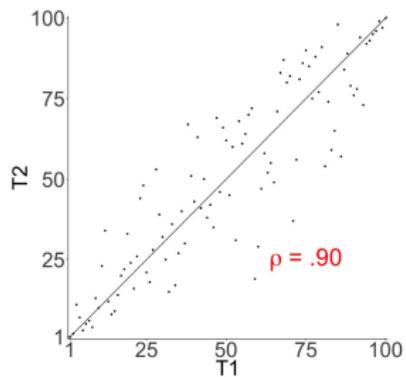
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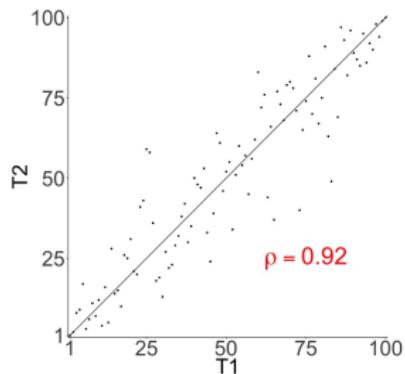
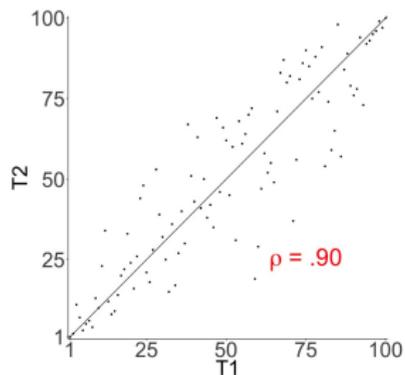
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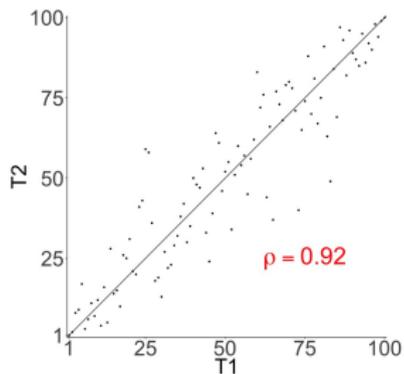
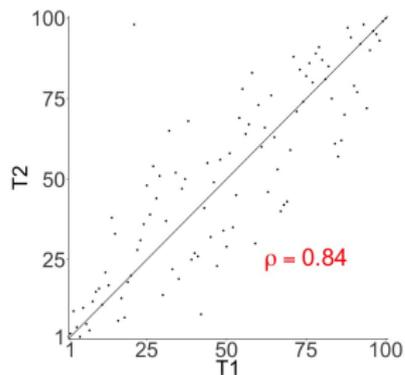
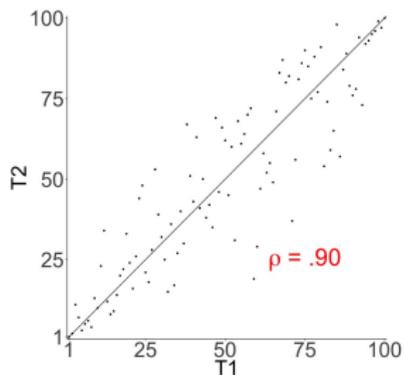
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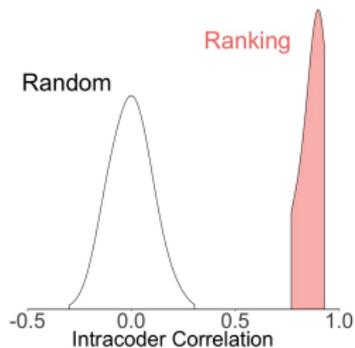
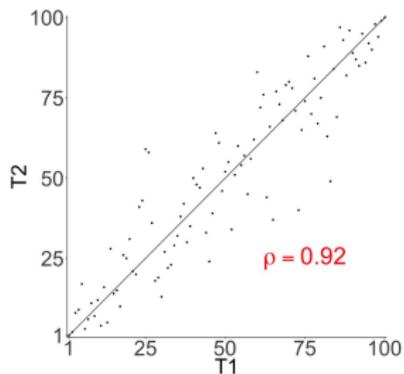
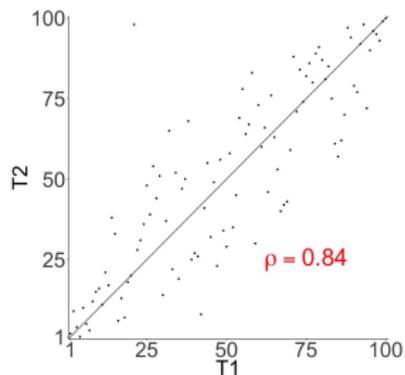
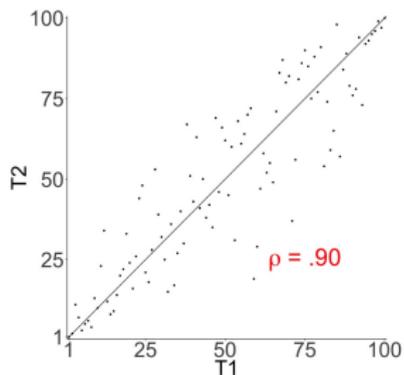
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- **Ensemble of predictive methods:** least squares, AdaBoosted decision trees, SVM, random forests. . .

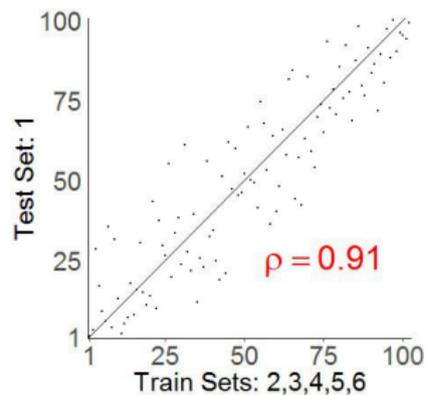
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Predict Test Set from 5 Training Sets

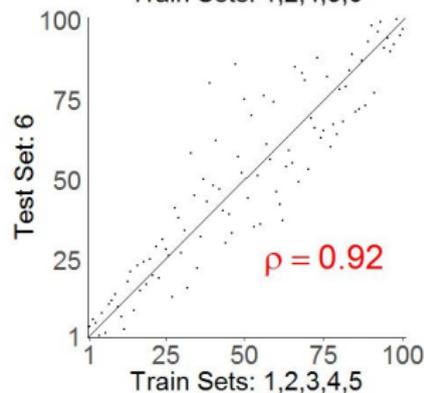
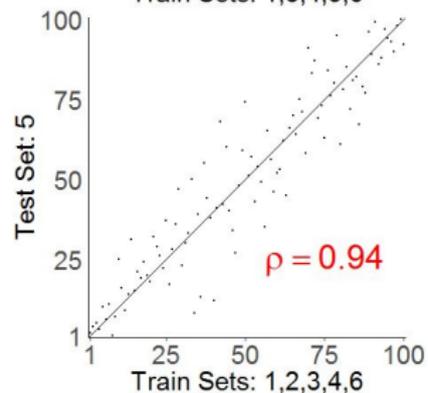
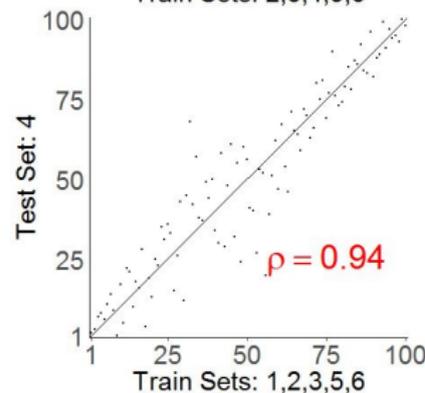
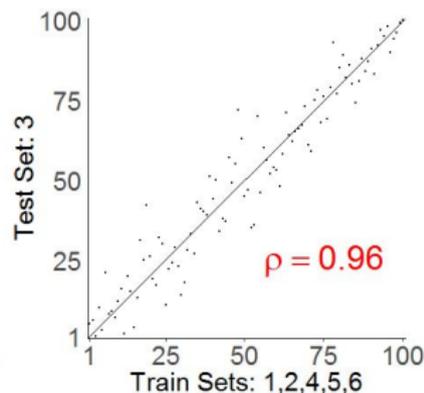
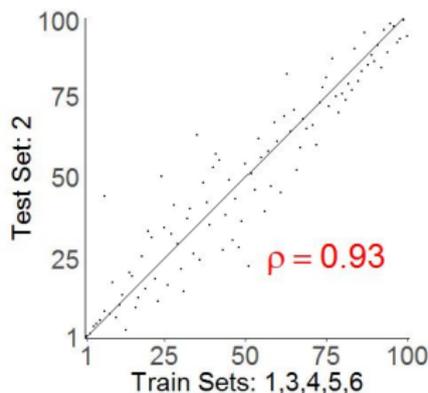
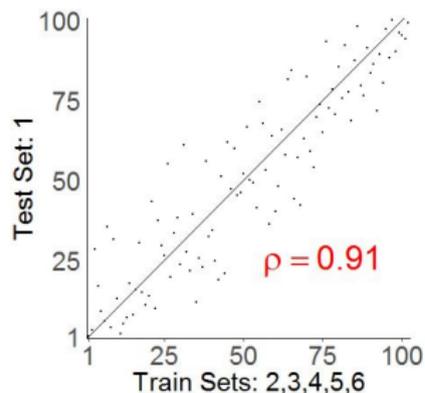
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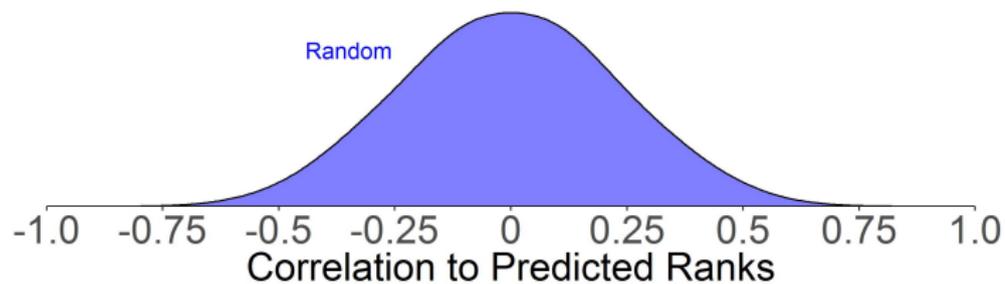
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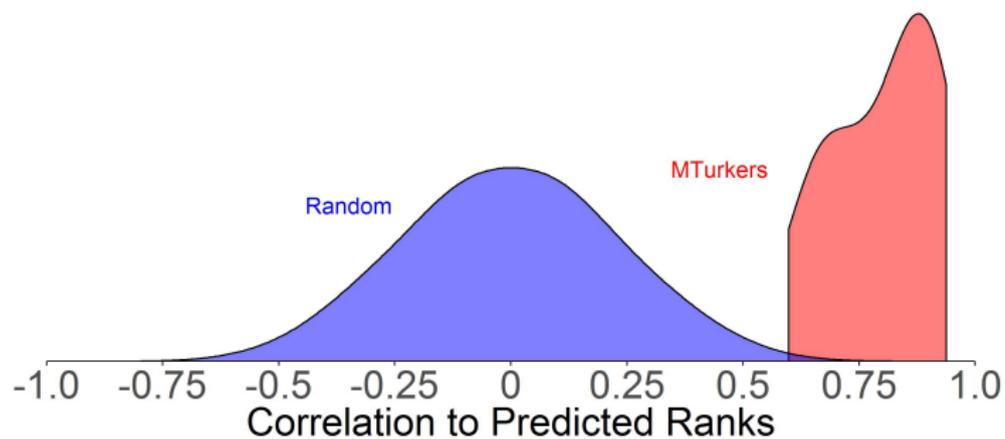
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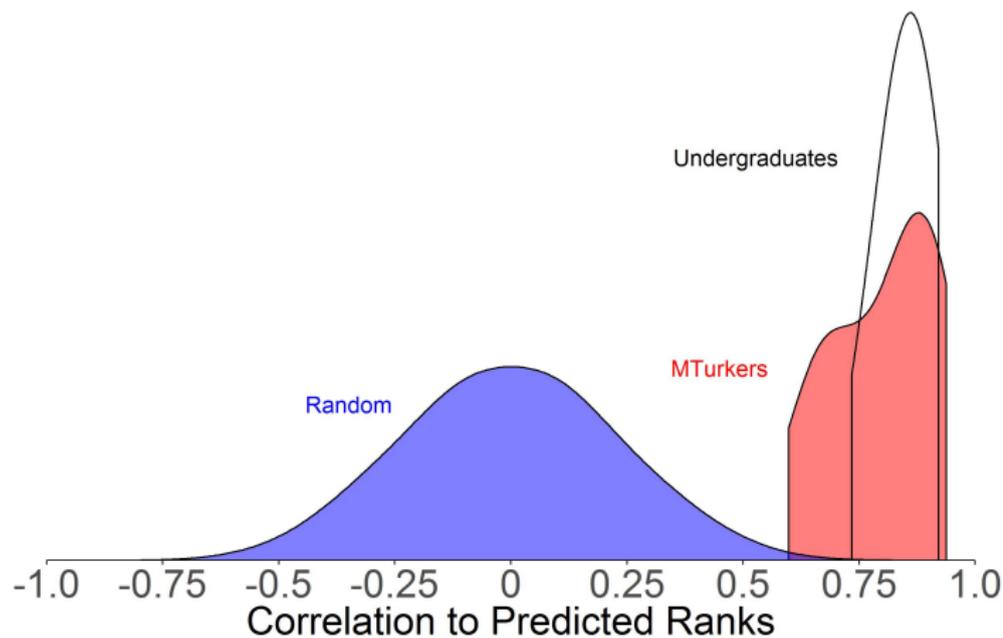
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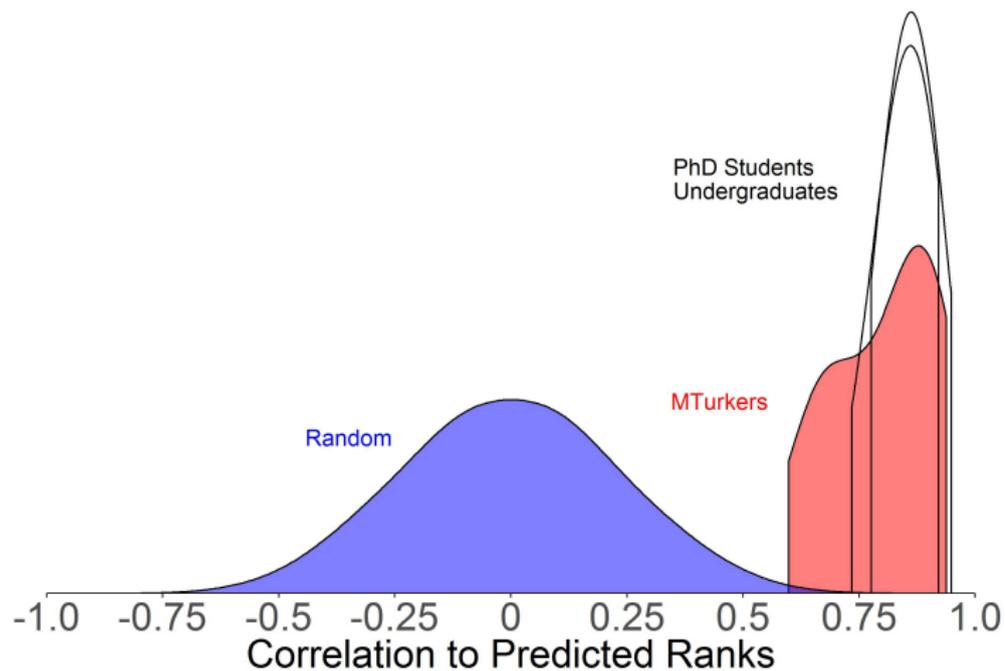
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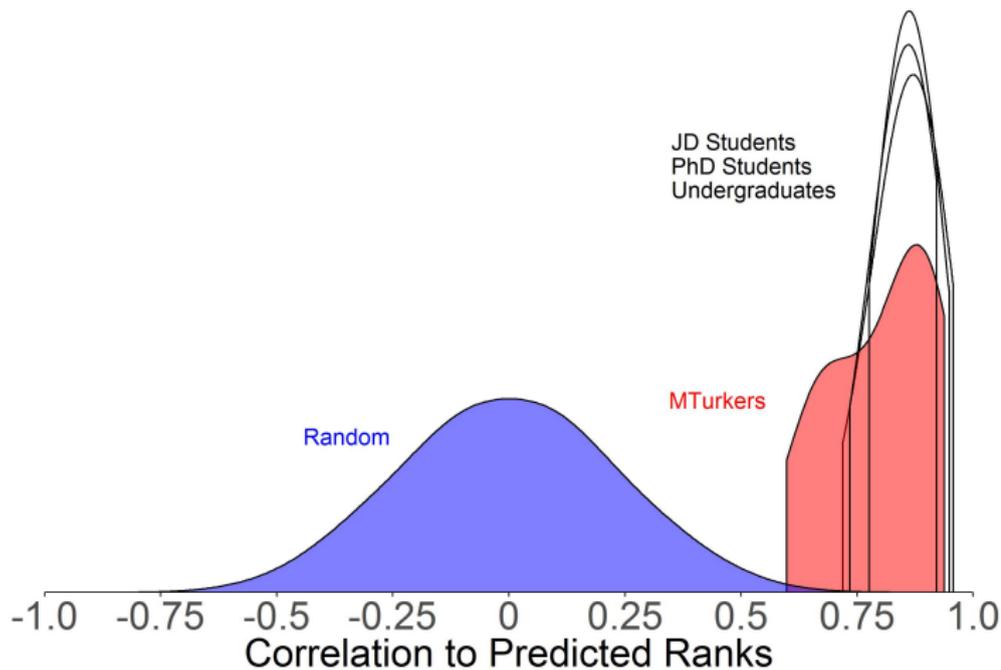
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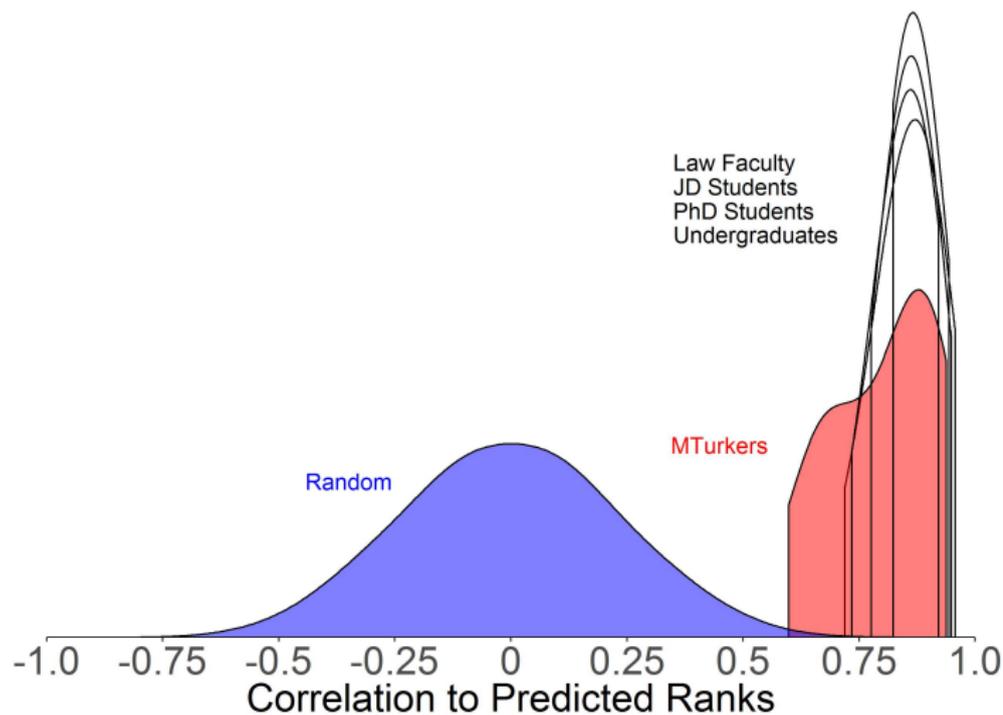
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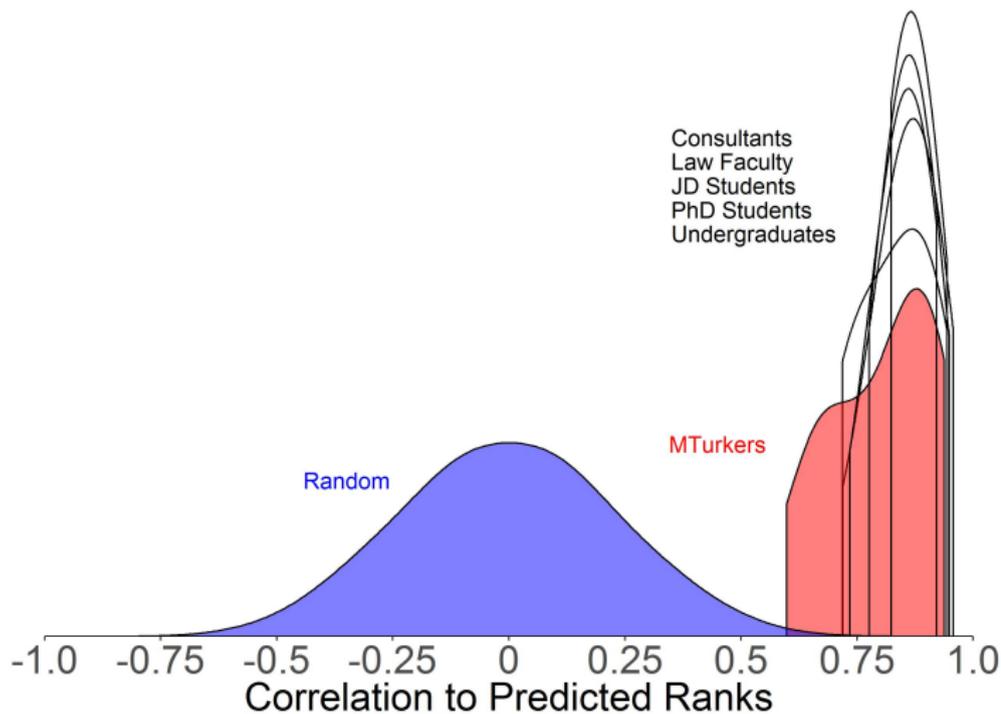
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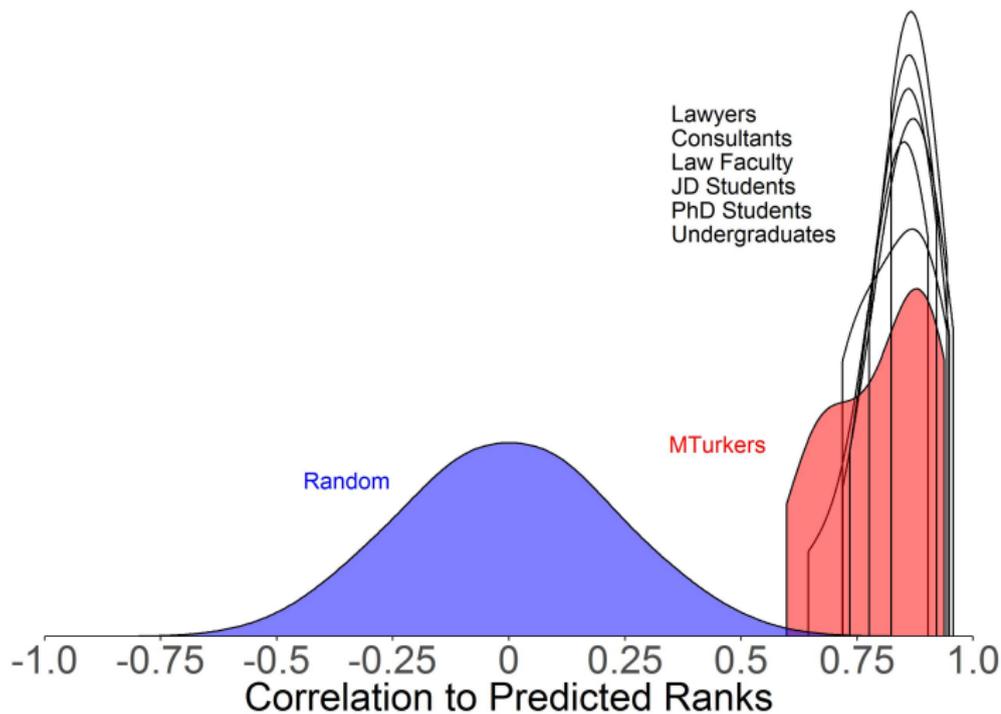
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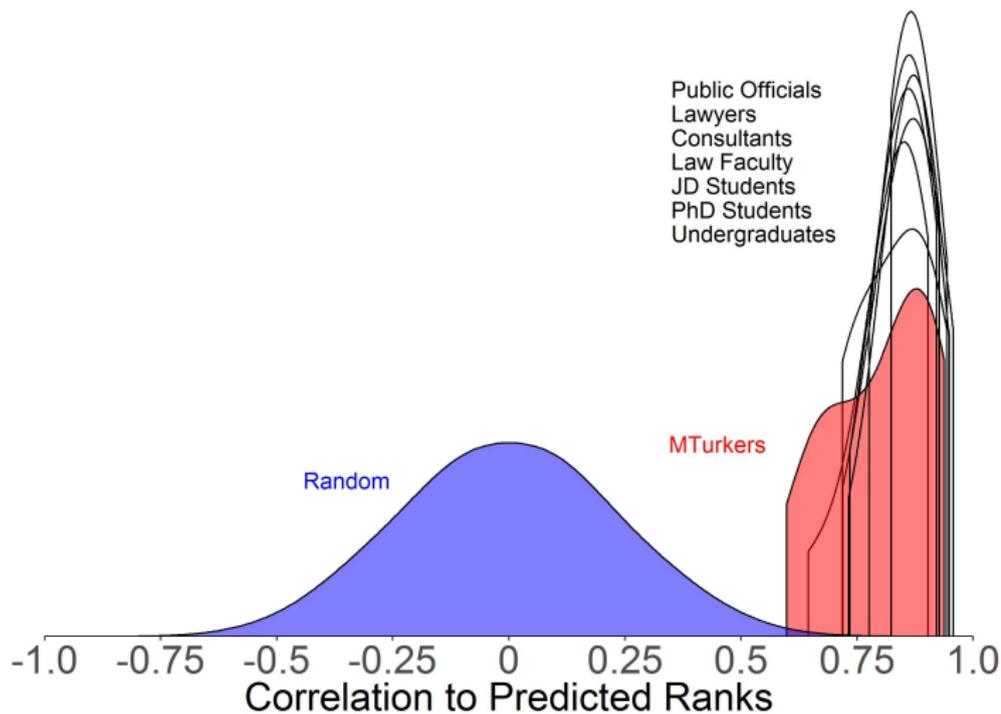
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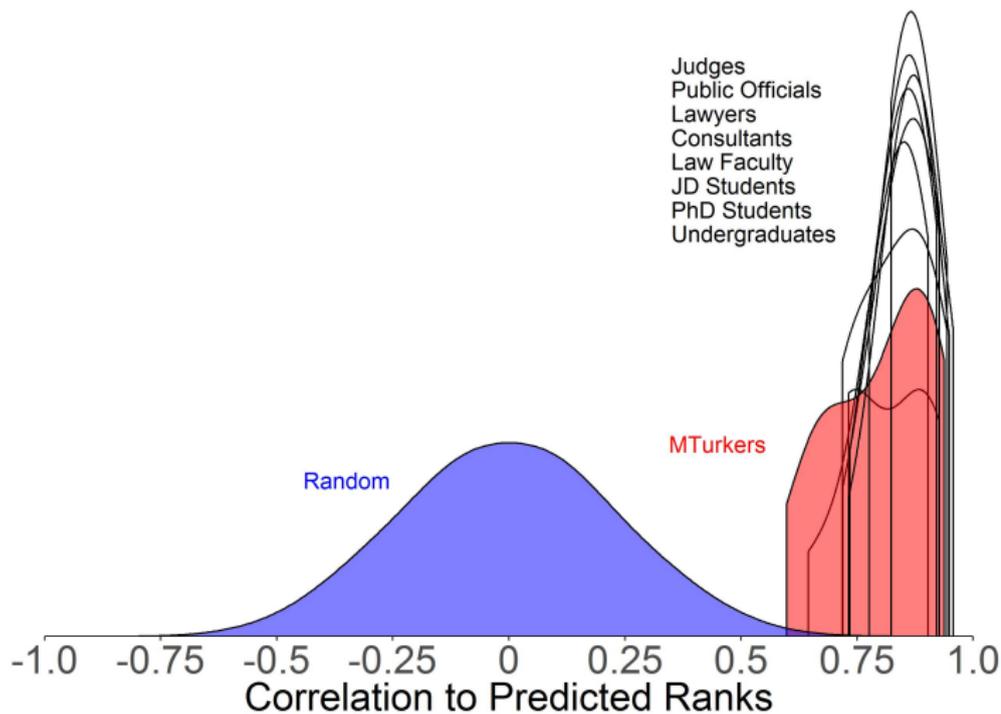
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## For more information



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[j.mp/MayyaKomisarchik](https://j.mp/MayyaKomisarchik)