Rethinking the Vote
The Politics and Prospects of American Election Reform

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14. Given that many counties in North Dakota contained fewer than fifteen precincts, that we had to obtain vote returns separately from the counties’ Boards of Elections, and that many counties failed to cooperate with our requests for vote returns, we worked to obtain data from enough counties to yield at least fifty precincts for each of the seven rotation orders for the race for U.S. president. Of the fifty-three counties in the state, we ended up with data from fourteen of the sixteen largest (excluding one county that did not rotate name order and one county that was unresponsive to repeated requests for vote returns).

15. The prevalence of primary effects also appears if we examine the direction of the nonsignificant effects. Of the 131 nonsignificant order effects, 90, or 69%, of them were in the direction of primary effects, and 31% were in the direction of recency effects. A sign test indicates that this is highly unlikely (p < .001) to have occurred by chance alone. Moreover, the average magnitude of the nonsignificant two-candidate primary effects (90%) was 24% greater than that of the nonsignificant two-candidate recency effects (73%). This leaning toward primary effects among the nonsignificant differences is unlikely to have occurred by chance alone and therefore suggests that there were more real primary effects in these races than we had statistical power to detect.

16. Characterizing the directions of name order effects in these races is a bit complex because the effect may not be monotonic. Although simple primacy or recency effects could certainly occur, a candidate could get more votes when listed first or last than when listed in the middle of an array. This would be what we will refer to as a “primacy and recency” effect. It is also conceivable that a candidate might get more votes when listed in the middle of an array than when listed first or last. This is what we will refer to as a “middle” effect.

17. This trend toward primacy effects was apparent even in the instances in which differences between name orders were not statistically significant or marginally so. Of the 86 nonsignificant order effects in races involving more than two candidates, 63 (or 73%) were in the direction of primary effects, and only 23 (or 27%) were in the direction of recency effects. A sign test again indicates that this is extremely unlikely (p < .001) to have occurred by chance alone. The average magnitude of the difference between the first and last positions was larger for the nonsignificant primary effects (65%) than the nonsignificant recency effects (54%).

18. Significant name order effects appeared in these analyses less often than in those reported by Miller and Krosnick (1998). This is most likely attributable importantly to the greater prevalence of partisan races among the states examined here (listing party affiliations of candidates on ballots reduces the likelihood of primary effects; see Miller and Krosnick 1998) and to the smaller sample sizes examined here for most races. For example, whereas 79% of the two-candidate races in Ohio analyzed here were partisan, only 57% of the two-candidate races analyzed by Miller and Krosnick (1998) were partisan. And whereas the average sample size for the two-candidate races analyzed here (excluding the two statewide races) was 443, the average sample size was 1,152 for the two-candidate races analyzed by Miller and Krosnick (1998).

INTRODUCTION

The 2000 U.S. presidential election rekindled interest in possible electoral reform. While most of the popular and academic accounts focused on ballotting irregularities in Florida, such as the now infamous “butterfly” ballot and mishandled absentee ballots, some also noted that this election marked only the fourth time in history that the candidate with a plurality of the popular vote did not also win the Electoral College (Posner 2001). This “anti-democratic” outcome has fueled desire for reform or even outright elimination of the Electoral College (Barnes, this volume; Crigler, Just, and Buhr, this volume; Ortíz, this volume).

This is not the first time that controversy has surrounded the Electoral College system. Perhaps the most scandalous presidential election in U.S. history was the 1876 race between Samuel J. Tilden, a Democrat, and Rutherford B. Hayes, a Republican. The nation was deeply divided; the rifts were caused partly by a deep economic recession and partly by a seemingly endless number of scandals involving graft and corruption in the incumbent Republican administration of Ulysses S. Grant. Making matters even more divisive were a number of third parties that contested the election. Similar to the 2000 elections, the outcome hinged on resolving potential vote count problems in Florida, Louisiana, and South Carolina. These states were so divided, as was the rest of the country, that they sent two slates each of electors to Congress—one set for Tilden and one set for Hayes. Congressional procedures for resolving disputed sets of electors had expired; Congress therefore established a fifteen-member commission to decide the issue of which set of electors to use. After much intrigue, the commission narrowly voted to use the electors for Hayes from all three disputed states, thus giving him the election. Hayes won...
the election despite the consensus that Tilden had won 51 percent of the popular vote to Hayes's 48 percent.

The Electoral College vote also went contrary to the popular vote in the 1888 election between the incumbent President Grover Cleveland and his Republican challenger, Benjamin Harrison. Cleveland garnered huge majorities in the eighteen states that supported him, whereas Harrison won slender majorities in some of the larger states that supported him. In the end, Cleveland won the popular vote by about 110,000 votes—constituting less than 1 percent of the total vote—but lost the Electoral College.

The other election in which the popular vote leader did not become president was in 1824, when Andrew Jackson won a plurality of both the popular and electoral votes. But because Jackson did not win an electoral vote majority, the election was decided by the House of Representatives, which voted for John Quincy Adams.

The popular vote was even closer in the 2000 election than in the 1888 election. Gore won the popular vote by approximately 541,000 votes—or about half a percentage point of the total vote cast—but, as with both Cleveland and Tilden before him, lost the Electoral College.

Most arguments against the Electoral College have either been based on these particular elections or on highly stylized formal models (see, for example, Banzhaf 1968). We take a different approach here. We develop a set of statistical models based on historical election results to evaluate the Electoral College as it has performed in practice. While we do not directly address the normative question of the value of the U.S. Electoral College, this chapter does provide the necessary tools and evidence to make such an evaluation.

There are two fundamental ways that the Electoral College could be flawed. First, it may be biased in favor of one party. That is, the distribution of votes could have a party's candidate systematically winning the popular vote but losing the Electoral College. For example, if it is likely that the Democratic candidate is to win with overwhelming majorities in a few states, then this will boost the overall Democratic vote share but not necessarily the odds of his or her winning the Electoral College. This is essentially what happened in the elections of 1876, 1888, and 2000. In order to know if this is a general problem, we need to ascertain the relationship between the average vote share a party's candidate receives and the likelihood of winning a majority of the Electoral College. Here we develop a statistical model based on an extension of Gelman and King's (1994) model of legislative elections to the case of the Electoral College.

The second possible effect of the Electoral College is on the voting power of individual citizens—that is, their influence on the outcome. A natural measure of voting power is the probability that a vote will be pivotal in determining the outcome of an election. This is the basis for almost all of the voting power measures considered in the literature (see Felsenthal and Machover 1998; Straffin 1978, for reviews). If the president were elected by straight popular vote, this measure would be the probability that one's vote would break a tie in favor of a candidate. Under the popular vote system every voter (ex ante) has an equal, but small, chance of casting the deciding vote and therefore has equal voting power.

Further, the popular vote system maximizes the average voting power across the electorate (see e.g., Felsenthal and Machover 1998; Gelman and Katz 2001). The Electoral College, on the other hand, divides the electorate into predetermined groups or coalitions by state. The states give all of their electoral votes to a given candidate based on majority vote in the state. Thus, a vote is pivotal both if it determines how the state's electoral votes are cast and if those electoral votes determine the winner in the Electoral College. Since states vary both in their sizes and in the likelihood of ties, voters in different states will, in general, have different voting power, and as a result the average voting power can be less than under plurality rule. In fact, this is the central critique raised by Banzhaf (1968). Again using the statistical model of presidential elections that we develop here, we can examine the empirical probability that an average voter is pivotal under both popular vote and Electoral College systems.

We show that after appropriate statistical analysis of the available historical electoral data, there is little basis to argue for reforming the Electoral College. We first show that while the Electoral College may once have been biased against the Democrats, the current distribution of voters advantages neither party. Further, the electoral vote will differ from the popular vote only when the average vote shares of the two major candidates are extremely close to 50 percent. As for individual voting power, we show that while there has been much temporal variation in relative voting power over the last several decades, the voting power of individual citizens would not likely increase under a popular vote system of electing the president.

The chapter proceeds as follows. In the next section we consider the partisan impact of the Electoral College. The following section examines the impact on average voting power, including what voting power would have been if electoral votes were allocated by congressional district or by a popular vote system. The final section concludes the chapter.

MEASURING THE PARTISAN IMPACT OF THE ELECTORAL COLLEGE

Most of the popular critiques of the Electoral College have focused on the possibility that the winner of the popular vote will not win the electoral vote, such as occurred in the elections of 1876, 1888, and 2000. That this discrepancy occurred is not surprising because the Electoral College and the popular vote are different electoral systems. In fact, the Constitutional Convention considered several alternative methods for electing the president, including direct popular vote, and rejected them all in favor of the Electoral College. The Convention delegates were concerned that elections would be dictated by the most populous states with little regard for the smaller ones (see also Madison 1788, Federalist No. 69, for further arguments in favor of the Electoral College system). This was thought particularly likely to occur if one of the presidential
candidates was a "favorite son" from one of the larger states, who would thus
draw large support from only one state or region but would still be able to win
the popular vote.

If a primary rationale for eliminating the Electoral College is that the re-

sults may be contrary to the outcome of the popular vote, we ought to know
how likely the popular vote.
draw large support from only one state or region but would still be able to win

tions). However, we can investigate the relationship between popular vote and
the probability that a party's candidate wins the Electoral College. We can, for
example, examine what popular vote share would be needed for the Demo-
cratic candidate to win the Electoral College 50 percent of the time, or even 95
percent of the time. If the vote needed to win 50 percent of the time were sub-
stantially greater than half of the popular vote, we would know that the dis-
tribution of votes across the states disadvantaged the Democrats. The Electoral
College would be biased in favor of the Republicans in this case.

How might the Electoral College favor one of the parties? Think of the Ele-
toral College as a legislative district map. A central concern about any dis-
tricting is how it affects the translation of votes into seats. The study of this
translation in political science is based on the idea of a seats-votes curve, which
has appeared in the academic literature for almost a century (see, e.g., Kendall
and Stuart 1950). A seats-votes curve is a mapping, stating for a given party's
average district vote what fraction of the seats they will receive. The Electoral
College represents just a slightly more complicated seats-votes curve. We
are interested in knowing the relationship between average popular vote and elec-
toral vote. The complication is that the districts in our case—that is, the states—
have different numbers of seats—that is, electoral seats.

As is well known, a legislative districting map may be a gerrymander in
favor of a particular party (Cox and Katz 2002). The classic way to engineer
such a partisan gerrymander is to pack as many of the other party's supporters
in as few districts as possible, thereby creating inefficiently safe districts,
while spreading one's own supporters across as many districts as possible,
thereby creating winnable but not inefficiently safe districts (see Cox and Katz
1999, 2002, for detailed discussion of this and other methods of partisan gerrymandering). Opposition voters in these districts are wasting their votes: They
are contributing to increasing the statewide vote share for their party but are
not increasing the number of seats they win. In the case of legislative maps,
this gerrymander is often intentional. With the Electoral College, such gerrym-
andering is not likely intentional because the allocation was set in the Con-
stitution, not by partisan state governments, as is the case with legislative maps.
Nevertheless, it could be the case that the distribution of voters across states
could create the conditions for a gerrymander favoring one of the parties in
the presidential election. If, for example, many Southern states vote over-
whelmingly for the Democrat, then the Democratic candidate may do well in
the popular vote but still may not win the Electoral College.

If we look at the 2000 election, the distribution of the popular vote was
problematic for Gore. Consider New York, the second most populous state,
average 51 percent of the two-party vote, the results from the Electoral College and popular vote will agree.

There is, however, a potential problem with the analysis presented in Figure 5.1. Implicitly, we are conditioning the findings presented there on turnout because the total vote is just the simple average across the states. However, if there are wide turnout differentials between states, this may mask some findings. As an alternative, we also ran the analysis constructing a national average by weighting state vote shares according to their actual population. These results appear in Figure 5.2.

We see some differences between the two figures. Most notably, early on the Electoral College seems somewhat biased against the Democrats, especially during the 1950s. At that time, a Democratic candidate needed larger popular vote shares to guarantee high chances of winning the election. This happened because the Democrats were winning Southern states with huge margins, and so they had lots of "wasted votes." The effect has reduced over time because the Republicans have started to win Southern states. This was not as obvious in Figure 5.1 because turnout was also much lower in the South, counteracting the wasted votes.

Thus, we see some evidence that in the 1940s and 1950s, the Electoral College could have led to results contrary to the popular vote, although it in fact did not. Under current conditions, this gap could only plausibly happen if the popular vote is extremely close, as it was in the 2000 election. In the 1980s, commentators talked about a Republican "lock" on the Electoral College, but really what was happening was that Republicans were winning presidential elections by getting many more popular votes than the Democrats. The Electoral College per se had nothing to do with it, as is clear from the segments of the graphs in Figures 5.1 and 5.2 showing the 1980s.

**ELECTORAL COLLEGE AND VOTING POWER**

In this section we explore how coalitional behavior induced by the Electoral College affects the probability that a given voter is decisive in an election, a natural measure to evaluate an electoral system.

The probability of a vote's being decisive is important directly because it represents a voter's influence on the electoral outcome, and this influence is crucial in a democracy, and also indirectly because it could influence campaigning. For example, one might expect campaign efforts to be proportional to the probability of a vote being decisive, multiplied by the expected number of votes changed per unit of campaign expense, although there are likely strategic complications since both sides are making campaign decisions (Brams and Davis 1974). The probability that a single vote is decisive in an election is also relevant in determining the utility of voting, the responsiveness of an electoral system to voter preferences, the efficacy of campaigning efforts, and comparisons of voting power (Aldrich 1993; Brams and Davis 1975; Ferejohn and Fiorina 1974; Riker and Ordeshook 1986; Norris, this volume).

Perhaps the simplest measure of decisiveness is the (absolute) Banzhaf (1965) index, which is the probability that an individual vote is decisive under the assumption that all voters are deciding their votes independently and at random, with probabilities 0.5 for each of two candidates. We shall refer to this
assumption as the random voting model. While clearly an unrealistic assumption, it does provide a benchmark to evaluate competing electoral rules and thereby make the problem theoretically tractable. The random voting model is, of course, a gross oversimplification, and in fact its implications for voting power in U.S. elections have been extremely misleading in the political science literature, as has been discussed by Gelman, King, and Boscardin (1998) and Gelman, Katz, and Baumberg (2002).

We offer an alternative approach here based on the empirical analysis of U.S. presidential elections. We use results from every election since 1960 as the basis for a set of simulations to calculate the average probability that a given voter is decisive under the popular vote, the Electoral College, and an alternative system in which each congressional district is worth one electoral vote.

**How the Electoral College Might Affect Voting Power**

Before continuing on it is probably best to consider how the Electoral College might affect voting power. In order to do this we need to consider a couple of different electoral systems. The simplest electoral system is the popular vote or majority rule, under which the candidate receiving the largest number of votes wins. On the other hand, the U.S. presidential system is an electoral vote or local winner-take-all rule, in which voters are grouped into several coalitions and the winner in each coalition gets a fixed number of “electoral votes” (with these electoral votes split or randomly assigned in the event of an exact tie within the coalition); the candidate with the most electoral votes is declared the winner.

In a popular vote system, the probability of one’s vote being decisive can be approximated with standard statistical techniques, given the number of voters, and assuming that the random voting model holds. It is just the chance that exactly half the electorate is voting for one candidate and the other half is voting for the other candidate. This probability declines as the number of voters gets larger.

Now consider the Electoral College system, in which the members of a state separately vote to allocate all of their electoral votes to the candidate that wins the majority of the state’s vote.

Under the random voting model, it is best to be in a large state. At one extreme, suppose a single state controlled a majority of the electoral votes. Then this state determines the election outcome, and if one is in that state, then her vote is decisive with approximate probability

$$\frac{2\sqrt{2}}{\pi} \times \left( \frac{n}{2} \right)^{-1/2},$$

which is approximately $\sqrt{2}$ times the probability of being decisive under the popular vote system. However, the $n - 1/2$ voters from other states have zero voting power. Therefore, it can be shown that the average probability of a decisive vote, averaging over all voters, is smaller than that under the popular vote system.
A. No Coalitions
A voter is decisive if the others are split 4-4:

\[
\Pr(\text{Voter is decisive}) = \binom{4}{2} \left( \frac{1}{2} \right)^4 = 0.273
\]

Average \( \Pr(\text{Voter is decisive}) = 0.273 \)

B. A Single Coalition of Three Voters
A voter in the coalition is decisive if others in the coalition are split 1-1 and the coalition is decisive:

\[
\Pr(\text{decisive}) = \frac{2}{2} \cdot 0.273 = 0.391
\]

A voter not in the coalition is decisive:

\[
\Pr(\text{decisive}) = \binom{5}{2} \left( \frac{1}{2} \right)^5 = 0.156
\]

Avg. \( \Pr(\text{decisive}) = \frac{1}{3} (0.391) + \frac{1}{3} (0.156) = 0.234 \)

C. A Single Coalition of Five Voters
A voter in the coalition is decisive if others in the coalition are split 2-2:

\[
\Pr(\text{decisive}) = \frac{4}{2} \left( \frac{1}{2} \right)^4 = 0.375
\]

A voter not in the coalition can never be decisive:

\[
\Pr(\text{decisive}) = 0
\]

Avg. \( \Pr(\text{decisive}) = \frac{2}{3} (0.375) + \frac{1}{3} (0) = 0.208 \)

D. Three Coalitions of Three Voters Each
A voter is decisive if his coalition is split 1-1 and the other two coalitions are split 1-1:

\[
\Pr(\text{decisive}) = \frac{1}{2} \cdot \frac{1}{2} = 0.250
\]

Avg. \( \Pr(\text{decisive}) = 0.250 \)

Figure 5.3 Coalitions and individual voting power

We consider a different approach to modeling whether a single vote is decisive. Consider a two-candidate election with majority rule in any given jurisdiction. Let \( V \) be the proportional vote differential (that is, the difference between the Democrat's and Republican's vote totals, divided by the number of voters, \( n \)). If a particular voter votes, that will add \( +1/n \) or \( -1/n \) to \( V \); the decisiveness of this vote is 0 if \( |V| > 1/n \), 1/2 if \( |V| = 1/n \), or 1 if \( V = 0 \).

Now suppose that the proportional vote differential has an approximate continuous probability distribution, \( p(V) \). This distribution can come from a theoretical model of voting (for example, the random voting model) or from empirical models based on election results or forecasts. Gelman et al. (1998) argue that, for modeling voting decisions, it is appropriate to use probabilities from forecasts, since these represent the information available to the voter before the election occurs. For retrospective analysis, it may also be interesting to use models based on perturbations of actual elections, as in Gelman and King (1994). In any case, all that is needed here is some probability distribution.

For any reasonably sized election, we can approximate the distribution \( p(V) \) of the proportional vote differential by a continuous function. In that case, the expected probability of decisiveness is simply \( 2p(V)/n \) evaluated at the point \( V = 0 \). For example, in a two-candidate election with ten thousand voters, if one candidate is forecast to get 54 percent of the vote with a standard error of 3 percent, then the vote differential is forecast at 8 percent with a standard error of 6 percent. The probability that an individual vote is decisive is then:

\[
\frac{1}{10000} \sqrt{2\pi(0.06)} \exp\left(-\frac{1}{2} \left(0.08/0.06\right)^2\right) = 0.0055
\]

using the statistical formula for the normal distribution.

The same ideas apply to more complicated elections, such as multicandidate contests, runoffs, and multistage systems (e.g., the Electoral College in the United States or the British parliamentary system, in which the goal is to win a majority of individually elected seats). In more complicated elections, it is simply necessary to specify a probability model for the entire range of possible outcomes and then work out the probability of the requisite combination events under which a vote is decisive. For example, in the Electoral College, one's vote is decisive if her state is tied (or within one vote of being tied) and if, conditional on this state being tied, no candidate has a majority based on the other states. Estimating the probability of this event requires a model for the joint distribution of the vote outcomes in all the states.

In order to estimate probabilities of close elections and decisiveness, it is therefore necessary to set up a probability model for vote outcomes. We want to go beyond the random voting model to set up a more realistic descriptor of vote outcomes. Gelman et al. (1998) fit a state-by-state election-forecasting model, with probabilities corresponding to the predictive uncertainty two months before the election. Here we use a simpler approach: We take the actual election outcome and perturb it, to represent possible alternative outcomes.
We label \( n_i \) as the observed outcome (the Democratic candidate's share of the two-party vote) in congressional district \( i \) in a given election year and obtain a probability distribution of hypothetical election outcomes \( y_i \) by adding normally distributed random errors at the national, regional, state, and congressional district levels, with a standard deviation of 2 percent at each level. We label \( n_i \) as the turnout in each district \( i \) and consider these as fixed—this is reasonable since uncertainty about election outcomes is driven by uncertainty about \( n \).

For any given election year, we use the multivariate normal distribution of the vector \( n \) of vote outcomes to compute the probability of a single vote's being decisive in the election. For the popular vote system, we determine this probability for any voter; for the electoral vote and congressional district vote systems, we determine the probability within each state or district and then compute national average probabilities, weighing by turnouts within states or districts. The actual probability calculations are done using the multivariate normal distribution described by Gelman et al. (1998).

Our results appear in Figure 5.4. The most striking feature of the figure is that the average probability of decisiveness changes dramatically from year to year but is virtually unaffected by changes in the electoral system. This may come as a surprise—given the theoretical results from the prior section, one might expect the average probability of decisiveness to be much higher for the popular vote system.

The results in Figure 5.4 are only approximate, not just because of the specific modeling choices made, but also because of the implicit assumption that the patterns of voting would not be affected by changes in the electoral system. For example, states such as California and Texas that were not close in the 2000 election might have had higher turnout under a popular vote system in which all votes counted equally. Thus, our results compare different electoral systems as applied to the actual observed votes and do not directly address counterfactual questions about what would happen if the electoral system were changed.

However, given these caveats, there does not seem to be strong evidence from Figure 5.4 to argue for moving from the Electoral College to a popular vote system. The average voter is not likely to affect the outcome of the presidential election under any of the proposed methods. Even in close elections, such as in 2000, a voter is more likely to win his or her state's lottery—or be struck by lightning—than to cast the deciding ballot. Further, there is not even that much difference in the voting power across the systems, even if the magnitudes were substantial.

CONCLUSIONS

We have presented statistical methodology to evaluate the two most common complaints against the Electoral College. When subjected to proper empirical analysis, neither of the complaints seem well justified.

With regard to the potential partisan bias in the Electoral College, we find no systematic effect, at least not in current elections (that is, those after the 1950s). We did find that it is possible for the Electoral College system to lead to different outcomes than the popular vote, but only when the nationwide vote is very close, say within half a percentage point, for the two top candidates. As we now know from the election of 2000, when elections are that close slight differences in electoral procedure can and will lead to different outcomes.

Our results about voting power and the Electoral College are perhaps a bit more surprising. There is a fairly significant theoretical literature suggesting that the Electoral College was unfair to certain voters. However, these claims are based on the random voting model, which is a highly stylized model of elections. When we looked at the empirical voting behavior, we do not find much difference between voting power under the Electoral College or popular vote systems. But these results are more tentative because in making our analysis we assumed that neither voters nor candidates would have behaved differently under a popular vote system. This is a suspect assumption, but less so than the random voting model that underlies the critique.

Of course, our empirical findings do not directly address normative questions, such as which electoral system should be used. However, our findings can be used to evaluate the positive claims that underlie the normative arguments.

NOTES

1. Actually two states, Nebraska and Maine, allocate Electoral College votes by congressional district.
Part I  Problems That Might Need To Be Fixed

2. A bit more formally, our probability model comes from a random components linear regression of democratic vote in state \( i \) in year \( t \), \( v_{i,t} \), on a set of observable regressors, \( X_{i,t} \).

\[ v_{i,t} = X_{i,t}\beta_i + \gamma_i + e_{i,t} \]

where \( \beta_i \) is a vector of \( k \) parameters that must be estimated from the data and \( \gamma_i \) and \( e_{i,t} \) are independent error terms. We further assume that both error terms are normally distributed with mean zero and variances that are estimated from the data. This implies that the vote shares themselves are also normally distributed around the expected conditional mean defined in the equation 1.

3. Formally, the probability model of the hypothetical vote proportions, \( \phi_{i,t} \), is determined by the analogous probability model:

\[ \phi_{i,t} = X_{i,t}\beta_i + \delta_{i,t} + \gamma_i + e_{i,t} \]

where \( \delta_{i,t} \) is a known constant used to model statewide partisan swings. If we want to have the average vote equal to some value, \( \bar{v} \), then this constant can be calculated by:

\[ \delta_{i,t} = \bar{v} - \sum_{i=1}^{N} (X_{i,t}\beta_i)T_{i,t}/\sum_{i=1}^{N} T_{i,t} \]

where \( N \) is the number of states and \( T_{i,t} \) is the voter turnout in state \( i \) in year \( t \).

4. Formally, in a popular vote system with \( n \) voters, the probability that your vote is decisive is \( (n - 1(n - 1)/2) \cdot 2^{-n} \); that is, the probability that \( x = n - 1/2 \) where \( x \) has a binomial distribution with parameters \( n - 1 \) and \( 1/2 \). For large (or even moderate) \( n \), this can be well approximated using the normal distribution as \( \sqrt{2/n}n^{-1/2} \), a standard result in probability (c.f. Woodroofe 1975).

5. Formally, the probability that an average voter is decisive under the Electoral College is \( \sqrt{1/nn^{-1/2}} \), which a factor of \( \sqrt{2} \) less than under the popular vote system.

6. We could also consider more elaborate cases. For example, suppose there are \( n = 3^d \) voters, where \( d \) is some integer, who are divided into three equal-sized coalitions, each of which is itself divided into three coalitions, and so forth, in a tree structure. Then all the \( n \) voters are symmetrically situated, and a given voter is decisive if the other two voters in his or her local coalition are split—this happens with probability \( 1/2 \)—and then the next two local coalitions must have opposite preferences—again, with a probability of \( 1/2 \)—and so on up to the top. The probability that all these splits happen, and thus the individual voter is decisive, is \( 1/2d = n^{-\log_3 2} = n^{-3^d} \), which is lower than the probability under the popular vote system (for large \( n \), that probability is approximately \( 0.8n^{-0.5} \)). For example, if \( n = 3^9 = 19683 \), then the probability of a decisive vote is \( 1/256 \) with the tree-structure of coalitions, compared to about \( 1/102 \) with majority rule.

7. If the number of voters \( n \) is odd, this approximates \( \Pr(V = 0) \); if \( n \) is even, it approximates \( 1/2\Pr(V = -1/n) + 1/2\Pr(V = 1/n) \)