Demographic Forecasting

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Joint work with Federico Girosi (RAND)
with contributions from Kevin Quinn and Gregory Wawro
What this Talk is About

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Mortality forecasts, which are studied in:

- demography & sociology
- public health & biostatistics
- economics & social security and retirement planning
- actuarial science & insurance companies
- medical research & pharmaceutical companies
- political science & public policy

A better forecasting method

Other results we needed to achieve this original goal

Approach: Formalizing qualitative insights in quantitative models
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- A better **farcasting** method
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Other Results (Needed to Develop Improved Forecasts)

- Output: same as linear regression
  - Estimates a set of linear regressions together (over countries, age groups, years, etc.)
  - Can include different covariates in each regression

- We demonstrate that most hierarchical and spatial Bayesian models with covariates misrepresent prior information

- Better ways of creating Bayesian priors

- Produces forecasts and farcasts using considerably more information
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A New Class of Statistical Models

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The Statistical Problem of Global Mortality Forecasting

779,799,281 deaths, in annual mortality rates

Multidimensional Data Structures: 24 causes of death, 17 age groups, 2 sexes, 191 countries, all for 50 annual observations.

One time series analysis for each of 155,856 cross-sections: with 1 minute to analyze each, one run takes 108 days

Every decision must be automated, systematized, and formalized: the same goal as including qualitative information in the model

Explanatory variables: Available in many countries: tobacco consumption, GDP, human capital, trends, fat consumption, total fertility rates, etc.

Numerous variables specific to a cause, age group, sex, and country

Most time series are very short. A majority of countries have only a few isolated annual observations; only 54 countries have at least 20 observations; Africa, AIDS, & Malaria are real problems
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Existing Method 1: Parameterize the Age Profile

Gompertz (1825): log-mortality is linear in age after age 20 reduces 17 age-specific mortality rates to 2 parameters (intercept and slope) Then forecast only these 2 parameters Reduces variance, constrains forecasts

Dozens of more general functional forms proposed But does it fit anything else?
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Mortality Age Profile: The Same Pattern?

Cardiovascular Disease (m)

ln(mortality)

Age

France

USA

Brazil

Demographic Forecasting
Mortality Age Profile: The Same Pattern?

Breast Cancer (f)

Japan
Venezuela
New Zealand

ln(mortality)

Age

Demographic Forecasting
Mortality Age Profile: The Same Pattern?

Other Infectious Diseases (f)

- Thailand
- Sri Lanka
- Barbados
- Italy
Mortality Age Profile: The Same Pattern?

Suicide (m)

-6
-7
-8
-9
-10

Age

ln(mortality)

Hungary
Canada
Colombia
Sri Lanka

15 20 25 30 35 40 45 50 55 60 65 70 75 80

10 20 30 40 50 60 70 80

ln(mortality)

10 9 8 7 6

Demographic Forecasting
Parameterizing Age Profiles Does Not Work

No mathematical form fits all or even most age profiles

Out-of-sample age profiles often unrealistic

The key empirical patterns are qualitative:
- Adjacent age groups have similar mortality rates
- Age profiles are more variable for younger ages
- We don't know much about levels or exact shapes

Key question: how to include this qualitative information

Also: Method ignores covariate information; the leading current method (McNown-Rogers) not replicable
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Existing Method 2: Deterministic Projections
Random walk with drift; Lee-Carter; least squares on linear trend
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Does it fit elsewhere?
The same pattern?
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Random Walk with Drift ≈ Lee-Carter ≈ Least Squares
The same pattern?
Random Walk with Drift \approx \text{Lee-Carter} \approx \text{Least Squares}

Suicide (m) USA

Data and Forecasts

Time

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Deterministic Projections Do Not Work

Linearity does not fit most time series data.
Out-of-sample age profiles become unrealistic over time.
Deterministic Projections Do Not Work

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Regression Approaches (Murray and Lopez, 1996)

Model mortality over countries ($c$) and ages ($a$) as:

$$m_{cat} = Z_{ca,t} - \ell \beta_{ca} + \epsilon_{cat}, t = 1, \ldots, T$$

$Z_{ca,t}$ covariates (GDP, tobacco...), lagged $\ell$ years.

$\beta_{ca}$ coefficients to be estimated.

Cannot estimate equation by equation (variance is too large);

Pool over countries: $\beta_{ca} \Rightarrow \beta_{a}$

Properties:

- Small variance (due to large $n$)
- Large biases (due to restrictive pooling over countries)
- Considerable information lost (due to no pooling over ages)

Demographic Forecasting
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  - same covariates required in all cross-sections
Likelihood for equation-by-equation least squares:

\[ P(m | \beta_i, \sigma_i) = \prod_t \mathcal{N}(m_{it} | Z_{it}\beta_i, \sigma_i^2) \]
Likelihood for equation-by-equation least squares:

\[ P(m \mid \beta_i, \sigma_i) = \prod_t N(m_{it} \mid Z_{it}\beta_i, \sigma_i^2) \]

Add priors and form a posterior

\[ P(\beta, \sigma, \theta \mid m) \propto P(m \mid \beta, \sigma) \times P(\beta \mid \theta) \times P(\theta)P(\sigma) \]

\[ = (\text{Likelihood}) \times (\text{Key Prior}) \times (\text{Other priors}) \]
Partial Pooling via a Bayesian Hierarchical Approach

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- Calculate point estimate for \( \beta \) (for \( \hat{y} \)) as the mean posterior:

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- The easy part: *easy-to-use software* to implement everything we discuss today.
The (Problematic) Classical Bayesian Approach

Assumption:
Similarities among cross-sections imply similarities among coefficients ($\beta$'s).

Requirements:
$s_{ij}$ measures the similarity between cross-section $i$ and $j$.

$\left(\beta_i - \beta_j\right)' \Phi (\beta_i - \beta_j) \equiv \| \beta_i - \beta_j \|^2 \Phi$

measures the distance between $\beta_i$ and $\beta_j$.

Natural choice for the prior:
$P(\beta | \Phi) \propto \exp \left[ -\frac{1}{2} \sum_{ij} s_{ij} \| \beta_i - \beta_j \|^2 \Phi \right]$
The (Problematic) Classical Bayesian Approach

**Assumption:** similarities among cross-sections imply similarities among coefficients (β’s).
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Natural choice for the prior: $P(\beta | \Phi) \propto \exp \left(- \frac{1}{2} \sum_{ij} s_{ij} \| \beta_i - \beta_j \|_\Phi^2 \right)$
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Requires the same covariates, with the same meaning, in every cross-section. Prior knowledge about $\beta$ exists for causal effects, not for control variables, or forecasting. Everything depends on $\Phi$, the normalization factor: $\Phi$ can't be estimated, and must be set. An uninformative prior for it would make Bayes irrelevant, An informative prior can't be used since we don't have information. Common practice: make some wild guesses.

The classical approach can be harmful: Making $\beta_i$ more smooth may make $\mu$ less smooth ($\mu = Z\beta$):

\[
\mu - \mu_{jt} = Z_{it}(\beta_i - \beta_j) + (Z_{it} - Z_{jt})\beta_j = \text{Coefficient variation} + \text{Covariate variation}
\]

Extensive trial-and-error runs, yielded no useful parameter values.
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Our Alternative Strategy: Priors on $\mu$

Three steps:

1. Specify a prior for $\mu$:

$$P(\mu | \theta) \propto \exp\left(-\frac{1}{2} H[\mu, \theta]\right),$$

for example,

$$H[\mu, \theta] \equiv \theta^T \sum_{t=1} A^{-1} \sum_{a=1} \left(\mu^a_t - \mu^a + 1, \sum\right).$$

2. To do Bayes, we need a prior on $\beta$.

Problem: How to translate a prior on $\mu$ into a prior on $\beta$ (a few-to-many transformation)?

3. Constrain the prior on $\mu$ to the subspace spanned by the covariates:

$$\mu = Z\beta$$

In the subspace, we can invert $\mu = Z\beta$ as

$$\beta = (Z^T Z)^{-1} Z^T \mu,$$

giving:

$$P(\beta | \theta) \propto \exp\left(-\frac{1}{2} H[Z\beta, \theta]\right)$$

the same prior on $\mu$, expressed as a function of $\beta$ (with constant Jacobian).
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the same prior on $\mu$, expressed as a function of $\beta$ (with constant Jacobian).
In other words, any prior information about \( \mu \) (the expected value of the dependent variable) is "translated" into information about the coefficients \( \beta \) via:

\[
\mu_{\text{cat}} = Z_{\text{cat}} \beta_{\text{ca}}
\]

A Simple Analogy

Suppose \( \delta = \beta_1 - \beta_2 \) and \( P(\delta) = N(\delta|0, \sigma^2) \).

What is \( P(\beta_1, \beta_2) \)?

It's a singular bivariate Normal.

It is defined over \( \beta_1, \beta_2 \) and constant in all directions but \( (\beta_1 - \beta_2) \).

We start with one-dimensional \( P(\mu_{\text{cat}}) \), and treat it as the multidimensional \( P(\beta_{\text{ca}}) \), constant in all directions but \( Z_{\text{cat}} \beta_{\text{ca}} \).
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- The normalization matrix $\Phi$ is unnecessary (task is performed by $Z$, which is known).
- Priors are based on knowledge rather than guesses.
An Age Prior

The prior is normal (and improper); Adjustable parameters: $n$ and $\theta$.

The choice of $n$ uniquely determines the "interaction" matrix $W_n$.

The variance of the prior is inversely proportional to $\theta$, which controls the "strength" of the prior.

Different age groups can have different covariates: the matrices $C_{aa'} \equiv 1^T Z_{a} Z_{a'}$ are rectangular ($d_a \times d_{a'}$).
An Age Prior

\[ \mathcal{P}(\mu \mid \theta) \sim \mathcal{P}(\beta \mid \theta) \propto \exp \left( -\theta \sum_{aa'} W_{aa'}^n \beta_a' C_{aa'} \beta_{a'} \right) \]

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- Different age groups can have different covariates: the matrices \( C_{aa'} \equiv \frac{1}{T} Z'_a Z_{a'} \) are rectangular \( (d_a \times d_{a'}) \).
Samples From Age Prior

All Causes \((m), n = 2\)

![Graph showing log-mortality over age](image)

Log-mortality

Age
All Causes (m) , n = 3
Samples From Age Prior

All Causes (m), n = 1

All Causes (m), n = 2

All Causes (m), n = 3

All Causes (m), n = 4
Prior Indifference

These priors are "indifferent" to transformations:

$$\mu(a, t) \rightarrow \mu(a, t) + p(a, t)$$

where $$p(a, t)$$ is a polynomial in $$a$$ (whose degree is the degree of the derivative in the prior)

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Formalizing (Prior) Indifference

equal \textcolor{red}{color} = equal \textcolor{red}{probability}
Formalizing (Prior) Indifference

equal color = equal probability

Level indifference
Formalizing (Prior) Indifference

equal color = equal probability

Level indifference

Level and slope indifference
The prior:

\[ P(\beta | \theta) \propto \exp(-\theta \sum a a^\prime W_n a a^\prime \beta^\prime a C a a^\prime \beta a^\prime a) \]

We figured out what \( n \) is but what is the smoothness parameter, \( \theta \)?

\( \theta \) controls the prior standard deviation.
Smoothness Parameter

- The prior:

\[ P(\beta \mid \theta) \propto \exp \left( -\theta \sum_{aa'} W_{aa'}^{n} \beta_{a} \cdot C_{aa'} \beta_{a'} \right) \]
Smoothness Parameter

- The prior:

\[ P(\beta | \theta) \propto \exp \left( -\theta \sum_{aa'} W_{aa'}^n \beta_{a'}^i C_{aa'} \beta_{a'}^i \right) \]

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- We figured out what \( n \) is
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- \( \theta \) controls the prior standard deviation
Samples from Age Prior

All Causes (f), n = 2

![Graph showing Age vs. Log-mortality for All Causes (f), n = 2](image-url)
Samples from Age Prior

All Causes (f), n = 2

Log-mortality vs Age
Samples from Age Prior

All Causes (f), n = 2

Age

Log-mortality

Age

Log-mortality

0 20 40 60 80

−10 −8 −6 −4 −2 0 2

All Causes (f), n = 2
 Samples from Age Prior

All Causes (f), n = 2

Log–mortality

Age
Samples from Age Prior

All Causes (f), n = 2

Age
Log-mortality

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All Causes (f), n = 2
Age
Log-mortality

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Demographic Forecasting
Samples from Age Prior

All Causes (f), n = 2

Age (x) vs. Log-mortality (y) for different samples.
Samples from Age Prior

All Causes (f), n = 2

Age

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0 20 40 60 80

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Demographic Forecasting
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Samples from Age Prior

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All Causes (f), n = 2

Demographic Forecasting
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All Causes (f), n = 2
Generalizations

The above tools: smooth over a (possibly discretized) continuous variable — age or age groups. We can also smooth over time (also a discretized continuous variable). Can smooth when cross-sectional unit \( i \) is a label, such as country. Can smooth simultaneously over different types of variables (age, country, and time).

We can smooth interactions:

- Smoothing trends over age groups.
- Smoothing trends over age groups as they vary across countries, etc.

The mathematical form for all these (separately or together) turns out to be the same:

\[
P(\beta | \theta) \propto \exp \left( -\theta^2 \sum_{ij} W_{ij} \beta_i' C_{ij} \beta_j \right) ,
\]

where

\[
C_{aa'} \equiv \frac{1}{T} \sum_a Z_a Z_{a'}
\]
Generalizations

- The above tools: smooth over a (possibly discretized) continuous variable — age or age groups.

The mathematical form for all these (separately or together) turns out to be the same:

\[ P(\beta | \theta) \propto \exp \left( -\theta^2 \sum_{ij} W_{ij} \beta_i' \beta_j \right), \]

where \( C_{aa} \equiv \frac{1}{T} Z_a Z_a' \)
Generalizations

- The above tools: smooth over a (possibly discretized) continuous variable — age or age groups.
- We can also smooth over time (also a discretized continuous variable).

The mathematical form for all these (separately or together) turns out to be the same:

$$P(β|θ) \propto \exp \left( -θ^2 \sum_{ij} W_{ij} β'_i C_{ij} β_j \right),$$

where $C_{aa} \equiv 1_T Z_a Z_a'$.
Generalizations

- The above tools: smooth over a (possibly discretized) continuous variable — age or age groups.
- We can also smooth over time (also a discretized continuous variable).
- Can smooth when cross-sectional unit \( i \) is a label, such as country.

The mathematical form for all these (separately or together) turns out to be the same:

\[
P(\beta | \theta) \propto \exp \left( -\theta^2 \sum_{ij} W_{ij} \beta_i \nu_{ij} \beta_j \right),
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where \( C_{aa} \equiv 1/T \mathbf{ZZ}' \).
Generalizations

- The above tools: smooth over a (possibly discretized) continuous variable — age or age groups.
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- Can smooth when cross-sectional unit $i$ is a label, such as country.
- Can smooth simultaneously over different types of variables (age, country, and time).

The mathematical form for all these (separately or together) turns out to be the same:

$$P(\beta|\theta) \propto \exp \left( -\theta^2 \sum_{ij} W_{ij} \beta_i^T C_{ij} \beta_j \right), \quad C_{aa} \equiv \frac{1}{T} Z_a Z_a'$$
The above tools: smooth over a (possibly discretized) continuous variable — age or age groups.

We can also smooth over time (also a discretized continuous variable).

Can smooth when cross-sectional unit \( i \) is a label, such as country.

Can smooth simultaneously over different types of variables (age, country, and time).

We can smooth interactions:

\[
P(\beta | \theta) \propto \exp \left( -\theta^2 \sum_{ij} W_{ij} \beta_i' C_{ij} \beta_j \right),
\]

\( C_{aa'} \equiv 1 \)
Generalizations

- The above tools: smooth over a (possibly discretized) continuous variable — age or age groups.
- We can also smooth over time (also a discretized continuous variable).
- Can smooth when cross-sectional unit $i$ is a label, such as country.
- Can smooth simultaneously over different types of variables (age, country, and time).
- We can smooth interactions:
  - Smoothing *trends* over age groups.
Generalizations

- The above tools: smooth over a (possibly discretized) continuous variable — age or age groups.
- We can also smooth over time (also a discretized continuous variable).
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  - Smoothing trends over age groups as they vary across countries, etc.

The mathematical form for all these (separately or together) turns out to be the same:

$$P(\beta|\theta) \propto \exp \left( -\theta^2 \sum_{ij} W_{ij} \beta_i C_{ij} \beta_j \right) ,$$

$$C_{aa}^\prime \equiv \frac{1}{T} Z_a Z_a^\prime$$
Generalizations

- The above tools: smooth over a (possibly discretized) continuous variable — age or age groups.
- We can also smooth over time (also a discretized continuous variable).
- Can smooth when cross-sectional unit \( i \) is a label, such as country.
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  - Smoothing trends over age groups as they vary across countries, etc.
- The mathematical form for *all* these (separately or together) turns out to be the same:

\[
P(\beta \mid \theta) \propto \exp \left( -\frac{\theta}{2} \sum_{ij} W_{ij} \beta_i' C_{ij} \beta_j \right), \quad C_{aa'} \equiv \frac{1}{T} Z_a Z_{a'}
\]
Mortality from Respiratory Infections, Males

Least Squares

Data and Forecasts

(m) Belize

1970 - 2030

Demographic Forecasting
Mortality from Respiratory Infections, males, $\sigma = 2.00$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 1.51$

Smoothing over Age Groups

Data and Forecasts

1970 2030

Age

(m) Belize

Data and Forecasts

Age
Mortality from Respiratory Infections, males, $\sigma = 1.15$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 0.87$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 0.66$

Smoothing over Age Groups

Data and Forecasts

1970 - 2030

Age

(m) Belize
Mortality from Respiratory Infections, males, $\sigma = 0.50$

Smoothing over Age Groups

Data and Forecasts

(\textit{m}) Belize

Age

1970

2030

12

4

Data and Forecasts

Age

12

8

6

4

2

0

20

40

60

80
Mortality from Respiratory Infections, males, $\sigma = 0.38$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 0.28$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 0.21$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 0.16$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 0.12$

Smoothing over Age Groups

Data and Forecasts

1970 - 2030
Mortality from Respiratory Infections, males, $\sigma = 0.09$

Smoothing over Age Groups

Data and Forecasts

(m) Belize

Age

1970 2030

Age

Data and Forecasts

Demographic Forecasting
Mortality from Respiratory Infections, males, $\sigma = 0.07$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 0.05$

Smoothing over Age Groups

Data and Forecasts

(m) Belize

Age

1970 2030
Mortality from Respiratory Infections, males, $\sigma = 0.04$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 0.03$
Mortality from Respiratory Infections, males, $\sigma = 0.02$

Smoothing over Age Groups

Data and Forecasts

(m) Belize

Age

Data and Forecasts

1970 - 2030

Age
Mortality from Respiratory Infections, males, $\sigma = 0.01$

Smoothing over Age Groups

Data and Forecasts

Age

(m) Belize

1970 2030

Age

Data and Forecasts

(m) Belize

1970 2030

Age
Mortality from Respiratory Infections, males

Least Squares

Data and Forecasts

(m) Belize

Time

Demographic Forecasting
Mortality from Respiratory Infections, males, $\sigma = 2.00$

Smoothing over Age Groups

Data and Forecasts

(m) Belize

Time


−12 −10 −8 −6 −4

Demographic Forecasting

65 / 98
Mortality from Respiratory Infections, males, $\sigma = 1.51$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 1.15$

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Mortality from Respiratory Infections, males, $\sigma = 0.38$

Smoothing over Age Groups

Data and Forecasts

Time

Demographic Forecasting
Mortality from Respiratory Infections, males, $\sigma = 0.28$

Smoothing over Age Groups

Data and Forecasts

(m) Belize

Time

Demographic Forecasting
Mortality from Respiratory Infections, males, $\sigma = 0.21$

Smoothing over Age Groups

Data and Forecasts

(m) Belize

Time

Demographic Forecasting
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Smoothing over Age Groups

Demographic Forecasting
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Mortality from Respiratory Infections, males, \( \sigma = 0.07 \)

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 0.05$

Smoothing over Age Groups

Demographic Forecasting
Mortality from Respiratory Infections, males, $\sigma = 0.04$

Smoothing over Age Groups

![Graph showing data and forecasts for mortality from respiratory infections in Belize over time from 1970 to 2030.]
Mortality from Respiratory Infections, males, $\sigma = 0.03$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 0.02$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 0.01$

Smoothing over Age Groups
Smoothing Trends over Age Groups

Demographic Forecasting
Smoothing Trends over Age Groups

Log-mortality in Belize males from respiratory infections
Smoothing Trends over Age Groups
Log-mortality in Belize males from respiratory infections

Least Squares
Smoothing Trends over Age Groups
Log-mortality in Belize males from respiratory infections

Least Squares
Smoothing Trends over Age Groups
Log-mortality in Belize males from respiratory infections

Least Squares
Smoothing Trends over Age Groups

Log-mortality in Belize males from respiratory infections

Least Squares

Smoothing Age Groups
Smoothing Trends over Age Groups
Log-mortality in Belize males from respiratory infections

Least Squares

Smoothing Age Groups
Smoothing Trends over Age Groups
Log-mortality in Belize males from respiratory infections

Least Squares

Smoothing Age Groups
Smoothing Trends over Age Groups and Time
Smoothing Trends over Age Groups and Time
Log-Mortality in Bulgarian males from respiratory infections
Smoothing Trends over Age Groups and Time
Log-Mortality in Bulgarian males from respiratory infections

Least Squares
Smoothing Trends over Age Groups and Time
Log-Mortality in Bulgarian males from respiratory infections

Least Squares
Smoothing Trends over Age Groups and Time
Log-Mortality in Bulgarian males from respiratory infections

Least Squares
Smoothing Trends over Age Groups and Time
Log-Mortality in Bulgarian males from respiratory infections

Least Squares

Smoothing Age and Time
Smoothing Trends over Age Groups and Time
Log-Mortality in Bulgarian males from respiratory infections

Least Squares

Smoothing Age and Time
Smoothing Trends over Age Groups and Time
Log-Mortality in Bulgarian males from respiratory infections

Least Squares

Smoothing Age and Time
Using Covariates (GDP, tobacco, trend, log trend)
Using Covariates (GDP, tobacco, trend, log trend)

Lung cancer in Korean Males
Using Covariates (GDP, tobacco, trend, log trend)
Lung cancer in Korean Males

Least Squares
Using Covariates (GDP, tobacco, trend, log trend)

Lung cancer in Korean Males

Least Squares

Data and Forecasts

Time

(m) Republic of Korea

Age

Data and Forecasts

1985–2030

Smooth over age, time, age/time

1990–2030
Using Covariates (GDP, tobacco, trend, log trend)

Lung cancer in Korean Males

Least Squares

Data and Forecasts

Demographic Forecasting
Using Covariates (GDP, tobacco, trend, log trend)

Lung cancer in Korean Males

Least Squares

Smooth over age, time, age/time
Using Covariates (GDP, tobacco, trend, log trend)
Lung cancer in Korean Males

Least Squares

Smooth over age, time, age/time
Using Covariates (GDP, tobacco, trend, log trend)
Lung cancer in Korean Males

Least Squares

Smooth over age, time, age/time
Using Covariates (GDP, tobacco, trend, log trend)
Using Covariates (GDP, tobacco, trend, log trend)

Lung cancer in Males, Singapore
Using Covariates (GDP, tobacco, trend, log trend)
Lung cancer in Males, Singapore

Least Squares
Using Covariates (GDP, tobacco, trend, log trend)
Lung cancer in Males, Singapore

Least Squares
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Lung cancer in Males, Singapore

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Using Covariates (GDP, tobacco, trend, log trend)

Lung cancer in Males, Singapore

Least Squares

Smooth over age, time, age/time
Using Covariates (GDP, tobacco, trend, log trend)
Lung cancer in Males, Singapore

Least Squares

Smooth over age, time, age/time
Using Covariates (GDP, tobacco, trend, log trend)

Lung cancer in Males, Singapore

Least Squares

Smooth over age, time, age/time

Data and Forecasts

1963 2030

(m) Singapore

Time

Age

Data and Forecasts

(m) Singapore

(m) Singapore

(m) Singapore

(m) Singapore
What about ICD Changes?

Other Infectious Diseases: USA, age 0 (m)

Other Infectious Diseases: France, age 0 (m)

Other Infectious Diseases: Australia, age 0 (m)

Other Infectious Diseases: United Kingdom, age 0 (m)
Fixing ICD Changes

Other Infectious Diseases: USA, age 0 (m)

Other Infectious Diseases: France, age 0 (m)

Other Infectious Diseases: Australia, age 0 (m)

Other Infectious Diseases: United Kingdom, age 0 (m)
http://GKing.Harvard.edu
<table>
<thead>
<tr>
<th>Outcome</th>
<th>% Improvement Over Best to Best</th>
<th>Conceivable Previous</th>
<th>Cardiac</th>
<th>Lung Cancer</th>
<th>Transportation</th>
<th>Respiratory Chronic</th>
<th>Other Infectious</th>
<th>Stomach Cancer</th>
<th>All-Cause</th>
<th>Suicide</th>
<th>Respiratory Infectious</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>22</td>
<td>49</td>
<td>16</td>
<td>31</td>
<td>13</td>
<td>30</td>
<td>22</td>
<td>17</td>
<td>3</td>
</tr>
</tbody>
</table>

Each row averages 6,800 forecast errors (17 age groups, 40 countries, and 10 out-of-sample years).

% to best conceivable = % of the way our method takes us from the best existing to the best conceivable forecast.

The new method out-performs with the same covariates, for most countries, causes, sexes, and age groups. Does considerably better with more informative covariates.
### Preview of Results: Out-of-Sample Evaluation

Mean Absolute Error in Males (over age and country)

<table>
<thead>
<tr>
<th>Cause</th>
<th>% Improvement Over Best</th>
<th>% to Best Conceivable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cardiovascular</td>
<td>22</td>
<td>49</td>
</tr>
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<td>24</td>
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<td>22</td>
</tr>
<tr>
<td>Suicide</td>
<td>7</td>
<td>17</td>
</tr>
<tr>
<td>Respiratory Infectious</td>
<td>3</td>
<td>7</td>
</tr>
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### Preview of Results: Out-of-Sample Evaluation

Mean Absolute Error in Males (over age and country)

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<tbody>
<tr>
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<table>
<thead>
<tr>
<th>Disease Category</th>
<th>% Improvement Over Best Previous</th>
<th>% Improvement to Best Conceivable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cardiovascular</td>
<td>22</td>
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- Each row averages 6,800 forecast errors (17 age groups, 40 countries, and 10 out-of-sample years).
## Preview of Results: Out-of-Sample Evaluation

Mean Absolute Error in Males (over age and country)

<table>
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<tr>
<th>% Improvement</th>
<th>Over Best Previous</th>
<th>to Best Conceivable</th>
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</thead>
<tbody>
<tr>
<td>Cardiovascular</td>
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## Preview of Results: Out-of-Sample Evaluation

Mean Absolute Error in Males (over age and country)

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<tr>
<th>Cause</th>
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<tr>
<td>Cardiovascular</td>
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### Preview of Results: Out-of-Sample Evaluation

Mean Absolute Error in Males (over age and country)

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Basic Prior: Smoothness over Age Groups

Prior knowledge: log-mortality age profile are smooth variations of a "typical" age profile $\bar{\mu}(a)$:

$$H[\mu, \theta] \equiv \theta \int_0^T dt \int_0^A da \left[ \mu(a, t) - \bar{\mu}(a) \right]^2$$

Discretize age and time:

$$P(\mu | \theta) \propto \exp \left( -\frac{1}{2} \theta \sum_{aa} t(a) (\mu(a, t) - \bar{\mu}(a))' W_{nn} (\mu(a', t) - \bar{\mu}(a')) \right)$$

where $W_{nn}$ is a matrix uniquely determined by $n$ and $\theta$. 

---

Demographic Forecasting
Prior knowledge: log-mortality age profile are smooth variations of a “typical” age profile \( \bar{\mu}(a) \):

\[
H[\mu, \theta] \equiv \int_0^T \int_0^A dt \int da d\mu_n da_n \left[ \mu_n(a,t) - \bar{\mu}(a) \right]^2
\]

\( P(\mu|\theta) \propto \exp\left( -\frac{1}{2} \sum_{aa'} \theta \left( \mu_{at} - \bar{\mu}_{a'} \right)^W_{aa'} (\mu_{a't} - \bar{\mu}_{a'}) \right) \)

where \( W_n \) is a matrix uniquely determined by \( n \) and \( \theta \).
Basic Prior: Smoothness over Age Groups

- Prior knowledge: log-mortality age profile are smooth variations of a “typical” age profile $\bar{\mu}(a)$:

$$H[\mu, \theta] \equiv$$
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Basic Prior: Smoothness over Age Groups

- Prior knowledge: log-mortality age profile are smooth variations of a “typical” age profile $\bar{\mu}(a)$:

$$H[\mu, \theta] \equiv \left( \frac{d^n}{da^n} [\mu(a, t) - \bar{\mu}(a)] \right)^2$$
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Discretize age and time:

$$P(\mu | \theta) \propto \exp \left( - \frac{1}{2} \theta \sum_{aa'} (\mu_{at} - \bar{\mu}_a)' W_{aa'}^n (\mu_{a't} - \bar{\mu}_{a'}) \right)$$
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- where $W^n$ is a matrix uniquely determined by $n$ and $\theta$
From a prior on $\mu$ to a prior on $\beta$

Add the specification $\mu = \bar{\mu} a + Z:\n
P(\beta | \theta) = \exp(-\theta^T \sum a a'^T W n a a'^T (Z a \beta a'^T) (Z a'^T \beta a'^T)) = \exp(-\theta \sum a a'^T W n a a'^T \beta a'^T C a a'^T)$

where we have defined:

$C a a'^T \equiv 1^T Z a Z a'^T$ is a $T \times d a$ matrix for age group $a$.
Add the specification $\mu_{at} = \bar{\mu}_a + Z_{at} \beta_a$:
From a prior on $\mu$ to a prior on $\beta$

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The Prior on the Coefficients $\beta$

$$\mathcal{P}(\beta \mid \theta) \propto \exp \left( -\theta \sum_{aa'} \omega_{aa'}^{n} \beta_{a}^{'} \mathbf{C}_{aa'} \beta_{a'}^{'} \right)$$

The prior is normal (and improper); the prior is uniquely determined by the choice of $n$, through the "interaction" matrix $W$. Different age groups can have different covariates: the matrices $\mathbf{C}_{aa'}$ are rectangular ($d_{a} \times d_{a'}^{'}$). The variance of the prior is inversely proportional to $\theta$, which controls the "strength" of the prior.
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With Country Smoothing

Transportation Accidents
(males)
Sri Lanka

Demographic Forecasting
Formalizing Similarity

Standard Bayesian Approach

- Assume coefficients are similar
- But we know little about the coefficients
- Requires the same covariates in each cross-section
- Why measure water quality in the U.S.?
- Requires covariates with the same meaning in each cross-section
- Does GDP mean the same thing in Botswana and the U.S.?
- Imposes no assumptions on covariates or mortality
- If covariates are dissimilar, then making coefficients similar makes mortality dissimilar [since $E(y_t) = X_t \beta$ in each cross-section]

Alternative Approach

- Assume expected mortality is similar
- Coefficients are unobserved, mortality patterns are well known
- Different covariates allowed in each cross-section
- Covariates with the same name can have different meanings

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**Since** $E(y_t) = X_t \beta$ **in each cross-section**

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Many Short Time Series

Coverage of WHO data base (age specific, all causes)

- % of world countries
- % of world population

# Observations

% of world countries
% of world population
<table>
<thead>
<tr>
<th>Disease</th>
<th>Mean Absolute Error</th>
<th>% Improvement</th>
<th>Previous Method Conceivable</th>
<th>Conceivable</th>
<th>Previous</th>
<th>% to Best Conceivable</th>
<th>% to Best</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.34</td>
<td>0.19</td>
<td>22</td>
<td>49</td>
<td></td>
<td></td>
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</tr>
<tr>
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<td>24</td>
<td>47</td>
<td></td>
<td></td>
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<tr>
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Each row averages 6,800 forecast errors (17 age groups, 40 countries, and 10 out-of-sample years). % to best conceivable = % of the way our method takes us from the best existing to the best conceivable forecast. The new method out-performs with the same covariates, for most countries, causes, sexes, and age groups. Does much better with better covariates.
### Preview of Results: Out-of-Sample Evaluation

**Mean Absolute Error in Males (over age and country)**

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<td>24</td>
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</tr>
<tr>
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<td>0.31</td>
<td>0.18</td>
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<td>31</td>
</tr>
<tr>
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<td>0.39</td>
<td>0.26</td>
<td>13</td>
<td>30</td>
</tr>
<tr>
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<td>0.48</td>
<td>0.32</td>
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<tr>
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<td>0.47</td>
<td>0.28</td>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

- Each row averages 6,800 forecast errors (17 age groups, 40 countries, and 10 out-of-sample years).
- % to best conceivable = % of the way our method takes us from the best existing to the best conceivable forecast.
- The new method out-performs with the same covariates, for most countries, causes, sexes, and age groups.
## Preview of Results: Out-of-Sample Evaluation

Mean Absolute Error in Males (over age and country)

<table>
<thead>
<tr>
<th></th>
<th>Mean Absolute Error</th>
<th>% Improvement</th>
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<td>Our Method</td>
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<tr>
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- Each row averages 6,800 forecast errors (17 age groups, 40 countries, and 10 out-of-sample years).
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- The new method out-performs with the same covariates, for most countries, causes, sexes, and age groups.
- Does much better with better covariates