Demographic Forecasting

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Harvard University

Joint work with Federico Girosi (RAND)
with contributions from Kevin Quinn and Gregory Wawro

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- Approach: Formalizing qualitative insights in quantitative models

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- Better ways of creating Bayesian priors
- Produces forecasts and farcasts using considerably more information

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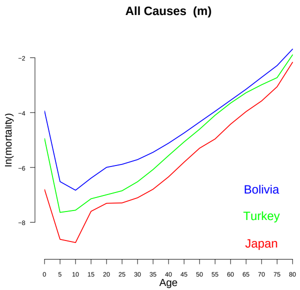
Existing Method 1: Parameterize the Age Profile



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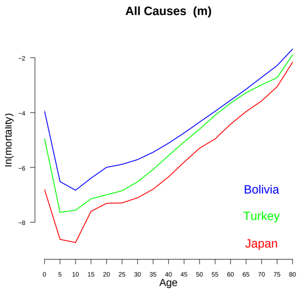

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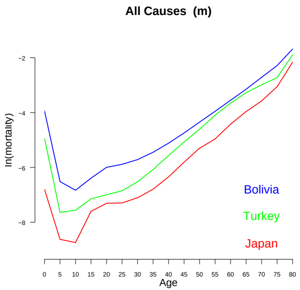

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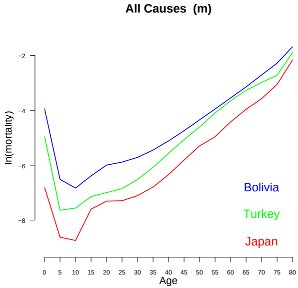

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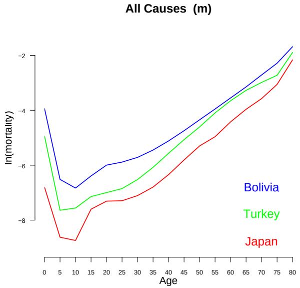

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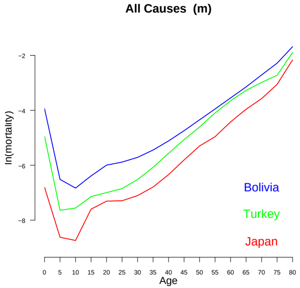

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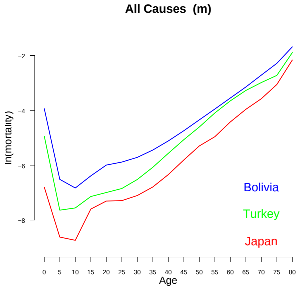

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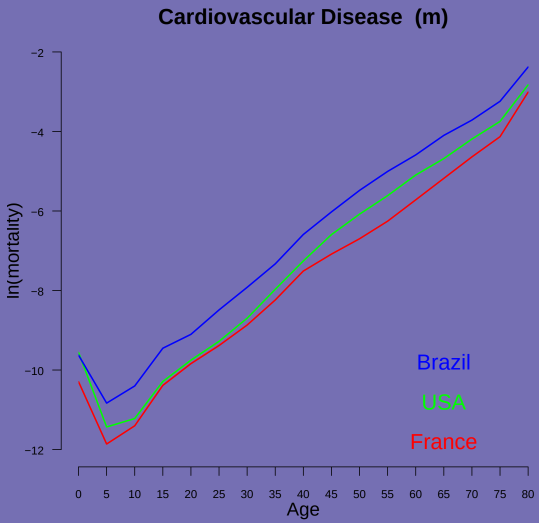
Mortality Age Profile: The Same Pattern?



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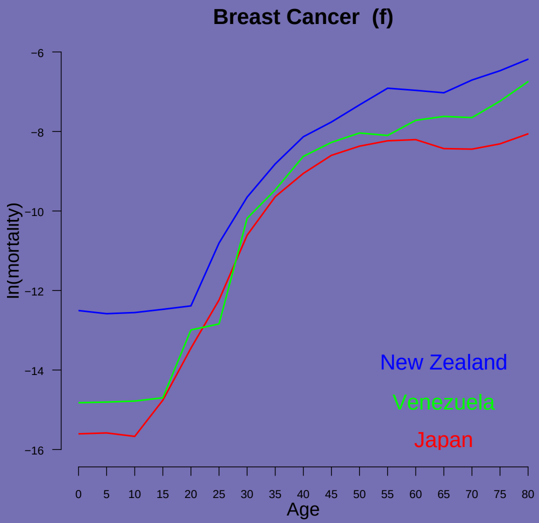

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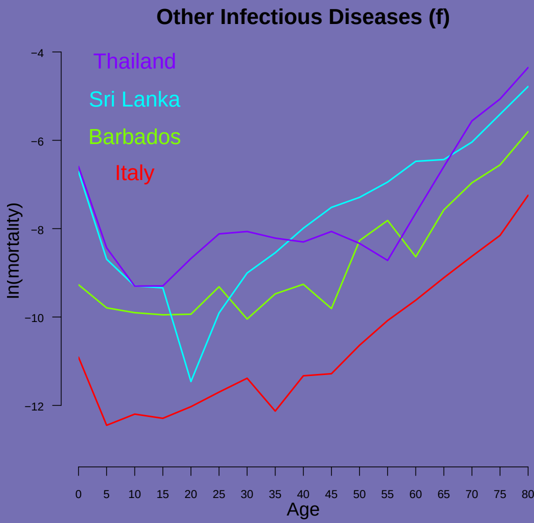

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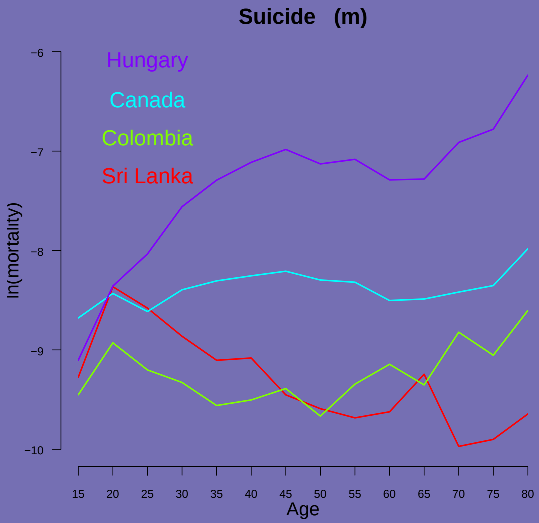

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- Also: Method ignores covariate information; the leading current method (McNown-Rogers) not replicable

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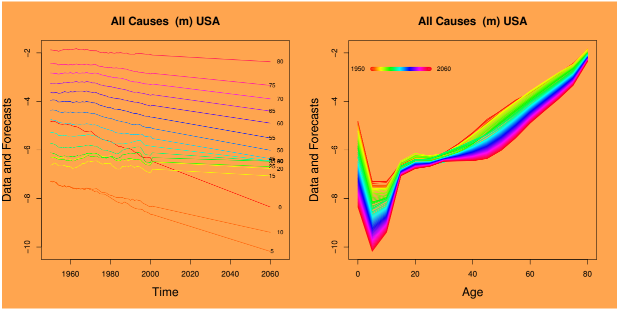

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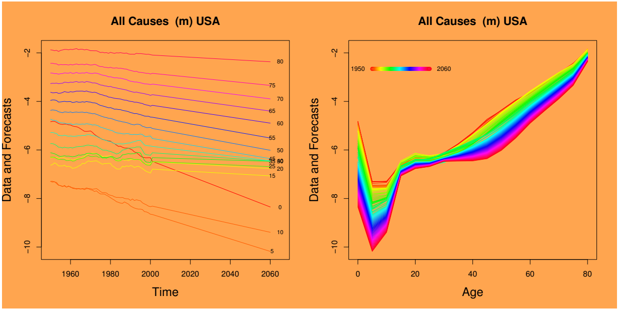

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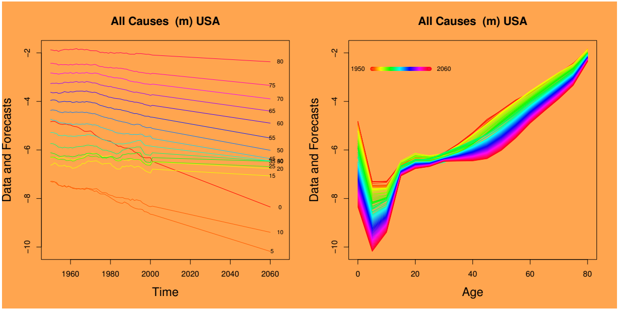

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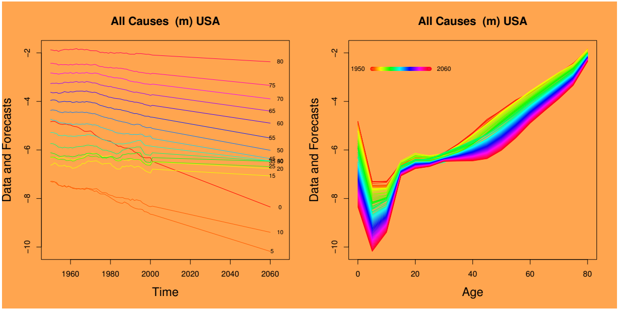

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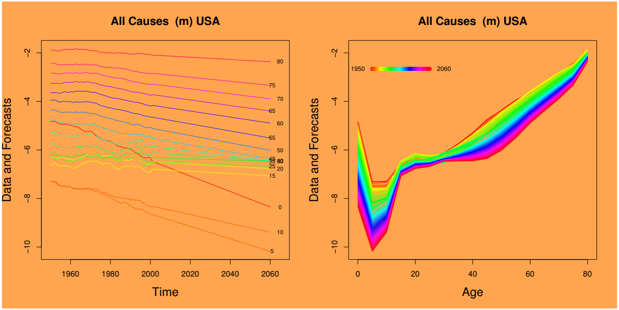

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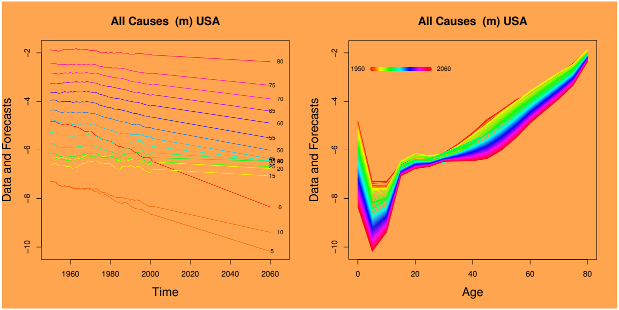

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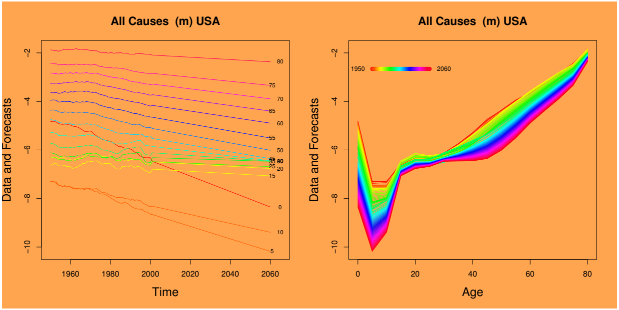

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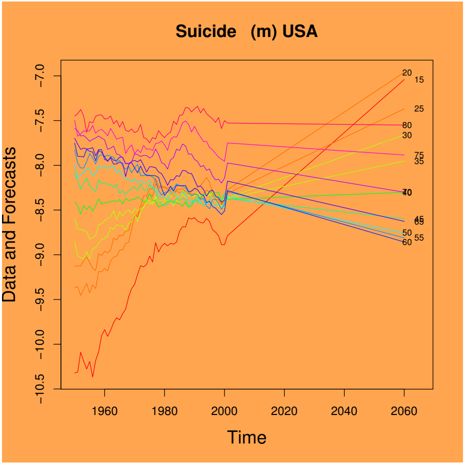

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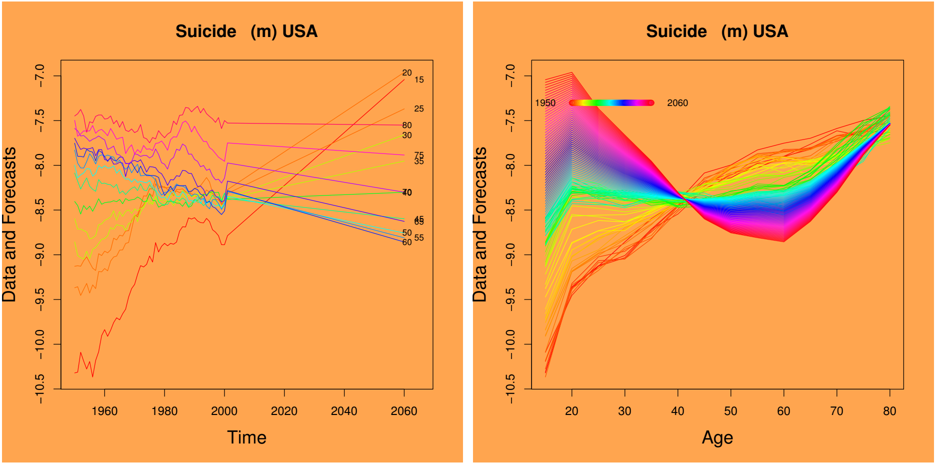

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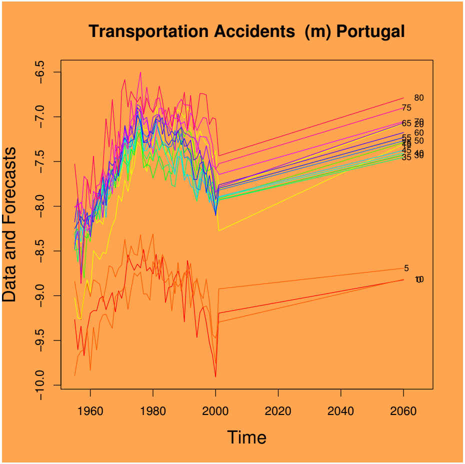

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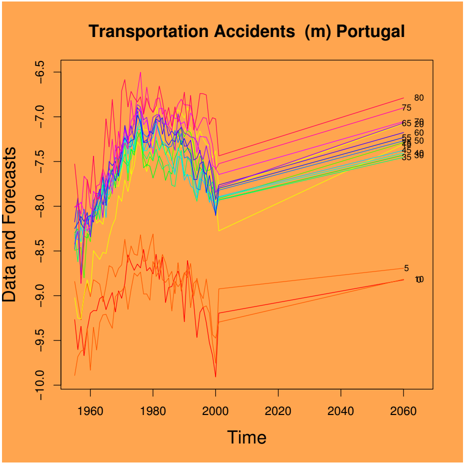

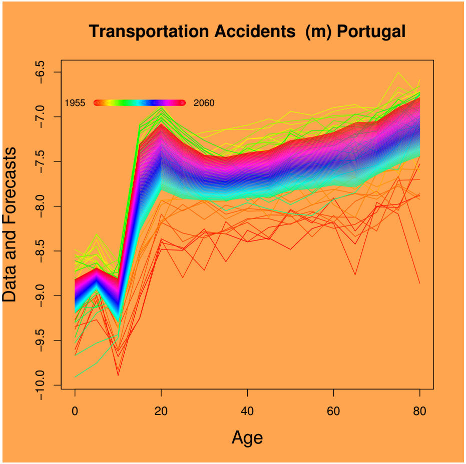

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Regression Approaches (Murray and Lopez, 1996)

- Model mortality over countries (c) and ages (a) as:

$$m_{cat} = \mathbf{Z}_{ca,t-\ell}\boldsymbol{\beta}_{ca} + \epsilon_{cat} \quad , \quad t = 1, \ldots, T$$

- $\mathbf{Z}_{ca,t-\ell}$ covariates (GDP, tobacco …) lagged ℓ years.
- $\boldsymbol{\beta}_{ca}$ coefficients to be estimated
- Cannot estimate equation by equation (variance is too large);
- Pool over countries: $\boldsymbol{\beta}_{ca} \Rightarrow \boldsymbol{\beta}_{a}$
- Properties:
  - Small variance (due to large n)
  - large biases (due to restrictive pooling over countries),
  - considerable information lost (due to no pooling over ages)
  - same covariates required in all cross-sections

Partial Pooling via a Bayesian Hierarchical Approach

- Likelihood for equation-by-equation least squares:

$$\mathcal{P}(m \mid \beta_i, \sigma_i) = \prod_t \mathcal{N}\left(m_{it} \mid \mathbf{Z}_{it}\beta_i, \sigma_i^2\right)$$

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- The easy part: *easy-to-use software* to implement everything we discuss today.

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Natural choice for the prior:

$$\mathcal{P}(\boldsymbol{\beta} \mid \Phi) \propto \exp\left(-\frac{1}{2} \sum_{ij} s_{ij} \|\boldsymbol{\beta}_i - \boldsymbol{\beta}_j\|^2_{\Phi} \right)$$

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- Extensive trial-and-error runs, yielded no useful parameter values.

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3. In the subspace, we can invert $\mu = \mathbf{Z}\boldsymbol{\beta}$ as $\boldsymbol{\beta} = (\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\mu$, giving:

$$\mathcal{P}(\boldsymbol{\beta} \mid \theta) \propto \exp\left(-\frac{1}{2}H[\mu,\theta]\right) = \exp\left(-\frac{1}{2}H[\mathbf{Z}\boldsymbol{\beta},\theta]\right)$$

   the same prior on μ, expressed as a function of $\boldsymbol{\beta}$ (with constant Jacobian).

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- We start with one-dimensional $P(\mu_{cat})$, and treat it as the multidimensional $P(\beta_{ca})$, constant in all directions but $Z_{cat}\beta_{ca}$.

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- Priors are based on knowledge rather than guesses.

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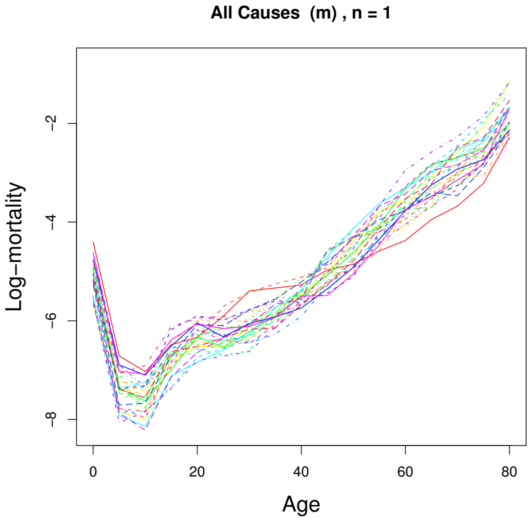
- Different age groups can have different covariates: the matrices $\mathbf{C}_{aa'} \equiv \frac{1}{T}\mathbf{Z}'_a\mathbf{Z}_{a'}$ are rectangular $(d_a \times d_{a'})$.

Samples From Age Prior



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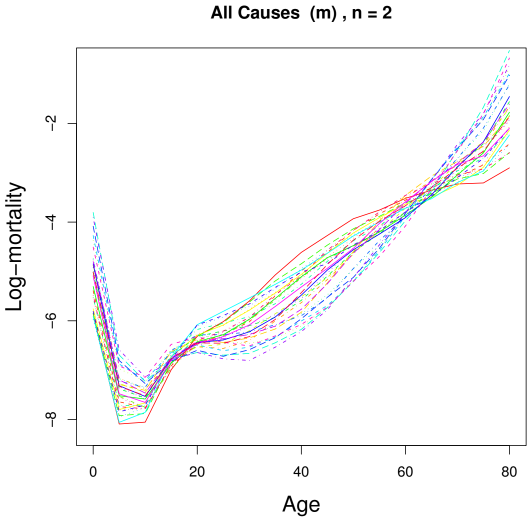

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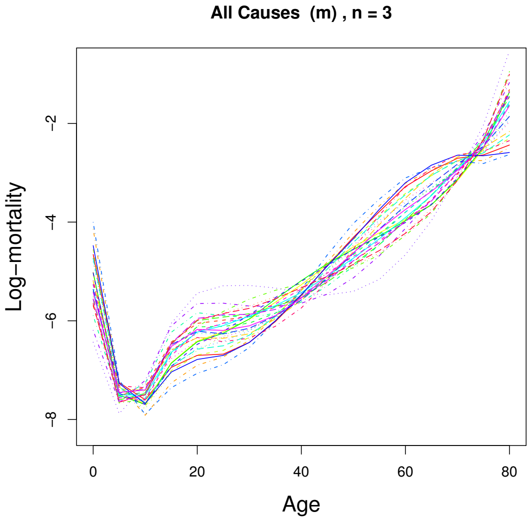

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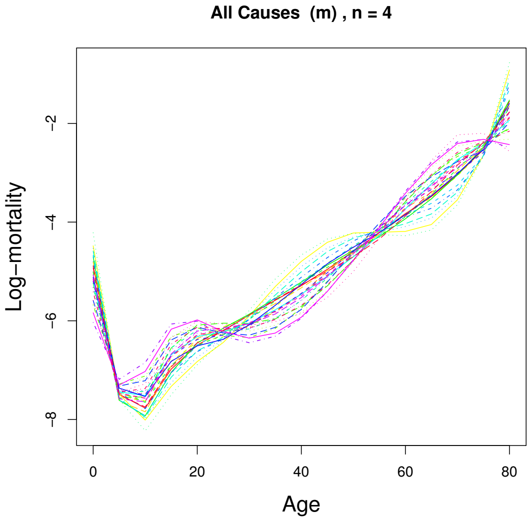

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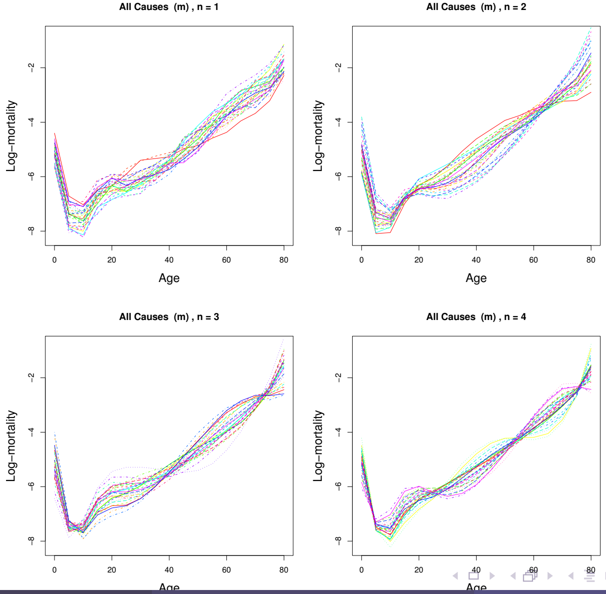

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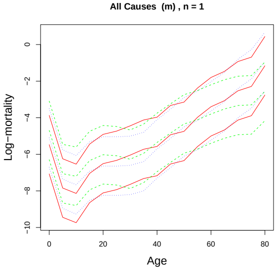

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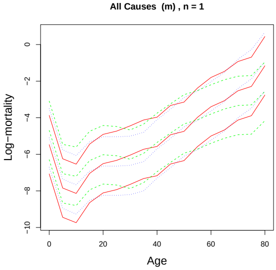
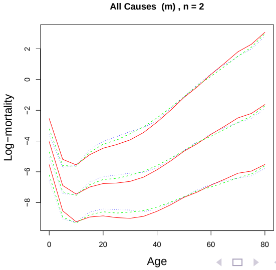


Level and slope indifference



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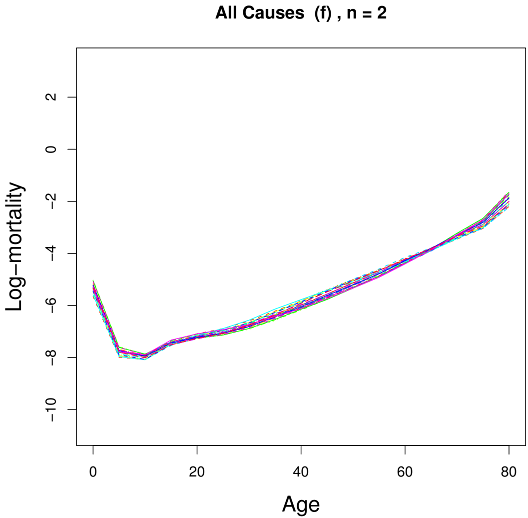

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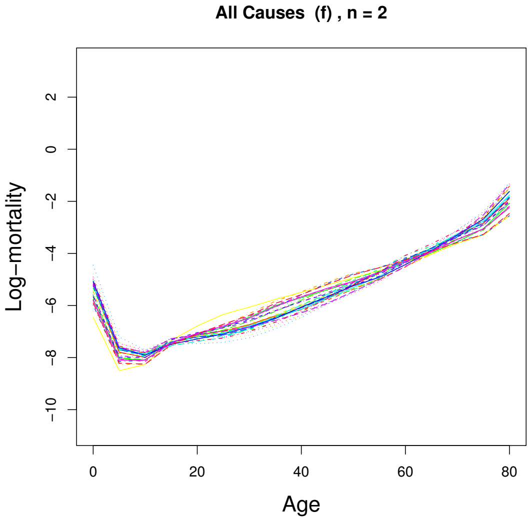

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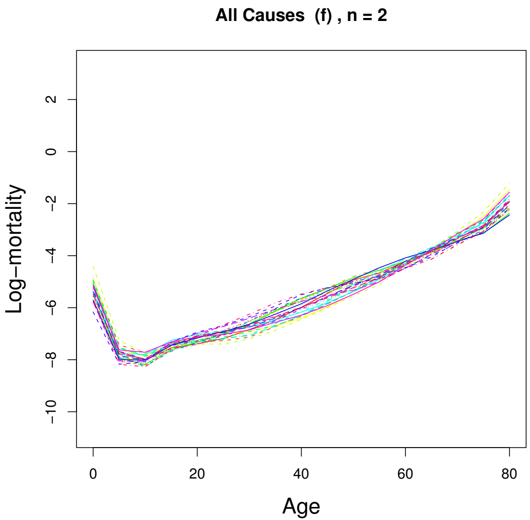

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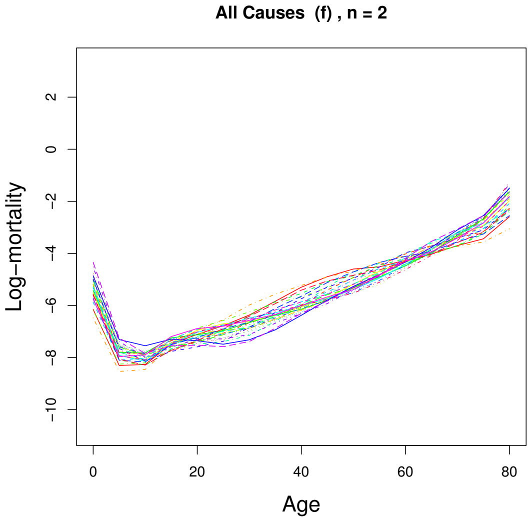

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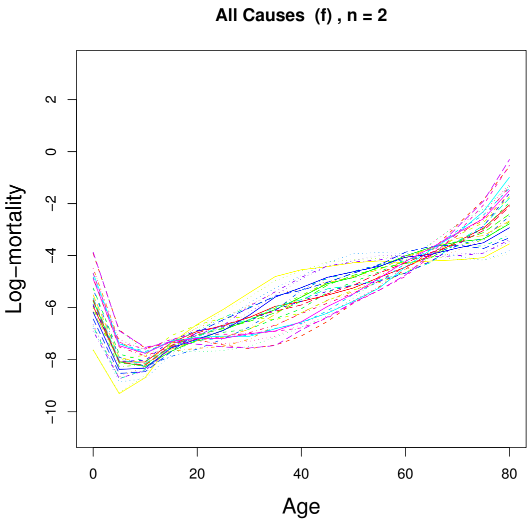

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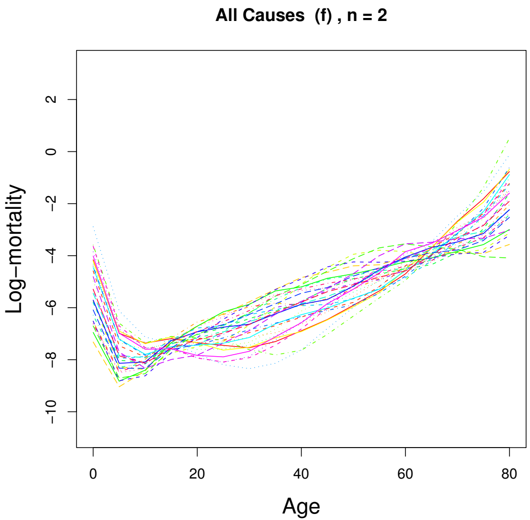

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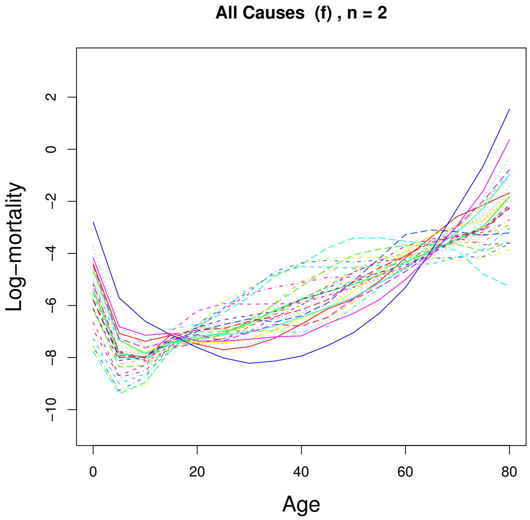

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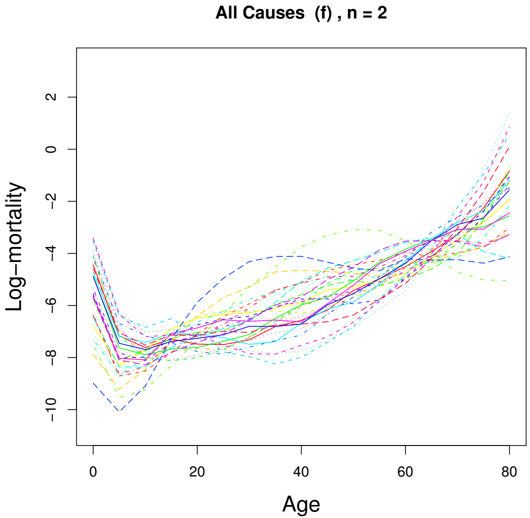

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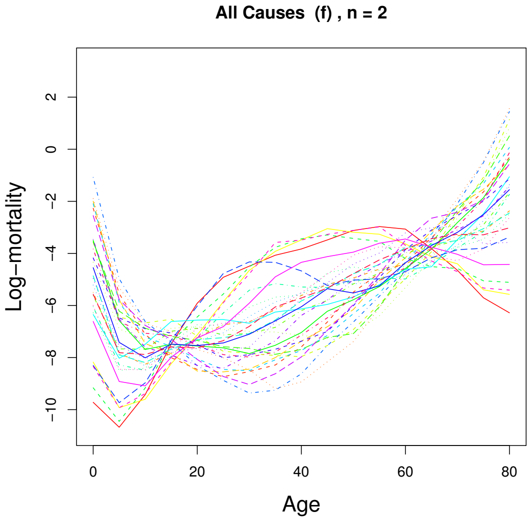

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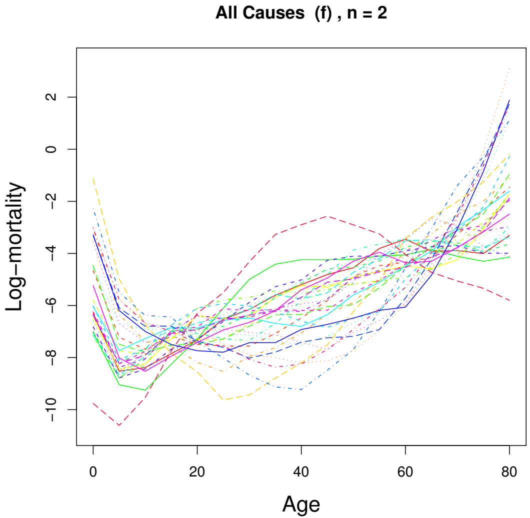

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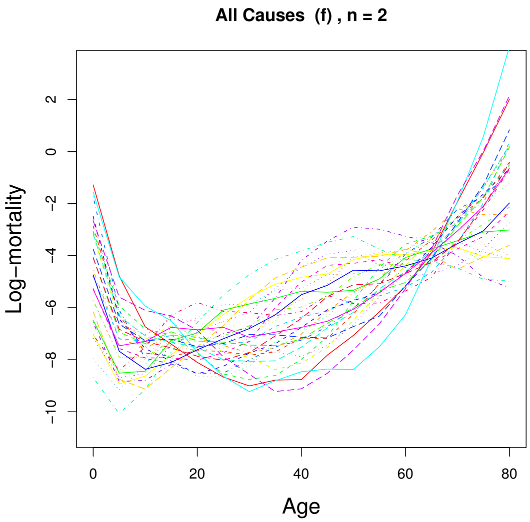

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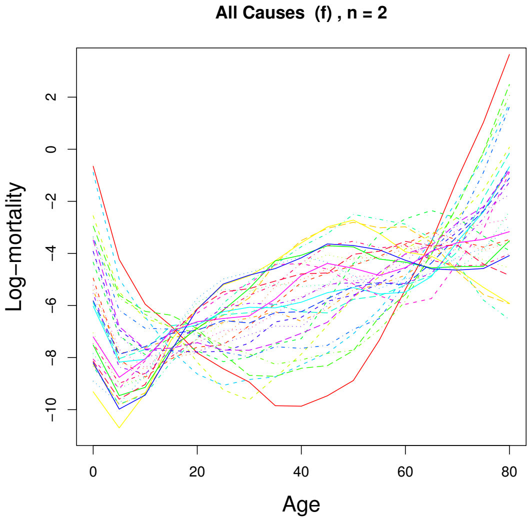

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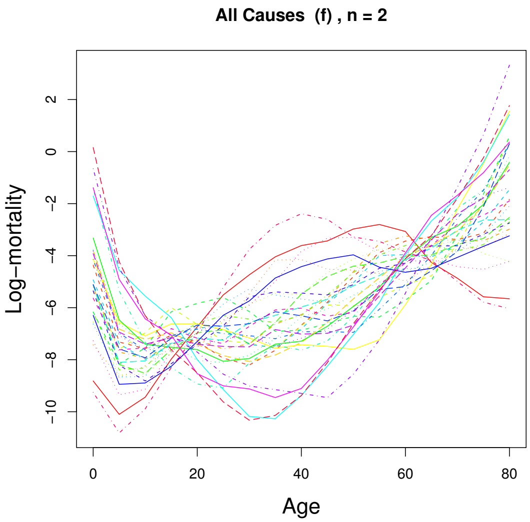

Generalizations

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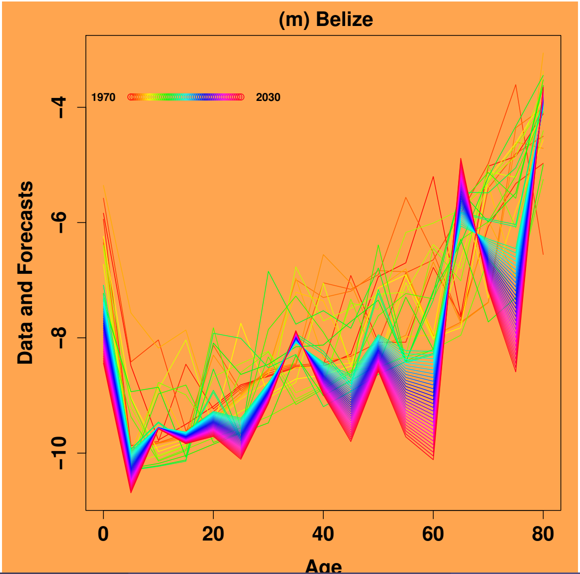
- The mathematical form for *all* these (separately or together) turns out to be the same:

$$\mathcal{P}(\beta \mid \theta) \propto \exp\left(-\frac{\theta}{2}\sum_{ij} W_{ij}\beta_i' \mathbf{C}_{ij}\beta_j\right), \qquad \mathbf{C}_{aa'} \equiv \frac{1}{T}\mathbf{Z}_a\mathbf{Z}_{a'}$$

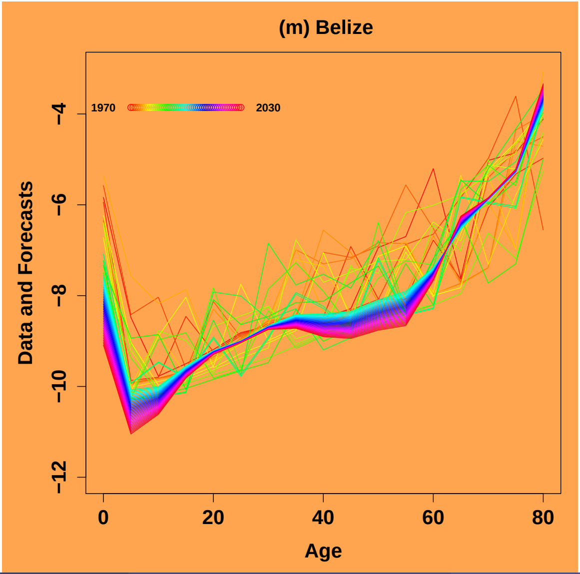
Mortality from Respiratory Infections, Males

Least Squares



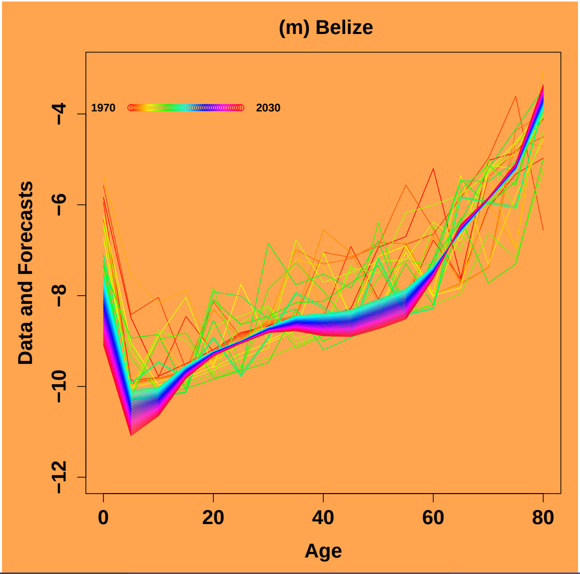
Mortality from Respiratory Infections, males, $\sigma = 2.00$

Smoothing over Age Groups



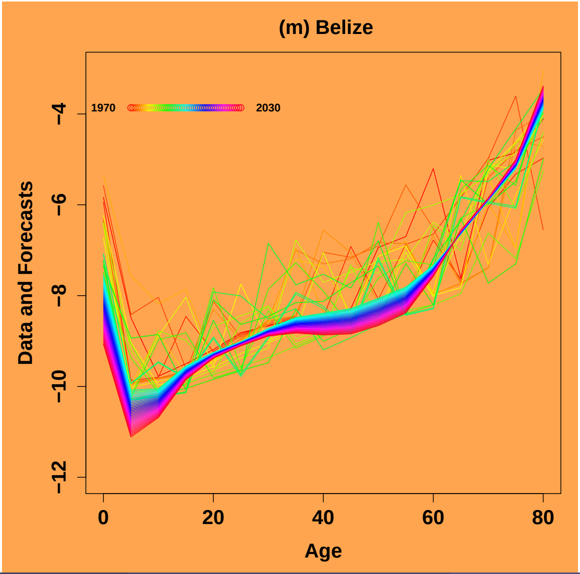
Mortality from Respiratory Infections, males, $\sigma = 1.51$

Smoothing over Age Groups



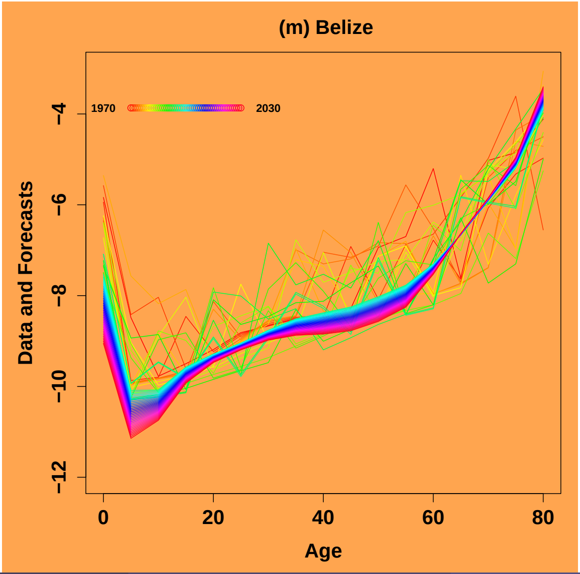
Mortality from Respiratory Infections, males, $\sigma = 1.15$

Smoothing over Age Groups



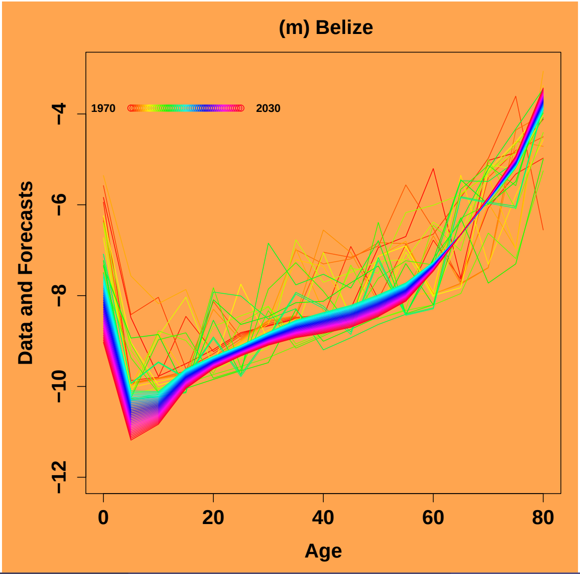
Mortality from Respiratory Infections, males, $\sigma = 0.87$

Smoothing over Age Groups



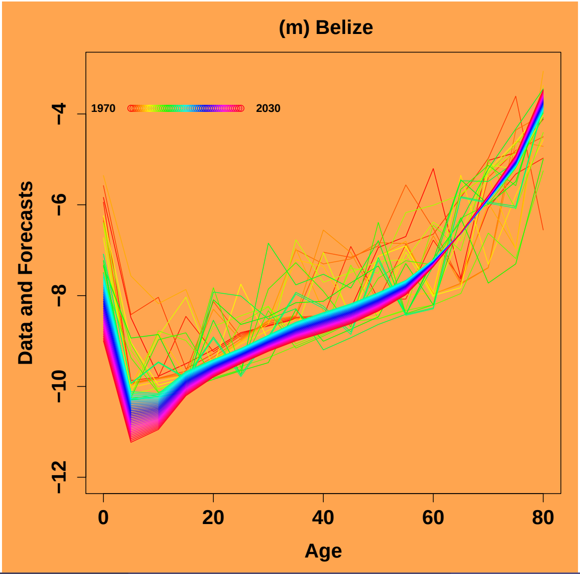
Mortality from Respiratory Infections, males, $\sigma = 0.66$

Smoothing over Age Groups



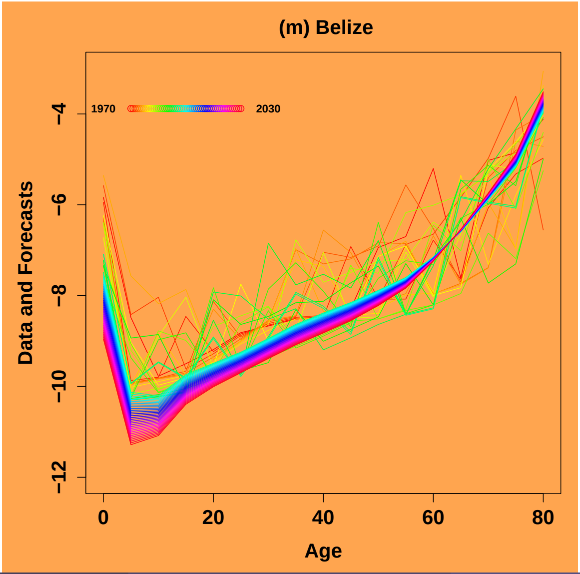
Mortality from Respiratory Infections, males, $\sigma = 0.50$

Smoothing over Age Groups



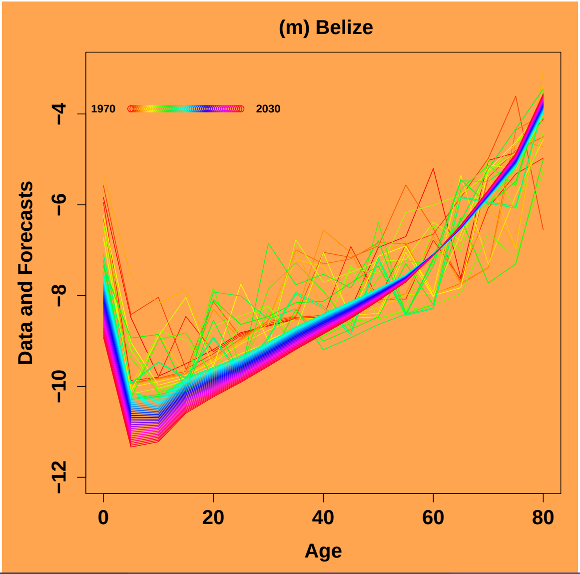
Mortality from Respiratory Infections, males, $\sigma = 0.38$

Smoothing over Age Groups



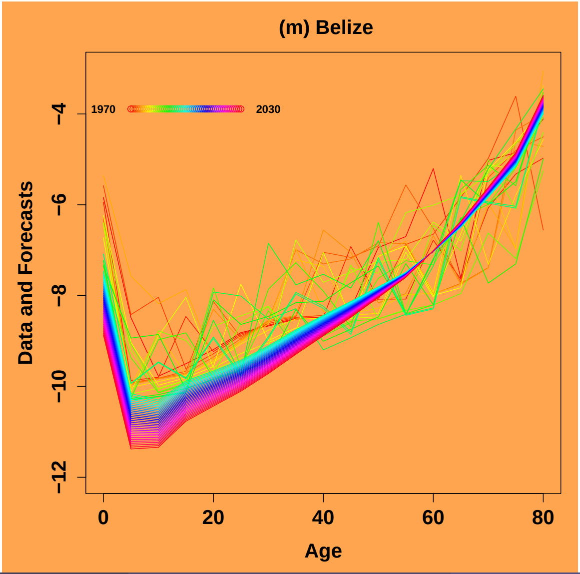
Mortality from Respiratory Infections, males, $\sigma = 0.28$

Smoothing over Age Groups



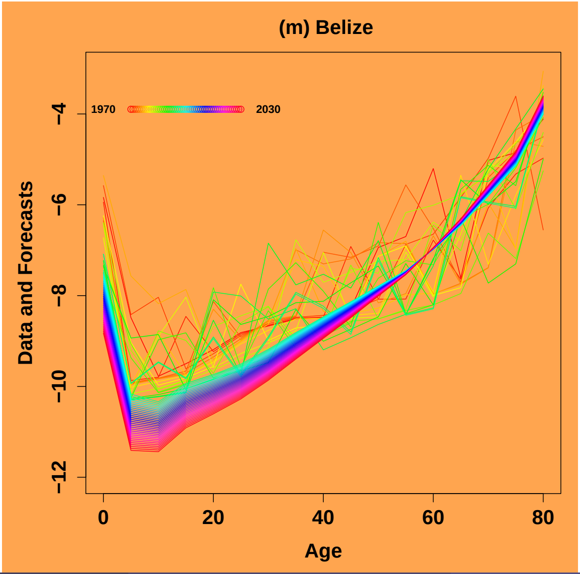
Mortality from Respiratory Infections, males, $\sigma = 0.21$

Smoothing over Age Groups



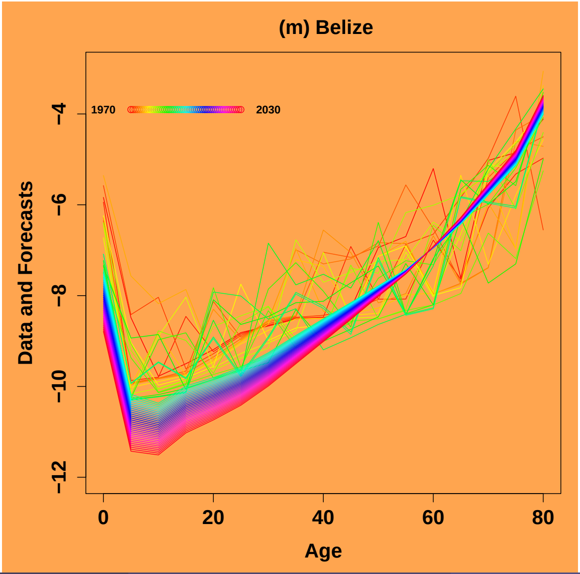
Mortality from Respiratory Infections, males, $\sigma = 0.16$

Smoothing over Age Groups



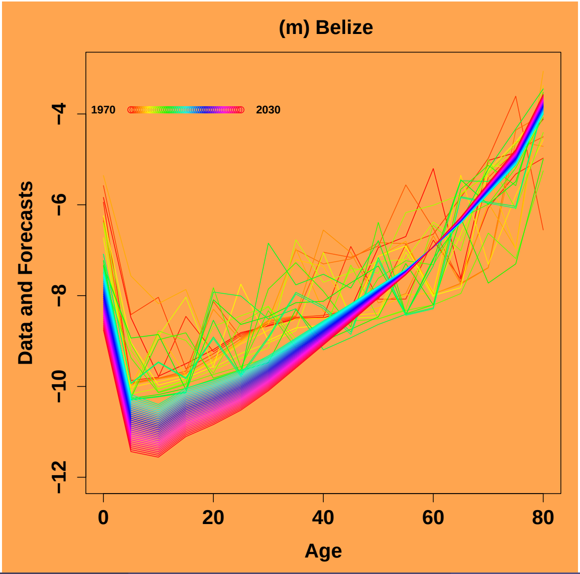
Mortality from Respiratory Infections, males, $\sigma = 0.12$

Smoothing over Age Groups



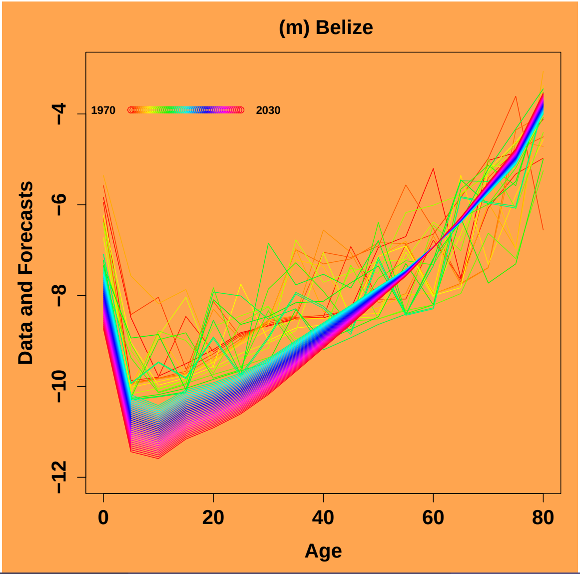
Mortality from Respiratory Infections, males, $\sigma = 0.09$

Smoothing over Age Groups



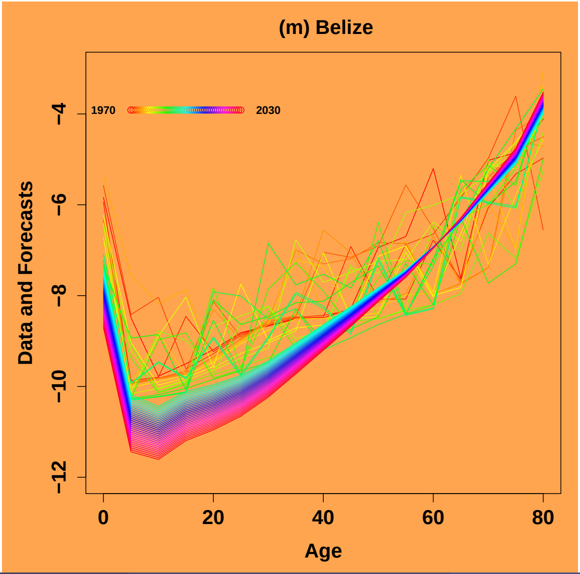
Mortality from Respiratory Infections, males, $\sigma = 0.07$

Smoothing over Age Groups



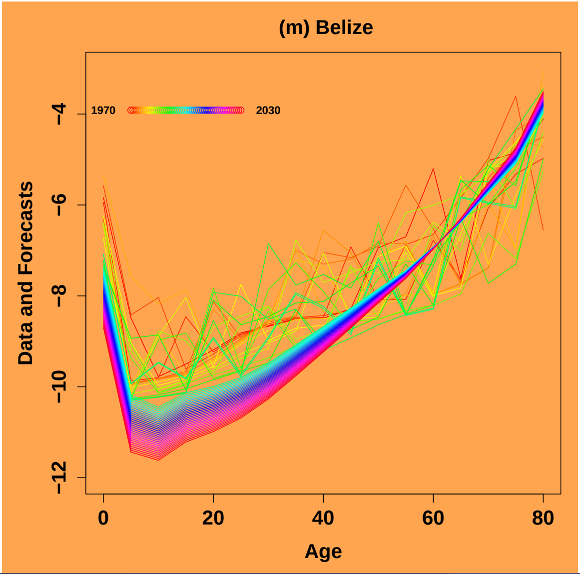
Mortality from Respiratory Infections, males, $\sigma = 0.05$

Smoothing over Age Groups



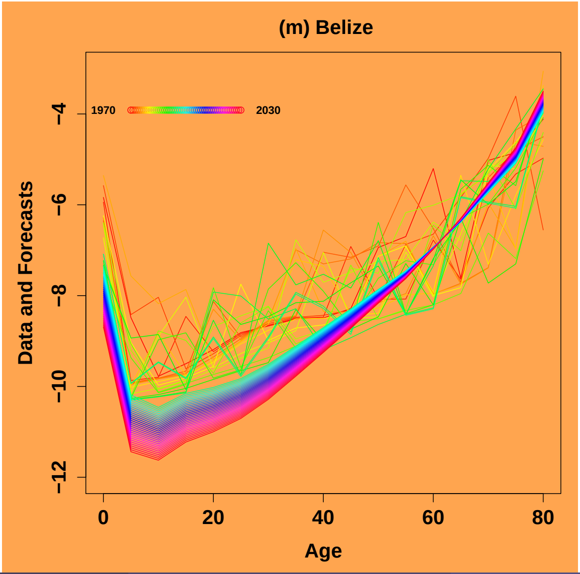
Mortality from Respiratory Infections, males, $\sigma = 0.04$

Smoothing over Age Groups



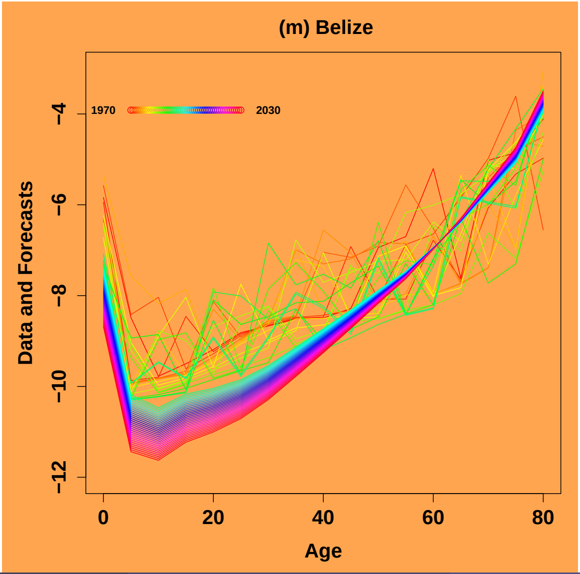
Mortality from Respiratory Infections, males, $\sigma = 0.03$

Smoothing over Age Groups



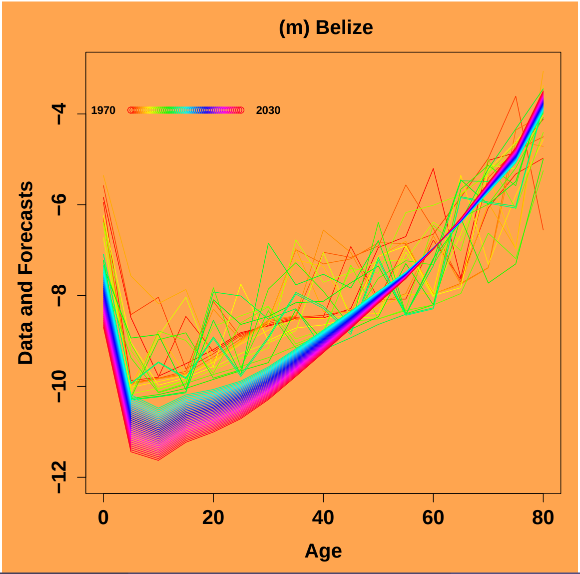
Mortality from Respiratory Infections, males, $\sigma = 0.02$

Smoothing over Age Groups



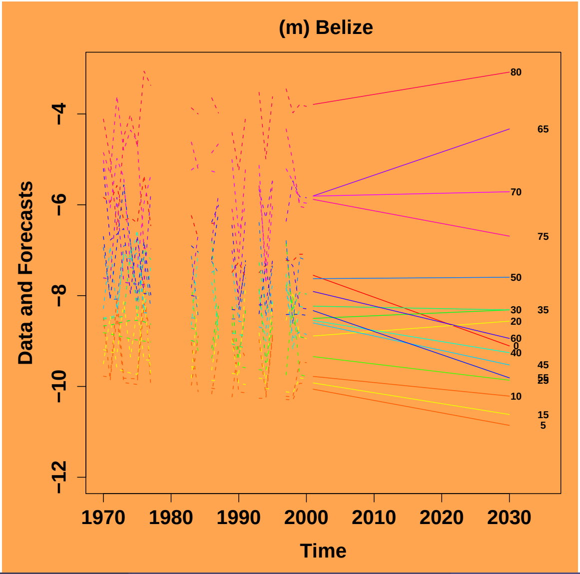
Mortality from Respiratory Infections, males, $\sigma = 0.01$

Smoothing over Age Groups



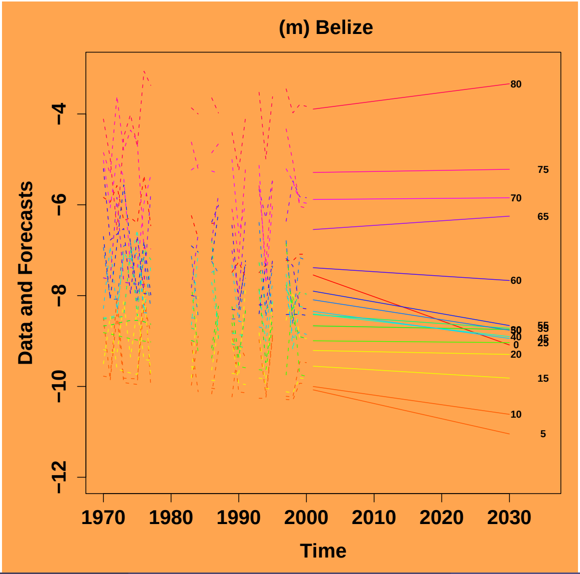
Mortality from Respiratory Infections, males

Least Squares



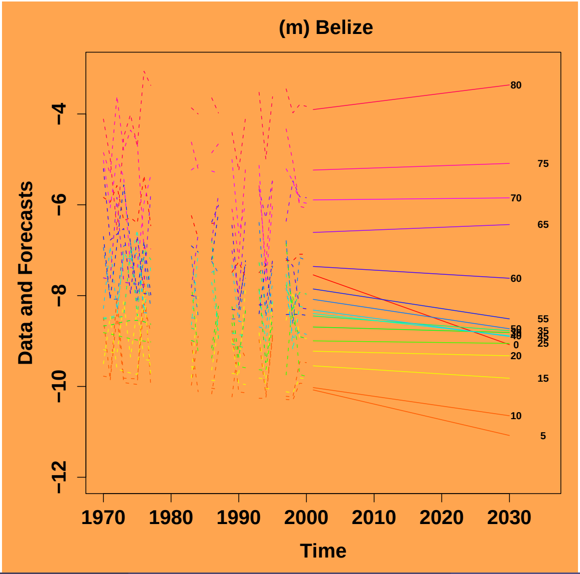
Mortality from Respiratory Infections, males, $\sigma = 2.00$

Smoothing over Age Groups



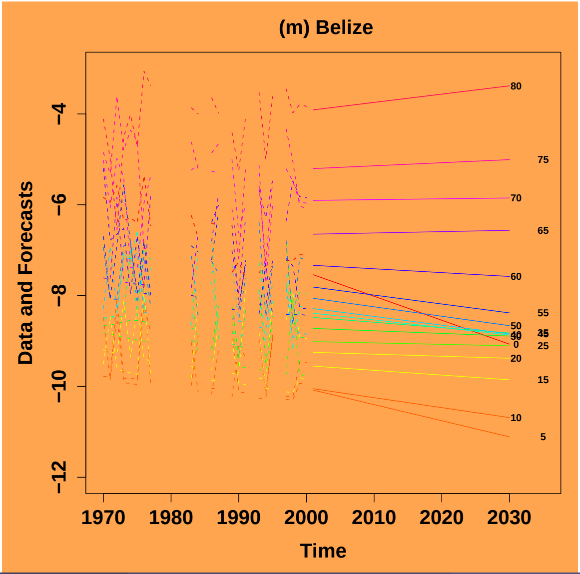
Mortality from Respiratory Infections, males, $\sigma = 1.51$

Smoothing over Age Groups



Mortality from Respiratory Infections, males, $\sigma = 1.15$

Smoothing over Age Groups



Mortality from Respiratory Infections, males, $\sigma = 0.87$

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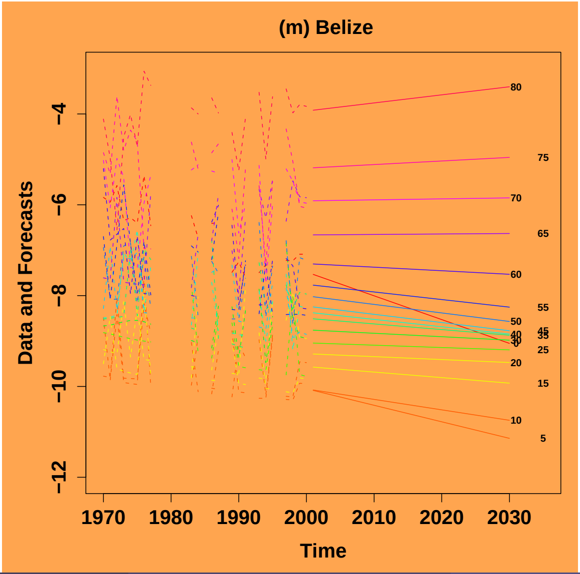

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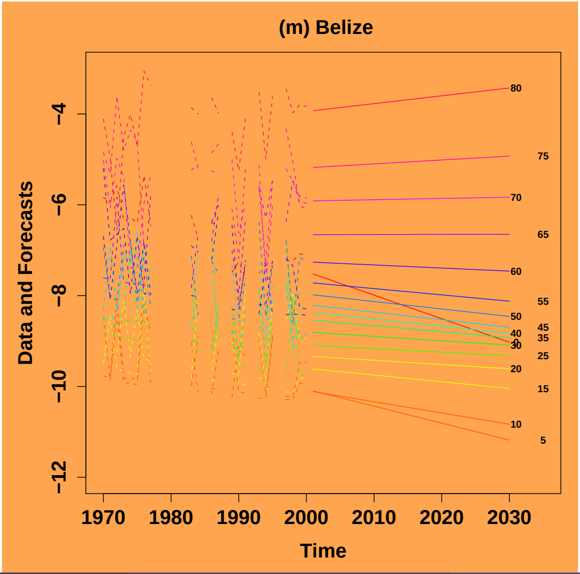

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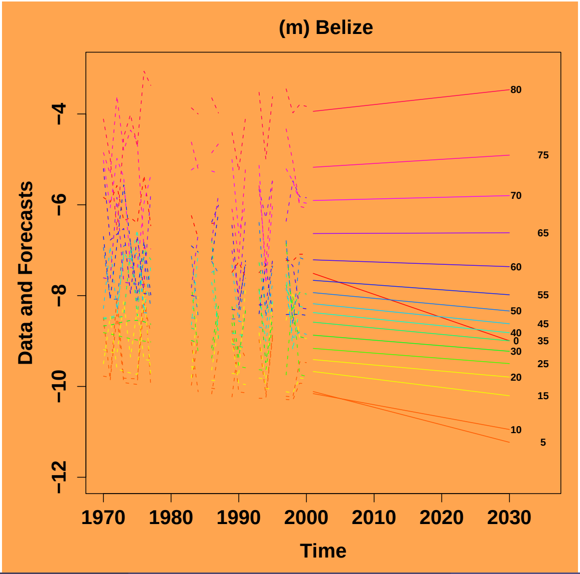

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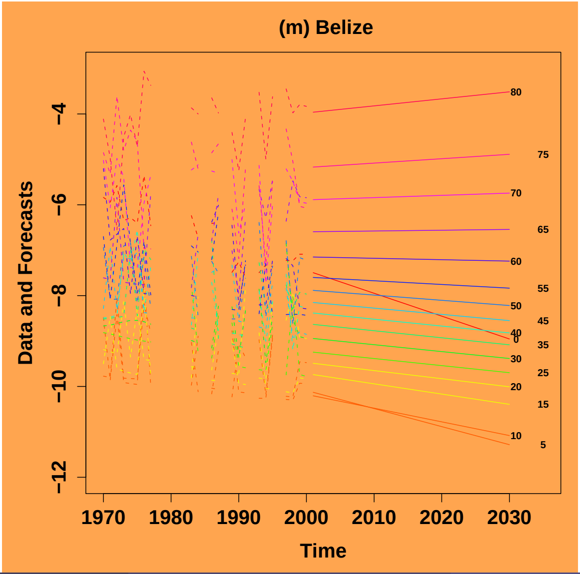

Mortality from Respiratory Infections, males, $\sigma = 0.28$

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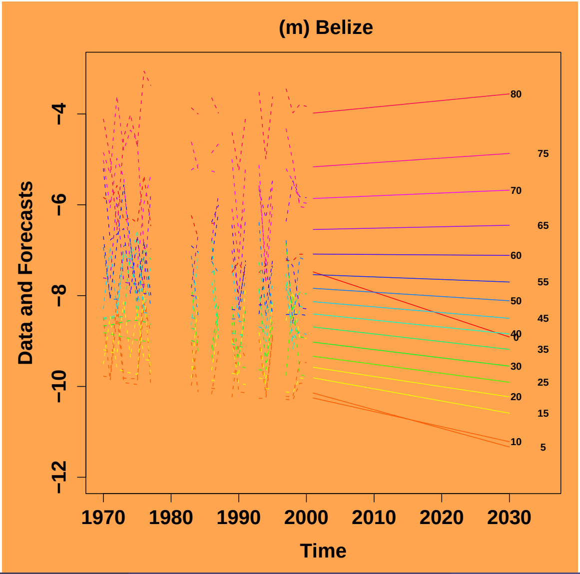

Mortality from Respiratory Infections, males, $\sigma = 0.21$

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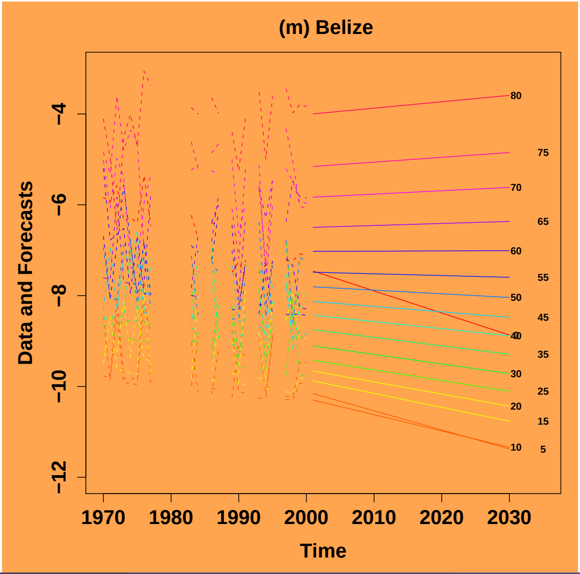

Mortality from Respiratory Infections, males, $\sigma = 0.16$

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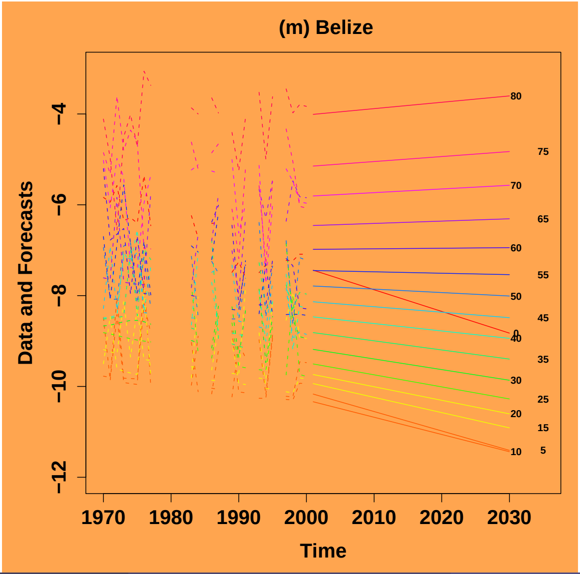
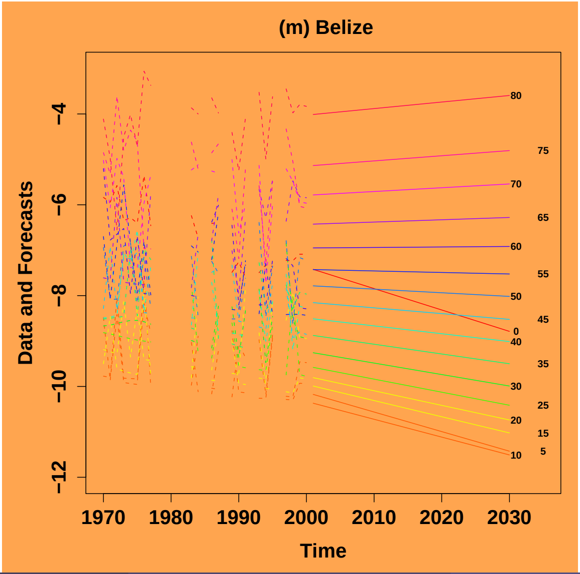


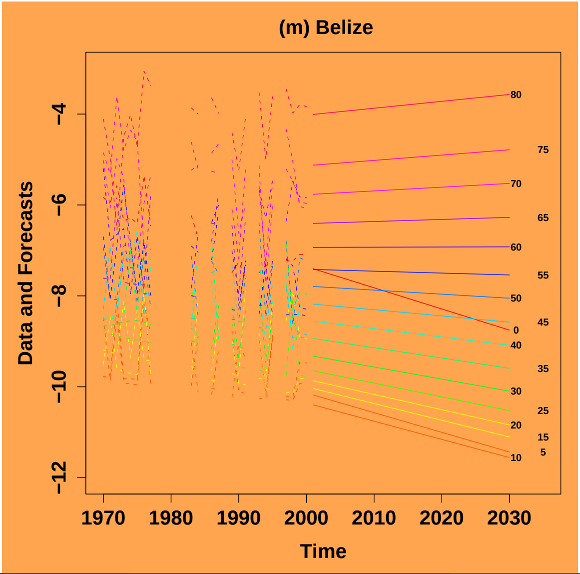
Mortality from Respiratory Infections, males, $\sigma = 0.12$

Smoothing over Age Groups



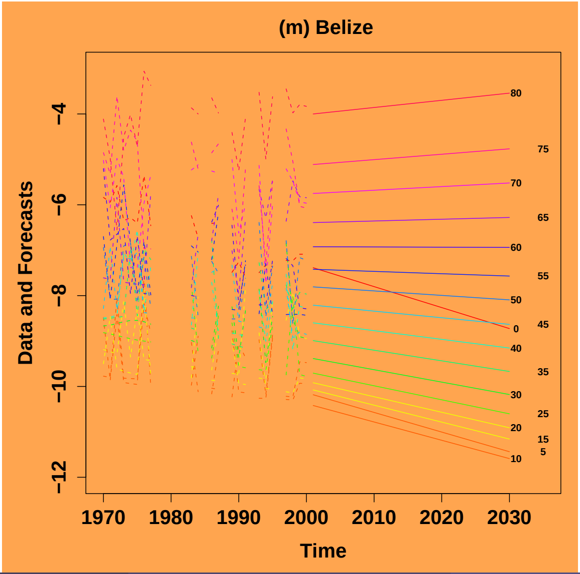
Mortality from Respiratory Infections, males, $\sigma = 0.09$

Smoothing over Age Groups



Mortality from Respiratory Infections, males, $\sigma = 0.07$

Smoothing over Age Groups



Mortality from Respiratory Infections, males, $\sigma = 0.05$

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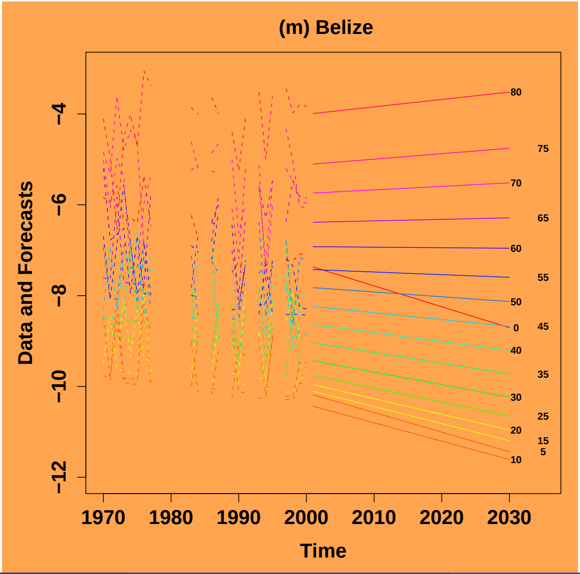

Mortality from Respiratory Infections, males, $\sigma = 0.04$

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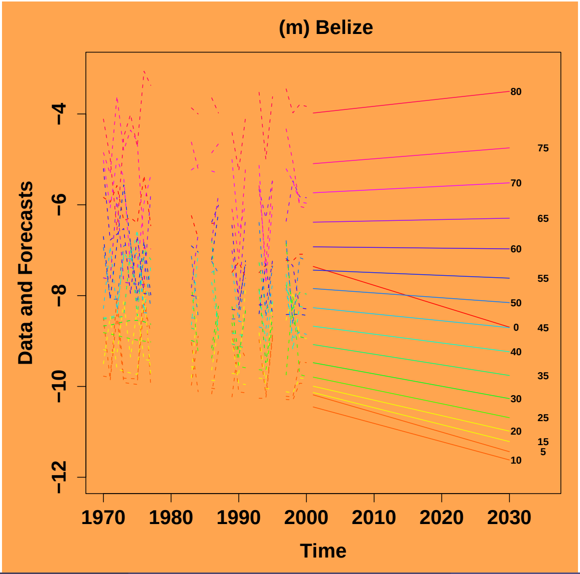

Mortality from Respiratory Infections, males, $\sigma = 0.03$

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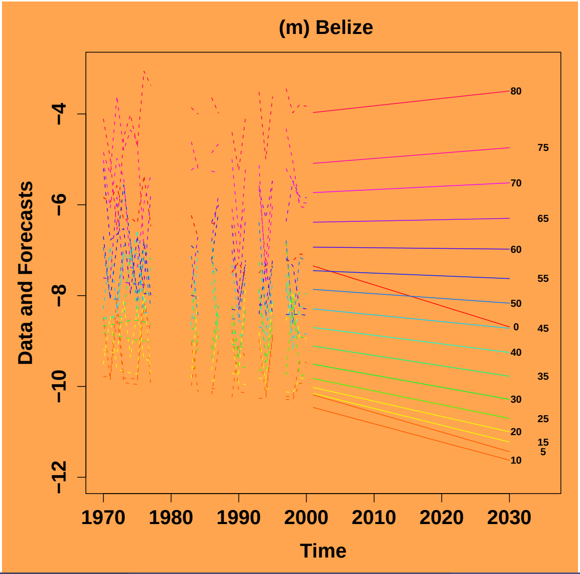

Mortality from Respiratory Infections, males, $\sigma = 0.02$

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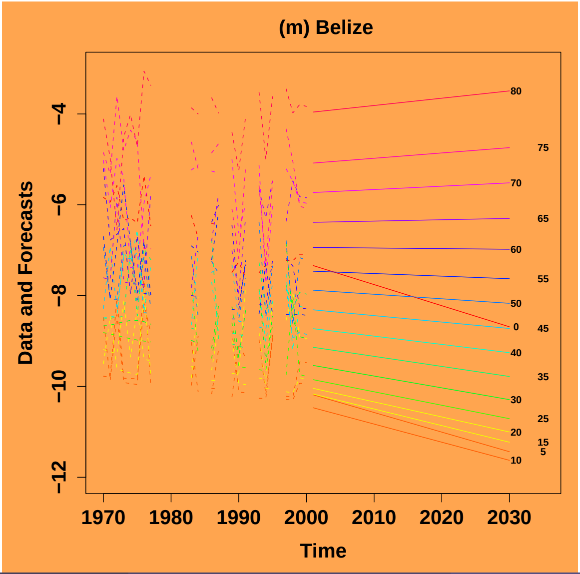

Mortality from Respiratory Infections, males, $\sigma = 0.01$

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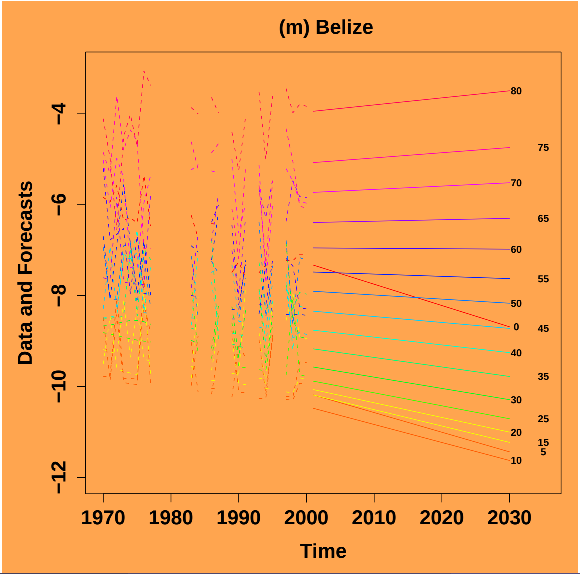

Smoothing Trends over Age Groups

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Log-mortality in Belize males from respiratory infections

Least Squares

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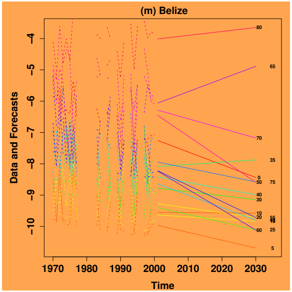

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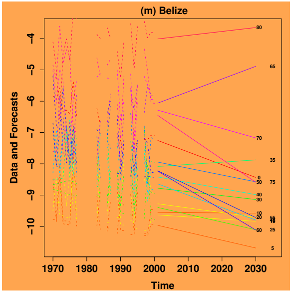

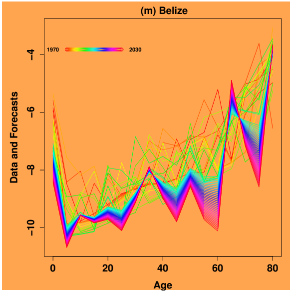

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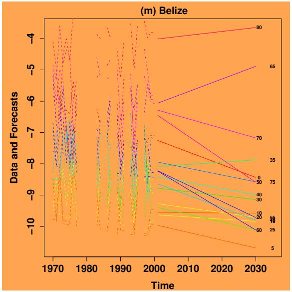

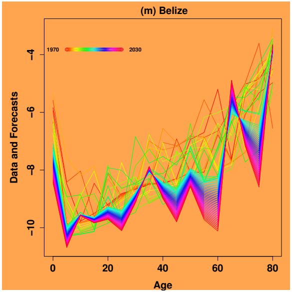

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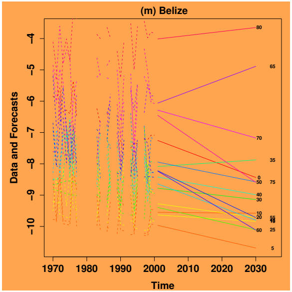

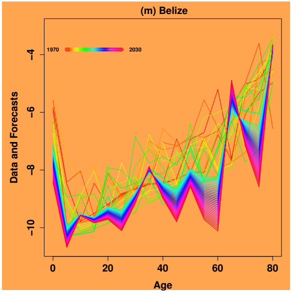

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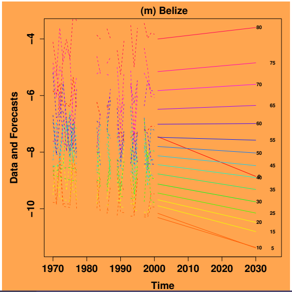

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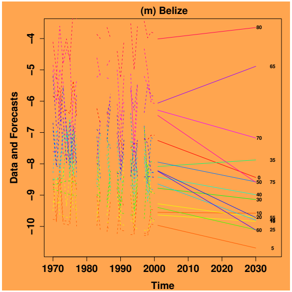
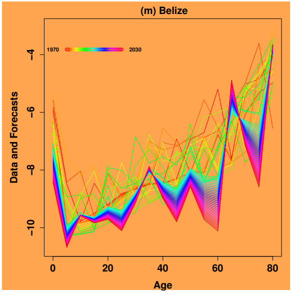
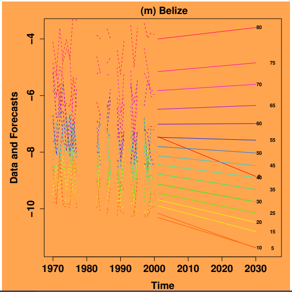
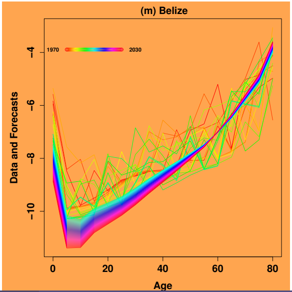




Smoothing
Age Groups





Smoothing Trends over Age Groups and Time

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Log-Mortality in Bulgarian males from respiratory infections

Least Squares

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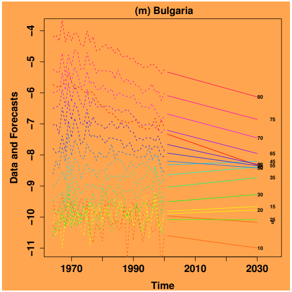

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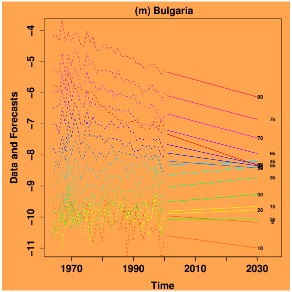

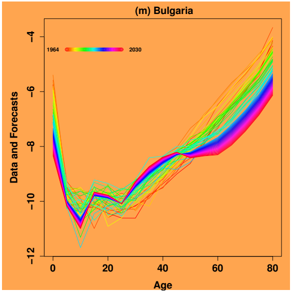

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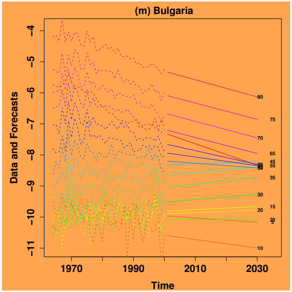

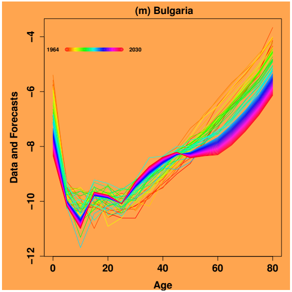

Smoothing
Age and Time

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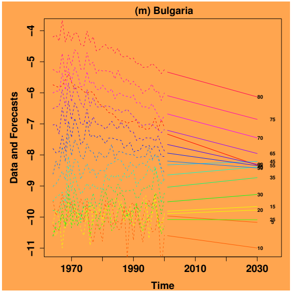

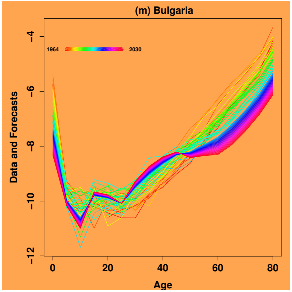

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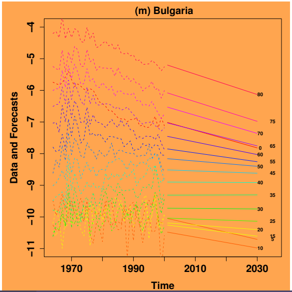

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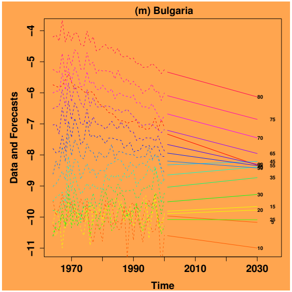

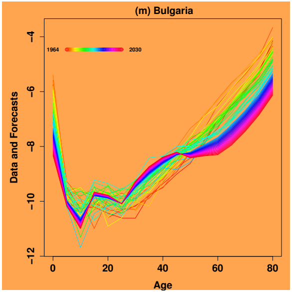

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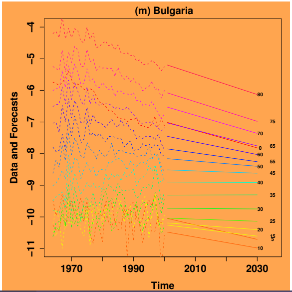

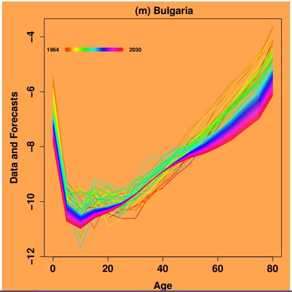

Using Covariates (GDP, tobacco, trend, log trend)

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Lung cancer in Korean Males

Least Squares

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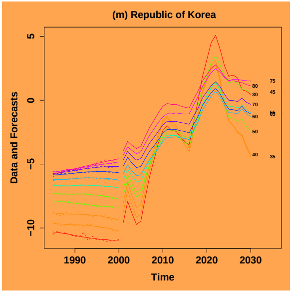

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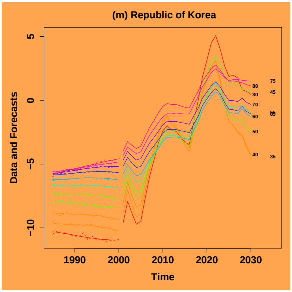

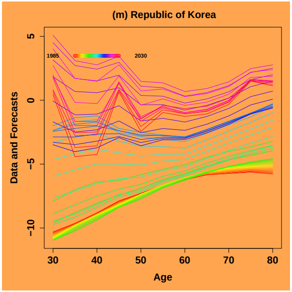

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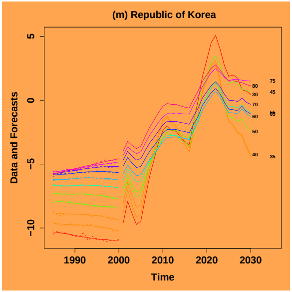

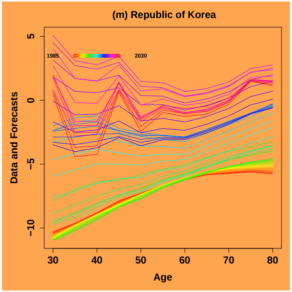

Smooth over age, time, age/time

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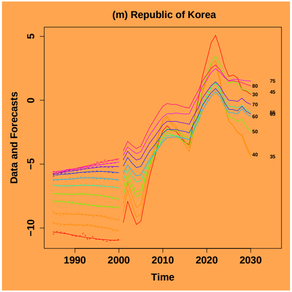

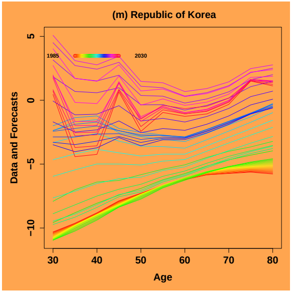

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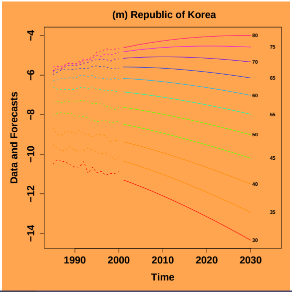

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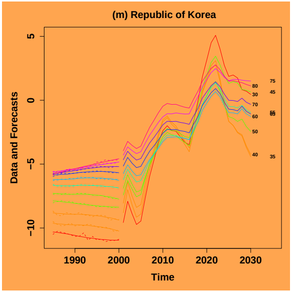

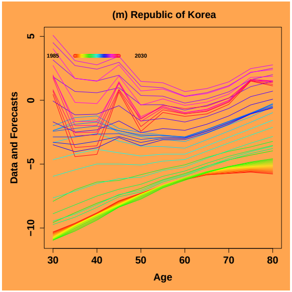

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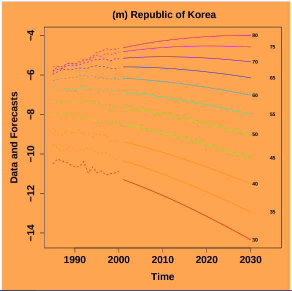

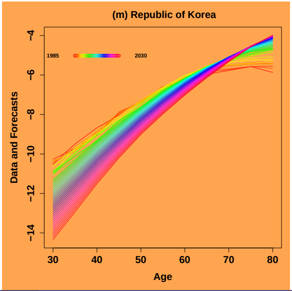

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Lung cancer in Males, Singapore

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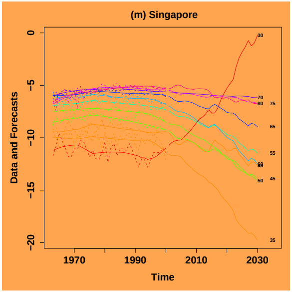

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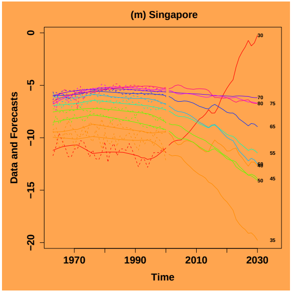

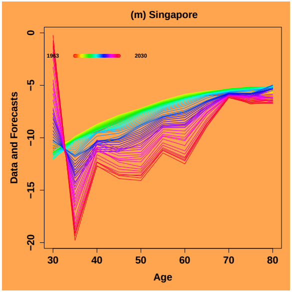

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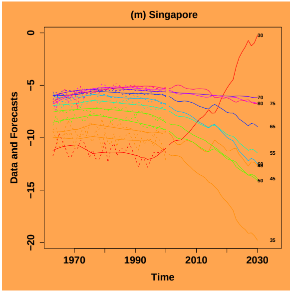

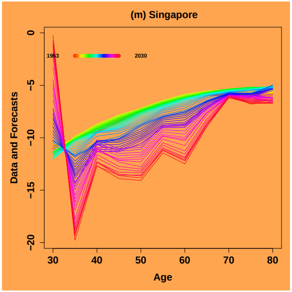

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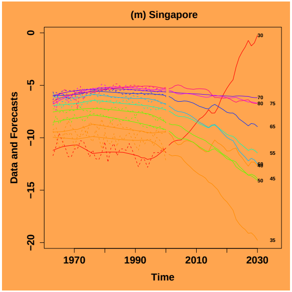

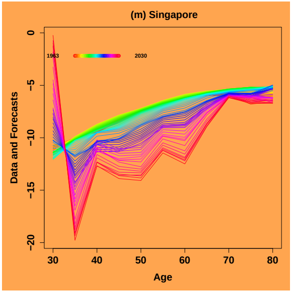

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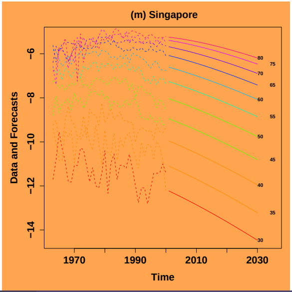

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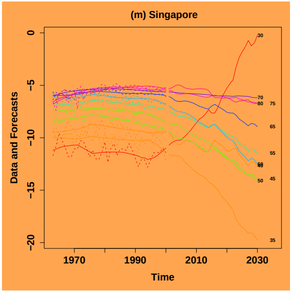

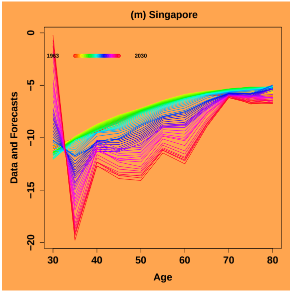

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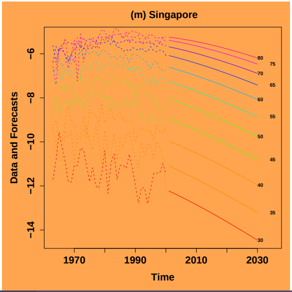
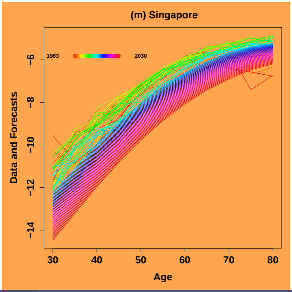
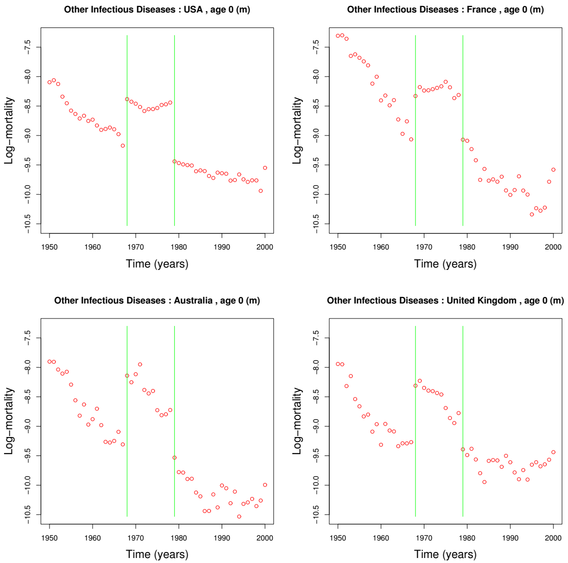
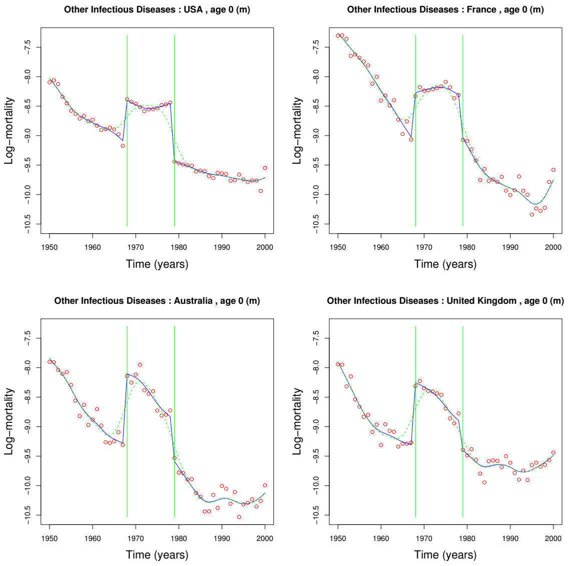




What about ICD Changes?



Fixing ICD Changes



A book manuscript, YourCast software, etc.

http://GKing.Harvard.edu

Preview of Results: Out-of-Sample Evaluation

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Mean Absolute Error in Males (over age and country)

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	% Improvement	
	Over Best Previous	to Best Conceivable
Cardiovascular	22	49
Lung Cancer	24	47
Transportation	16	31
Respiratory Chronic	13	30
Other Infectious	12	30
Stomach Cancer	8	24
All-Cause	12	22
Suicide	7	17
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- Does *considerably* better with more informative covariates

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- where W^n is a matrix uniquely determined by n and θ

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where we have defined:

$$\mathbf{C}_{aa'} \equiv \frac{1}{T}\mathbf{Z}'_a\mathbf{Z}_{a'}$$

$\mathbf{Z}_a$ is a $T \times d_a$ data matrix for age group a

The Prior on the Coefficients $\boldsymbol{\beta}$

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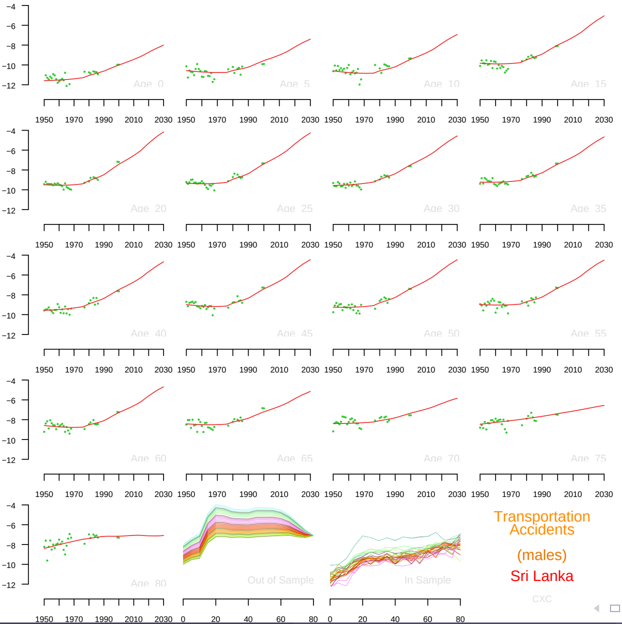
- The variance of the prior is inversely proportional to θ, which controls the "strength" of the prior.

Without Country Smoothing



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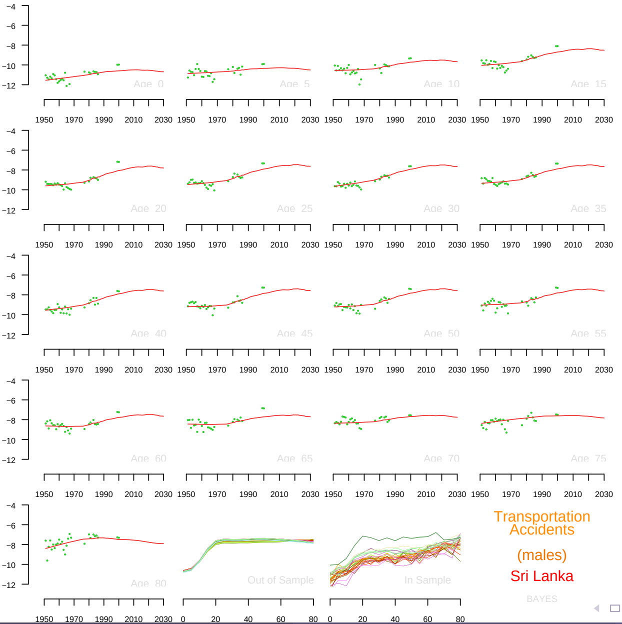

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Alternative Approach

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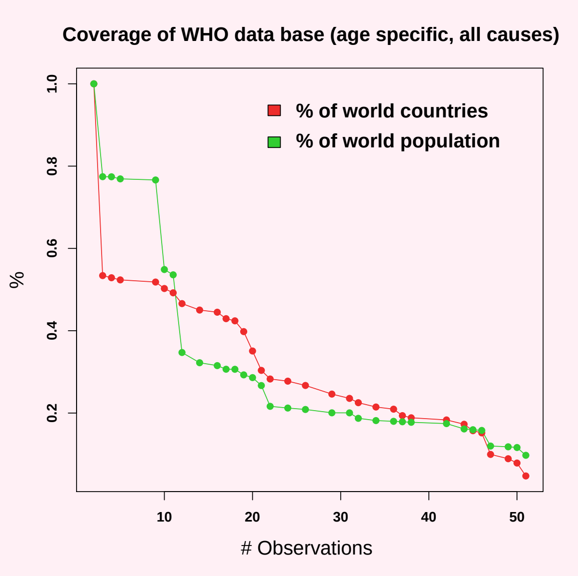
- Covariates with the same name can have different meanings

Many Short Time Series



Preview of Results: Out-of-Sample Evaluation

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Mean Absolute Error in Males (over age and country)

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	Mean Absolute Error			% Improvement	
	Best Previous	Our Method	Best Conceivable	Over Best Previous	to Best Conceivable
Cardiovascular	0.34	0.27	0.19	22	49
Lung Cancer	0.36	0.27	0.17	24	47
Transportation	0.37	0.31	0.18	16	31
Respiratory Chronic	0.45	0.39	0.26	13	30
Other Infectious	0.55	0.48	0.32	12	30
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- Does much better with better covariates