Demographic Forecasting

Gary King
Harvard University

Joint work with Federico Girosi (RAND)
with contributions from Kevin Quinn and Gregory Wawro
What this Talk is About

Mortality forecasts, which are studied in:
- demography & sociology
- public health & biostatistics
- economics & social security and retirement planning
- actuarial science & insurance companies
- medical research & pharmaceutical companies
- political science & public policy

A better forecasting method

Other results we needed to achieve this original goal

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Other Results (Needed to Develop Improved Forecasts)

Output: same as linear regression
Estimates a set of linear regressions together (over countries, age groups, years, etc.)
Can include different covariates in each regression
We demonstrate that most hierarchical and spatial Bayesian models with covariates misrepresent prior information
Better ways of creating Bayesian priors
Produces forecasts and farcasts using considerably more information
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The Statistical Problem of Global Mortality Forecasting

- 779,799,281 deaths, in annual mortality rates
- Multidimensional Data Structures: 24 causes of death, 17 age groups, 2 sexes, 191 countries, all for 50 annual observations.
- One time series analysis for each of 155,856 cross-sections: with 1 minute to analyze each, one run takes 108 days
- Every decision must be automated, systematized, and formalized: the same goal as including qualitative information in the model
- Explanatory variables: Available in many countries: tobacco consumption, GDP, human capital, trends, fat consumption, total fertility rates, etc.
- Numerous variables specific to a cause, age group, sex, and country
- Most time series are very short. A majority of countries have only a few isolated annual observations; only 54 countries have at least 20 observations; Africa, AIDS, & Malaria are real problems
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Each row averages 6,800 forecast errors (17 age groups, 40 countries, and 10 out-of-sample years).

% to best conceivable = % of the way our method takes us from the best existing to the best conceivable forecast.

The new method out-performs with the same covariates, for most countries, causes, sexes, and age groups.

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### Preview of Results: Out-of-Sample Evaluation

#### Mean Absolute Error in Males (over age and country)

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All-Cause Mortality Age Profile Patterns

All Causes (m)

Japan
Turkey
Bolivia

Age
ln(mortality)

0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80

Demographic Forecasting
Existing Method 1: Parameterize the Age Profile

- **Gompertz (1825):** log-mortality is linear in age after age 20
  - Reduces 17 age-specific mortality rates to 2 parameters (intercept and slope)
  - Then forecast only these 2 parameters
  - Reduces variance, constrains forecasts

- Dozens of more general functional forms proposed
- But does it fit anything else?
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Mortality Age Profile: The Same Pattern?

Cardiovascular Disease (m)

ln(mortality) vs Age for France, USA, Brazil
Mortality Age Profile: The Same Pattern?

Breast Cancer (f)

Japan
Venezuela
New Zealand

ln(mortality)
Age
Mortality Age Profile: The Same Pattern?

Other Infectious Diseases (f)

Thailand
Sri Lanka
Barbados
Italy

ln(mortality) vs. Age
Mortality Age Profile: The Same Pattern?

![Graph showing mortality trend across different ages for different countries.](image)
Parameterizing Age Profiles Does Not Work

No mathematical form fits all or even most age profiles
Out-of-sample age profiles often unrealistic
The key empirical patterns are qualitative:
- Adjacent age groups have similar mortality rates
- Age profiles are more variable for younger ages
- We don't know much about levels or exact shapes

Key question: how to include this qualitative information
Also: Method ignores covariate information; the leading current method (McNown-Rogers) not replicable
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- Adjacent age groups have similar mortality rates
- Age profiles are more variable for younger ages
- We don't know much about levels or exact shapes

Key question: how to include this qualitative information

Also: Method ignores covariate information; the leading current method (McNown-Rogers) not replicable
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Deterministic Projections

Data and Forecasts

All Causes (m) USA

Time

Age

Demographic Forecasting

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Deterministic Projections

Data and Forecasts

All Causes (m) USA

Time

Age

1950
2060

Data and Forecasts

All Causes (m) USA

1960 1980 2000 2020 2040 2060

−10 −8 −6 −4 −2

All Causes (m) USA

Time

0
5
10
15
2025
3035 4045
50
55
60
65
70
75
80

1950
2060

0 20 40 60 80

−10 −8 −6 −4 −2

Demographic Forecasting
Existing Method 2: Deterministic Projections

Data and Forecasts

Random walk with drift; Lee-Carter; least squares on linear trend

Pros: simple, fast, works well in appropriate data

Cons: omits covariates; forecasts fan out; age profile becomes less smooth

Does it fit elsewhere?
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Does it fit elsewhere?
The same pattern?
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Random Walk with Drift $\approx$ Lee-Carter $\approx$ Least Squares

Data and Forecasts

Demographic Forecasting

15 / 99
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Data and Forecasts

Suicide (m) USA

Time

Age

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Demographic Forecasting
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Data and Forecasts

Transportation Accidents (m) Portugal
Time

Transportation Accidents (m) Portugal
Age

Demographic Forecasting
Deterministic Projections Do Not Work

Linearity does not fit most time series data.
Out-of-sample age profiles become unrealistic over time.
Deterministic Projections Do Not Work

- Linearity does not fit most time series data
Deterministic Projections Do Not Work

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Regression Approaches (Murray and Lopez, 1996)

Model mortality over countries ($c$) and ages ($a$) as:

$$m_{ca,t} = Z_{c,a,t-\ell} \beta_{ca} + \epsilon_{ca,t},$$

$Z_{c,a,t-\ell} \in \mathbb{R}^{d_{ca}}$: covariates (GDP, tobacco . . . ) lagged $\ell$ years.

$\beta_{ca} \in \mathbb{R}^{d_{ca}}$: coefficients to be estimated.

Cannot estimate equation by equation (variance is too large); pool over countries:

$\beta_{ca} \Rightarrow \beta_{a}$

Properties:
Small variance (due to large $n$)
large biases (due to restrictive pooling over countries),
considerable information lost (due to no pooling over ages)
same covariates required in all cross-sections
Model mortality over countries \((c)\) and ages \((a)\) as:

\[
m_{ca} = Z_{ca,t-\ell} \beta_{ca} + \epsilon_{cat}, \quad t = 1, \ldots, T
\]
Model mortality over countries \((c)\) and ages \((a)\) as:

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Likelihood for equation-by-equation least squares:

\[ P(m | \beta_i, \sigma_i) = \prod_t \mathcal{N}(m_{it} | Z_{it}\beta_i, \sigma_i^2) \]
Partial Pooling via a Bayesian Hierarchical Approach

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- Add priors and form a posterior

\[ P(\beta, \sigma, \theta | m) \propto P(m | \beta, \sigma) \times P(\beta | \theta) \times P(\theta)P(\sigma) \]

\[ = (\text{Likelihood}) \times (\text{Key Prior}) \times (\text{Other priors}) \]
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- Calculate point estimate for \( \beta \) (for \( \hat{y} \)) as the mean posterior:
  \[
  \beta^{\text{Bayes}} \equiv \int \beta P(\beta, \sigma, \theta \mid m) \, d\beta d\theta d\sigma
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- The easy part: easy-to-use software to implement everything we discuss today.
The (Problematic) Classical Bayesian Approach

Assumption: similarities among cross-sections imply similarities among coefficients ($\beta$'s).

Requirements: $s_{ij}$ measures the similarity between cross-section $i$ and $j$.

$(\beta_i - \beta_j)'\Phi(\beta_i - \beta_j) \equiv \|\beta_i - \beta_j\|_2\Phi$ measures the distance between $\beta_i$ and $\beta_j$.

Natural choice for the prior: $P(\beta | \Phi) \propto \exp\left(-\frac{1}{2}\sum_{ij} s_{ij}\|\beta_i - \beta_j\|_2\Phi\right)$
Assumption: similarities among cross-sections imply similarities among coefficients (β's).
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$$P(\beta | \Phi) \propto \exp \left( -\frac{1}{2} \sum_{ij} s_{ij} \|\beta_i - \beta_j\|^2 \Phi \right)$$
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Requires the same covariates, with the same meaning, in every cross-section.

Prior knowledge about $\beta$ exists for causal effects, not for control variables, or forecasting.

Everything depends on $\Phi$, the normalization factor:

$\Phi$ can't be estimated, and must be set. An uninformative prior for it would make Bayes irrelevant, an informative prior can't be used since we don't have information.

Common practice: make some wild guesses.

The classical approach can be harmful: Making $\beta_i$ more smooth may make $\mu$ less smooth ($\mu = Z\beta$):

$$\mu - \mu_{jt} = Z_{it}(\beta_i - \beta_j) + (Z_{it} - Z_{jt})\beta_j$$

Coefficient variation + Covariate variation

Extensive trial-and-error runs, yielded no useful parameter values.
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- Extensive trial-and-error runs, yielded no useful parameter values.
Our Alternative Strategy: Priors on $\mu$

Three steps:

1. Specify a prior for $\mu$:
   \[ P(\mu | \theta) \propto \exp \left( -\frac{1}{2} H[\mu, \theta] \right), \]
   e.g., \[ H[\mu, \theta] \equiv \theta^T \sum_{t=1}^{T} A^{-1} \sum_{a=1}^{A} (\mu_a t - \mu_a + 1, t)^2 \]

2. To do Bayes, we need a prior on $\beta$.
   Problem: How to translate a prior on $\mu$ into a prior on $\beta$ (a few-to-many transformation)?

3. Constrain the prior on $\mu$ to the subspace spanned by the covariates:
   \[ \mu = Z \beta \]

   In the subspace, we can invert $\mu = Z \beta$ as
   \[ \beta = (Z^T Z)^{-1} Z^T \mu, \]
   giving:
   \[ P(\beta | \theta) \propto \exp \left( -\frac{1}{2} H[Z \beta, \theta] \right) \]
   the same prior on $\mu$, expressed as a function of $\beta$ (with constant Jacobian).
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\mathcal{P}(\mu | \theta) \propto \exp\left(-\frac{1}{2} H[\mu, \theta]\right), \text{ e.g., } H[\mu, \theta] \equiv \theta \frac{T}{A-1} \sum_{t=1}^{T} \sum_{a=1}^{A-1} (\mu_{at} - \mu_{a+1,t})^2
$$

   - To do Bayes, we need a prior on $\beta$
   - Problem: How to translate a prior on $\mu$ into a prior on $\beta$ (a few-to-many transformation)?

2. Constrain the prior on $\mu$ to the subspace spanned by the covariates:

$$
\mu = Z\beta
$$
Our Alternative Strategy: Priors on $\mu$

Three steps:

1. Specify a prior for $\mu$:

   $$P(\mu | \theta) \propto \exp \left( -\frac{1}{2} H[\mu, \theta] \right)$$
   
   e.g., $H[\mu, \theta] \equiv \frac{\theta}{T} \sum_{t=1}^{T} \sum_{a=1}^{A-1} (\mu_{at} - \mu_{a+1,t})^2$

   To do Bayes, we need a prior on $\beta$

   Problem: How to translate a prior on $\mu$ into a prior on $\beta$
   (a few-to-many transformation)?

2. Constrain the prior on $\mu$ to the subspace spanned by the covariates:

   $\mu = Z\beta$

3. In the subspace, we can invert $\mu = Z\beta$ as $\beta = (Z'Z)^{-1}Z'\mu$, giving:

   $$P(\beta | \theta) \propto \exp \left( -\frac{1}{2} H[\mu, \theta] \right) = \exp \left( -\frac{1}{2} H[Z\beta, \theta] \right)$$

   the same prior on $\mu$, expressed as a function of $\beta$ (with constant Jacobian).
Say that again?

In other words, any prior information about \( \mu \) (the expected value of the dependent variable) is “translated” into information about the coefficients \( \beta \) via:

\[
\mu_{\text{cat}} = Z_{\text{cat}} \beta_{\text{ca}}
\]

A Simple Analogy

Suppose \( \delta = \beta_1 - \beta_2 \) and \( P(\delta) = N(\delta|0, \sigma^2) \).

What is \( P(\beta_1, \beta_2) \)?

It's a singular bivariate Normal.

It's defined over \( \beta_1, \beta_2 \) and constant in all directions but \((\beta_1 - \beta_2)\).

We start with one-dimensional \( P(\mu_{\text{cat}}) \), and treat it as the multidimensional \( P(\beta_{\text{ca}}) \), constant in all directions but \( Z_{\text{cat}} \beta_{\text{ca}} \).
Say that again?

In other words

Any prior information about $\mu$ (the expected value of the dependent variable) is “translated” into information about the coefficients $\beta$ via

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- What is $P(\beta_1, \beta_2)$?
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- Suppose $\delta = \beta_1 - \beta_2$ and $P(\delta) = N(\delta|0, \sigma^2)$
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- It's defined over $\beta_1, \beta_2$ and constant in all directions but $(\beta_1 - \beta_2)$. 
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Any prior information about $\mu$ (the expected value of the dependent variable) is “translated” into information about the coefficients $\beta$ via

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**A Simple Analogy**

- Suppose $\delta = \beta_1 - \beta_2$ and $P(\delta) = N(\delta|0, \sigma^2)$
- What is $P(\beta_1, \beta_2)$?
- It's a *singular* bivariate Normal
- It's defined over $\beta_1, \beta_2$ and constant in all directions but $(\beta_1 - \beta_2)$.
- We start with one-dimensional $P(\mu_{\text{cat}})$, and treat it as the multidimensional $P(\beta_{\text{ca}})$, constant in all directions but $Z_{\text{cat}} \beta_{\text{ca}}$. 
Advantages of the resulting prior over $\beta$, created from prior over $\mu$

- **Fully Bayesian**: The same theory of inference applies.
- Can use standard Bayesian machinery for estimation.
- $\mu_i$ and $\mu_j$ can always be compared, even with different covariates.
- The normalization matrix $\Phi$ is unnecessary (task is performed by $Z$, which is known).
- Priors are based on knowledge rather than guesses.
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Basic Prior: Smoothness over Age Groups

Prior knowledge: log-mortality age profile are smooth variations of a "typical" age profile $\mu(a)$:

$$H[\mu, \theta] \equiv \theta^T \int^T_0 \int^A_0 \left[ d\mu(a, t) - \bar{\mu}(a) \right]_2$$

Discretize age and time:

$$P(\mu | \theta) \propto \exp\left(-\frac{1}{2} \theta^T \sum_{a a'} \int \left[ \mu(a, t) - \bar{\mu}(a) \right]_W \left[ \mu(a', t) - \bar{\mu}(a') \right]_W \right)$$

where $W_n$ is a matrix uniquely determined by $n$ and $\theta$. 
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Discretize age and time:

$$\mathcal{P}(\mu \mid \theta) \propto \exp \left( - \frac{1}{2} \theta \sum_{aa't} (\mu_{at} - \bar{\mu}_a)' W^n_{aa'} (\mu_{a't} - \bar{\mu}_{a'}) \right)$$
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- Discretize age and time:

$$P(\mu | \theta) \propto \exp \left( - \frac{1}{2} \theta \sum_{aa'} (\mu_{at} - \bar{\mu}_a)' W^n_{aa'} (\mu_{a't} - \bar{\mu}_{a'}) \right)$$

- where $W^n$ is a matrix uniquely determined by $n$ and $\theta$
From a prior on $\mu$ to a prior on $\beta$

Add the specification $\mu$ at $\mu = \bar{\mu} + \mathbf{Z}^\top \mathbf{a}$:

$$P(\beta | \theta) = \exp\left(-\theta \sum \mathbf{a}^\top \mathbf{W}_n \mathbf{a} \left(\mathbf{Z}^\top \mathbf{a} \mathbf{a} \left(\mathbf{Z}^\top \mathbf{a} \mathbf{Z}^\top \mathbf{a} \right)^{-1} \mathbf{Z}^\top \mathbf{a} \right)ight)$$

where we have defined:

$$C_{aa} \equiv \mathbf{Z}^\top \mathbf{a} \mathbf{Z}^\top \mathbf{a}$$

$\mathbf{Z}_a$ is a $T \times d$ data matrix for age group $a$. 
From a prior on $\mu$ to a prior on $\beta$

Add the specification $\mu_{at} = \bar{\mu}_a + Z_{at} \beta_a$:
From a prior on $\mu$ to a prior on $\beta$

Add the specification $\mu_{at} = \bar{\mu}_a + Z_{at} \beta_a$:

$$P(\beta \mid \theta) = \exp \left( -\frac{\theta}{T} \sum_{aa't} W^{n}_{aa'} (Z_{at} \beta_a) (Z_{a't} \beta_{a'}) \right)$$

$$= \exp \left( -\theta \sum_{aa'} W^{n}_{aa'} \beta'_a C_{aa'} \beta_{a'} \right)$$
From a prior on $\mu$ to a prior on $\beta$

Add the specification $\mu_{at} = \bar{\mu}_a + Z_{at} \beta_a$:

$$p(\beta \mid \theta) = \exp \left( -\frac{\theta}{T} \sum_{aa'} W_{aa'}(Z_{at}\beta_a)(Z_{a't}\beta_a') \right)$$

$$= \exp \left( -\theta \sum_{aa'} W_{aa'} \beta_a' c_{aa'} \beta_a' \right)$$

where we have defined:

$$c_{aa'} \equiv \frac{1}{T} Z_a' Z_a' \quad Z_a \text{ is a } T \times d_a \text{ data matrix for age group } a$$
The Prior on the Coefficients $\beta$

The prior is normal (and improper); the prior is uniquely determined by the choice of $n$ through the "interaction" matrix $W_n$. Different age groups can have different covariates: the matrices $C_{aa'}$ are rectangular ($d_a \times d_{a'}$).

The variance of the prior is inversely proportional to $\theta$, which controls the "strength" of the prior.

$$P(\beta | \theta) \propto \exp \left( -\theta \sum_{aa'} W_{aa'}^n \beta'_a C_{aa'} \beta_{a'} \right)$$
The Prior on the Coefficients $\beta$

$$P(\beta \mid \theta) \propto \exp \left( -\theta \sum_{aa'} W_{aa'}^n \beta'_a C_{aa'} \beta_{a'} \right)$$

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The Prior on the Coefficients $\beta$

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\[
\mathcal{P}(\beta \mid \theta) \propto \exp \left( -\theta \sum_{aa'} W_{aa'}^{n} \beta'_a C_{aa'} \beta_{a'} \right)
\]
Samples From Age Prior

All Causes (m), n = 1

Age

Log-mortality

Age

0 20 40 60 80

-8 -6 -4 -2

Demographic Forecasting
Samples From Age Prior

All Causes (m), n = 2

Age

Log-mortality

Age

Log-mortality

0 20 40 60 80

−8 −6 −4 −2
Samples From Age Prior

All Causes (m), n = 3

Age
Log-mortality

Age

Demographic Forecasting
Samples From Age Prior

All Causes (m), n = 1

All Causes (m), n = 2

All Causes (m), n = 3

All Causes (m), n = 4
These priors are "indifferent" to transformations:

\[ \mu(a, t) \mapsto \mu(a, t) + p(a, t) \]

where \( p(a, t) \) is a polynomial in \( a \) (whose degree is the degree of the derivative in the prior).

Prior information is about relative (not absolute) levels of log-mortality.
These priors are “indifferent” to transformations:

$$\mu(a, t) \sim \mu(a, t) + p(a, t)$$
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where $p(a, t)$ is a polynomial in $a$ (whose degree is the degree of the derivative in the prior)
Prior Indifference

- These priors are “indifferent” to transformations:

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- where \( p(a, t) \) is a polynomial in \( a \) (whose degree is the degree of the derivative in the prior)

- Prior information is about relative (not absolute) levels of log-mortality
Formalizing (Prior) Indifference

equal color = equal probability

<table>
<thead>
<tr>
<th>Level indifference</th>
<th>Age</th>
<th>Log-mortality</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 20 40 60 80</td>
<td></td>
<td>-10 -8 -6 -4 -2 0</td>
</tr>
<tr>
<td>All Causes (m), n = 1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Level and slope indifference</th>
<th>Age</th>
<th>Log-mortality</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 20 40 60 80</td>
<td></td>
<td>-8 -6 -4 -2 0 2</td>
</tr>
<tr>
<td>All Causes (m), n = 2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Demographic Forecasting
Formalizing (Prior) Indifference

equal \textcolor{red}{\text{color}} = equal probability

Level indifference
Formalizing (Prior) Indifference

equal color = equal probability

Level indifference

Level and slope indifference
Smoothness Parameter

The prior:

\[ P(\beta | \theta) \propto \exp\left(-\theta \sum a a' W_n a a' C a a' \beta \right) \]

We figured out what \( n \) is, but what is the smoothness parameter, \( \theta \)?

\( \theta \) controls the prior standard deviation.
Smoothness Parameter

The prior:

\[ P(\beta \mid \theta) \propto \exp \left( -\theta \sum_{aa'} W_{aa'}^{n} \beta_a \beta_{a'} C_{aa'}^{n} \right) \]
The prior:

\[ \mathcal{P}(\beta \mid \theta) \propto \exp \left( -\theta \sum_{aa'} W_{aa'}^n \beta_a' C_{aa'} \beta_{a'} \right) \]

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$$P(\beta | \theta) \propto \exp \left( -\theta \sum_{aa'} W^n_{aa'/\beta_a' c_{aa'/\beta_a'}} \right)$$

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Smoothness Parameter

The prior:

\[ P(\beta \mid \theta) \propto \exp \left( -\theta \sum_{aa'} W_{nn'}^{aa'} \beta_{aa'}^\prime C_{aa'} \beta_{aa'}^\prime \right) \]

- We figured out what \( n \) is
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- \( \theta \) controls the prior standard deviation
Samples from Age Prior

All Causes (f), n = 2

Age

Log-mortality

Age
Samples from Age Prior

All Causes (f), n = 2
Samples from Age Prior

All Causes (f), n = 2

Age

Log-mortality

0 20 40 60 80
−10 −8 −6 −4 −2 0 2

All Causes (f), n = 2

Age
Samples from Age Prior

All Causes (f), n = 2

![Graph showing the relationship between log-mortality and age for all causes of death for females with n = 2. The graph shows a trend of increasing log-mortality with age.]
Samples from Age Prior

All Causes (f), n = 2
Samples from Age Prior

All Causes (f), n = 2

Log-mortality vs Age

Demographic Forecasting
Samples from Age Prior

All Causes (f), n = 2

Log-mortality vs. Age

Demographic Forecasting
Samples from Age Prior

All Causes (f), n = 2

Log-mortality vs Age

Demographic Forecasting
Samples from Age Prior

All Causes (f), n = 2

Log-mortality vs. Age

Demographic Forecasting
Samples from Age Prior

All Causes (f), n = 2

Log-mortality vs Age
Samples from Age Prior

All Causes (f), n = 2

Log-mortality

Age
Samples from Age Prior

All Causes (f), n = 2

Log-mortality vs. Age

Demographic Forecasting
Generalizations

The above tools: smooth over a (possibly discretized) continuous variable — age or age groups. We can also smooth over time (also a discretized continuous variable). Can smooth when cross-sectional unit \( i \) is a label, such as country. Can smooth simultaneously over different types of variables (age, country, and time). We can smooth interactions:

- Smoothing trends over age groups.
- Smoothing trends over age groups as they vary across countries, etc.

The mathematical form for all these (separately or together) turns out to be the same:

\[
P(\beta | \theta) \propto \exp \left( \sum_{ij} W_{ij} \beta_i C_{ij} \beta_j \right),
\]

where \( C_{aa'} \equiv 1 \).
Generalizations

- The above tools: smooth over a (possibly discretized) continuous variable — age or age groups.
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- The above tools: smooth over a (possibly discretized) continuous variable — age or age groups.
- We can also smooth over time (also a discretized continuous variable).

The mathematical form for all these (separately or together) turns out to be the same:

$$P(\beta | \theta) \propto \exp \left( -\theta \sum_{ij} W_{ij} \beta_i C_{ij} \beta_j \right),$$

where $C_{aa} \equiv 1_T Z_a Z_a'$.
Generalizations

- The above tools: smooth over a (possibly discretized) continuous variable — age or age groups.
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The mathematical form for all these (separately or together) turns out to be the same:

$$P(\beta | \theta) \propto \exp \left[ -\theta^2 \sum_{ij} W_{ij} \beta_i' C_{ij} \beta_j \right], \quad C_{aa'} \equiv \frac{1}{T} Z_a Z_{a'}'$$
Generalizations

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\[C_{aa} = \frac{1}{T} Z_a Z_a'.\]
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\]
Mortality from Respiratory Infections, Males

Least Squares

Data and Forecasts

(m) Belize

1970

2030

Age

Data and Forecasts

(m) Belize

1970

2030

Age

Data and Forecasts

(m) Belize

1970

2030

Age
Mortality from Respiratory Infections, males, $\sigma = 2.00$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 1.51$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 1.15$

Smoothing over Age Groups

Data and Forecasts

Demographic Forecasting
Mortality from Respiratory Infections, males, \( \sigma = 0.87 \)

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 0.66$

Smoothing over Age Groups

Data and Forecasts

(m) Belize

Age

Demographic Forecasting
Mortality from Respiratory Infections, males, $\sigma = 0.50$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 0.38$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 0.28$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 0.21$

Smoothing over Age Groups

Data and Forecasts

(m) Belize

Age

0 20 40 60 80

−12 −10 −8 −6 −4

Data and Forecasts

1970 2030

0 20 40 60 80

−12 −10 −8 −6 −4

(m) Belize

1970 2030

0 20 40 60 80

−12 −10 −8 −6 −4

Demographic Forecasting
Mortality from Respiratory Infections, males, $\sigma = 0.16$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 0.12$

Smoothing over Age Groups

(m) Belize

(Data and Forecasts 1970 2030)

(Age) Demographic Forecasting
Mortality from Respiratory Infections, males, $\sigma = 0.09$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 0.07$

Smoothing over Age Groups

![Data and Forecasts](image-url)
Mortality from Respiratory Infections, males, $\sigma = 0.05$

Smoothing over Age Groups

![Graph showing mortality from Respiratory Infections over age groups from 1970 to 2030 in Belize.](image)
Mortality from Respiratory Infections, males, $\sigma = 0.04$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 0.03$

Smoothing over Age Groups

![Graph showing mortality data and forecasts for Belize from 1970 to 2030, with age ranges on the x-axis and mortality rates on the y-axis.](Image)
Mortality from Respiratory Infections, males, $\sigma = 0.02$

Smoothing over Age Groups

![Graph showing mortality data and forecasts for Belize from 1970 to 2030.](image-url)
Mortality from Respiratory Infections, males, $\sigma = 0.01$

Smoothing over Age Groups
Mortality from Respiratory Infections, males

Least Squares

Data and Forecasts

(m) Belize

Time


-12 -10 -8 -6 -4

50 55 60 65 70 75 80
Mortality from Respiratory Infections, males, $\sigma = 2.00$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, \( \sigma = 1.51 \)

Smoothing over Age Groups

![Graph](image-url)
Mortality from Respiratory Infections, males, $\sigma = 1.15$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 0.87$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 0.66$

Smoothing over Age Groups

Data and Forecasts

Time

(m) Belize

Demographic Forecasting
Mortality from Respiratory Infections, males, $\sigma = 0.50$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 0.38$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 0.28$

Smoothing over Age Groups

![Graph showing mortality data and forecasts for Belize over time from 1970 to 2030.](image)
Mortality from Respiratory Infections, males, $\sigma = 0.21$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 0.16$

Smoothing over Age Groups

![Graph showing mortality data and forecasts over time for Belize](image-url)
Mortality from Respiratory Infections, males, $\sigma = 0.12$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 0.09$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 0.07$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 0.05$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 0.04$

Smoothing over Age Groups

Data and Forecasts
Mortality from Respiratory Infections, males, $\sigma = 0.03$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 0.02$

Smoothing over Age Groups
Mortality from Respiratory Infections, males, $\sigma = 0.01$

Smoothing over Age Groups
Smoothing Trends over Age Groups
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Log-mortality in Belize males from respiratory infections
Smoothing Trends over Age Groups

Log-mortality in Belize males from respiratory infections

Least Squares
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Least Squares

Smoothing Age Groups
Smoothing Trends over Age Groups and Time

Demographic Forecasting
Smoothing Trends over Age Groups and Time
Log-Mortality in Bulgarian males from respiratory infections
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Smoothing
Age and Time
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Log-Mortality in Bulgarian males from respiratory infections

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Least Squares

Smoothing Age and Time
Using Covariates (GDP, tobacco, trend, log trend)
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Lung cancer in Korean Males
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Lung cancer in Korean Males

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Smooth over age, time, age/time
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Smooth over age, time, age/time
What about ICD Changes?
Fixing ICD Changes

Other Infectious Diseases: USA, age 0 (m)

Other Infectious Diseases: France, age 0 (m)

Other Infectious Diseases: Australia, age 0 (m)

Other Infectious Diseases: United Kingdom, age 0 (m)
http://GKing.Harvard.edu
With Country Smoothing

Transportation Accidents (males) Sri Lanka

BAYES

Demographic Forecasting
Formalizing Similarity

Standard Bayesian Approach

- Assume coefficients are similar
- But we know little about the coefficients
- Requires the same covariates in each cross-section
  - Why measure water quality in the U.S.?
  - Requires covariates with the same meaning in each cross-section
  - Does GDP mean the same thing in Botswana and the U.S.?
- Imposes no assumptions on covariates or mortality
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Alternative Approach

- Assume expected mortality is similar
- Coefficients are unobserved, mortality patterns are well known
- Different covariates allowed in each cross-section
- Covariates with the same name can have different meanings

Demographic Forecasting
Formalizing Similarity

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Assume coefficients are similar — But we know little about the coefficients

Requires the same covariates in each cross-section — Why measure water quality in the U.S.?

Requires covariates with the same meaning in each cross-section — Does GDP mean the same thing in Botswana and the U.S.?

Imposes no assumptions on covariates or mortality — If covariates are dissimilar, then making coefficients similar makes mortality dissimilar \( \text{since } E(y_t) = X_t \beta \text{ in each cross-section} \)

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Demographic Forecasting
## Formalizing Similarity

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<th>Mean Absolute Error</th>
<th>% Improvement</th>
<th>% to Best Conceivable</th>
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Each row averages 6,800 forecast errors (17 age groups, 40 countries, and 10 out-of-sample years).

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## Preview of Results: Out-of-Sample Evaluation

Mean Absolute Error in Males (over age and country)

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<tr>
<td>All-Cause</td>
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<td>0.15</td>
<td>0.08</td>
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<tr>
<td>Suicide</td>
<td>0.31</td>
<td>0.29</td>
<td>0.18</td>
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<tr>
<td>Respiratory Infectious</td>
<td>0.49</td>
<td>0.47</td>
<td>0.28</td>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

- Each row averages 6,800 forecast errors (17 age groups, 40 countries, and 10 out-of-sample years).
- % to best conceivable = % of the way our method takes us from the best existing to the best conceivable forecast.
- The new method out-performs with the same covariates, for most countries, causes, sexes, and age groups.
<table>
<thead>
<tr>
<th>Category</th>
<th>Mean Absolute Error</th>
<th>% Improvement</th>
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<td>Best Previous</td>
<td>Our Method</td>
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<tr>
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<tr>
<td>Lung Cancer</td>
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<td>Transportation</td>
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<tr>
<td>Other Infectious</td>
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<td>Stomach Cancer</td>
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<tr>
<td>All-Cause</td>
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<td>0.15</td>
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</tbody>
</table>

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- \% to best conceivable = \% of the way our method takes us from the best existing to the best conceivable forecast.
- The new method out-performs with the same covariates, for most countries, causes, sexes, and age groups.
- Does much better with better covariates.