How to Measure Legislative District Compactness If You Only Know it When You See it¹

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¹Based on joint work with Aaron Kaufman and Mayya Komisarchik ²GaryKing.org

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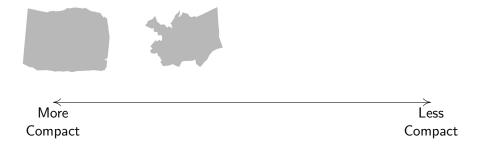
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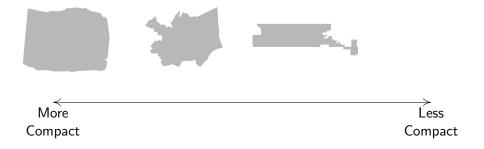
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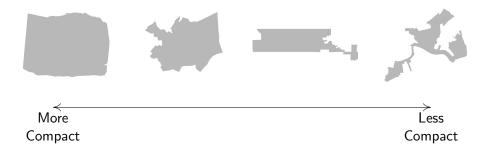
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 - Required in many other jurisdictions



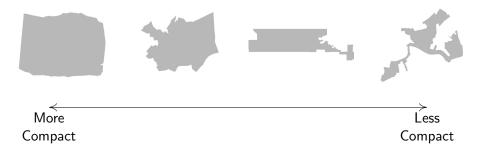




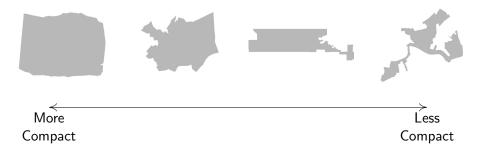




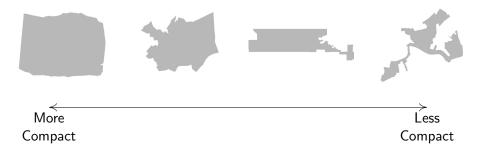
A simple single compactness dimension that you know when you see



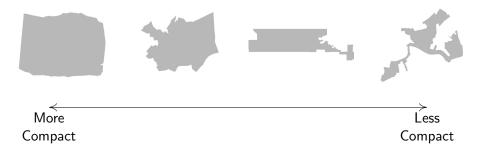
• The dimension is intuitive



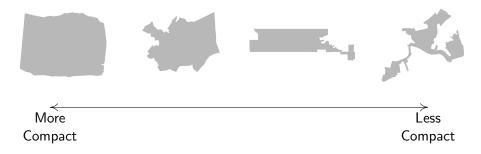
- The dimension is intuitive
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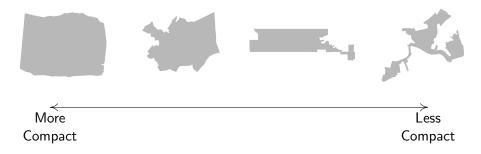
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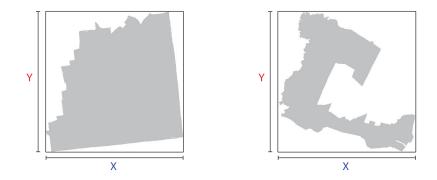
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- $\bullet \, \rightsquigarrow$ Let's start with existing measures by social scientists











Squarish districts more compact than long thin ones



In both districts: $X/Y \approx 1.30$

Circular districts are most compact

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Measure 2: Reock, District / Bounding Circle Areas Circular districts are most compact

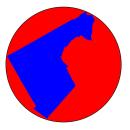


Circular districts are most compact





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In both cases, $X/(Y + X) \approx 0.37$









Measure 3: Boyce-Clark, Variation in Centroid Deviations

All travel distances from center should be similar



In both cases, $MAD(r)/\bar{r} \approx 0.31$

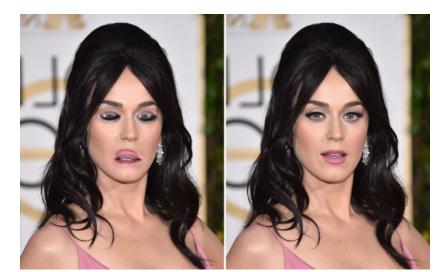
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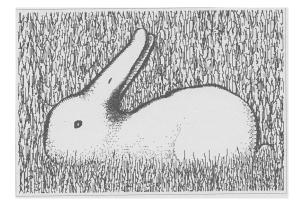
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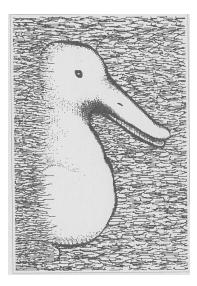
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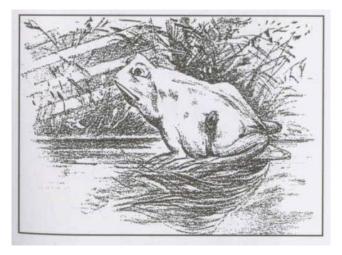
A Brief Interlude on Perception: See the Rabbit?



A Brief Interlude on Perception: See the Rabbit Duck?



A Brief Interlude on Perception: See the Frog?



A Brief Interlude on Perception: See the Frog Horse?



• Existing measures of compactness:

• Nearly 100 proposed

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- Almost all are rotationally invariant

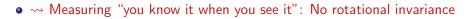
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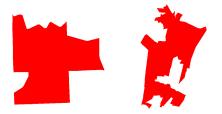
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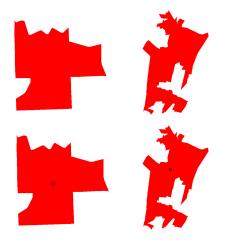


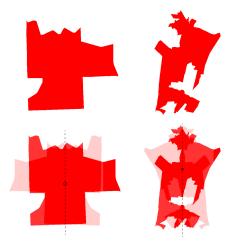
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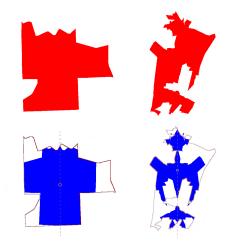


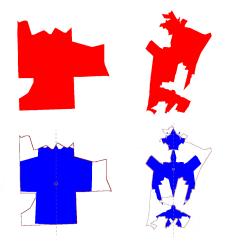
New Measure: Y-Symmetry, area of symmetric reflection











In both cases, Overlap/Original Area ≈ 0.34

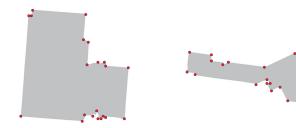
New Measure 2: Number of Visually Significant Corners Computer vision algorithm identifies "objects" in photos

Computer vision algorithm identifies "objects" in photos \rightsquigarrow Fewer corners is more compact

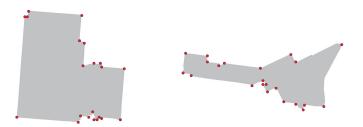
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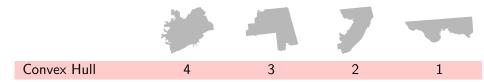


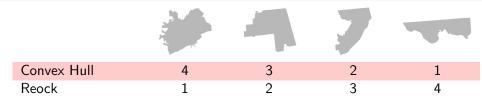
Both districts have 21 significant corners

Which is more compact?

* - 7 -







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Reock	1	2	3	4
Polsby-Popper	4	1	2	3

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- Many more inconsistencies on individual districts

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 - Which dimension? The one we know when we see

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Paired Comparisons (Fechner 1860; Thurstone 1912) v Ranking (very old, rarely used)

Paired Comparison



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Paired Comparison



Utterly fails on inter- and intra-coder reliability

Paired Comparisons (Fechner 1860; Thurstone 1912) v Ranking (very old, rarely used)

Full Ranking



Paired Comparisons (Fechner 1860; Thurstone 1912) v Ranking (very old, rarely used)

Full Ranking — on line

MOST Compact Here



LEAST Compact Here

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We show: very high reliability

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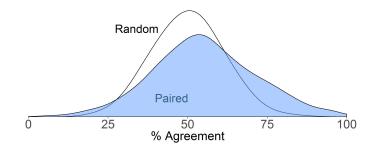
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Intercoder Reliability of Pairs

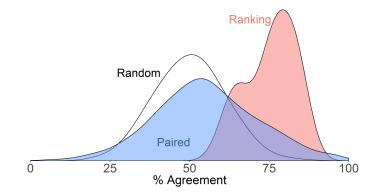
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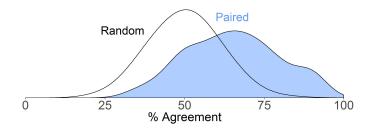
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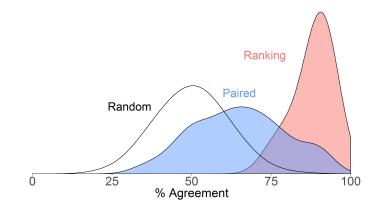
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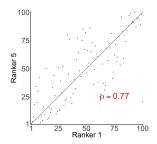
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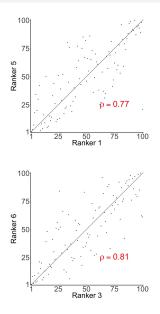


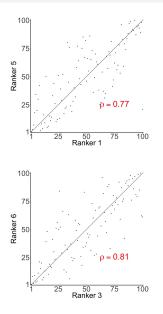
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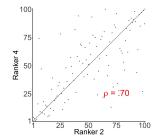
Paired Comparisons: better than chance; Pairs implied by ranks: much better

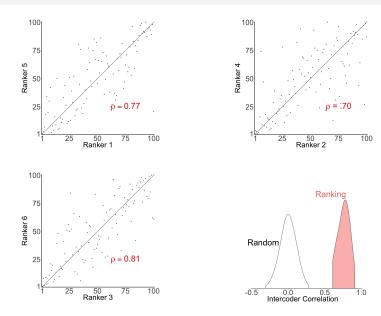


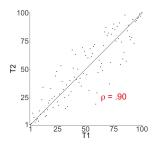


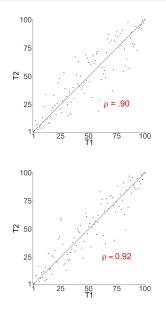


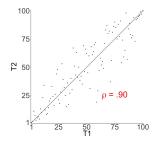


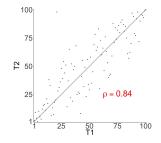


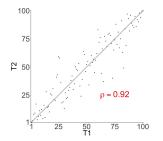


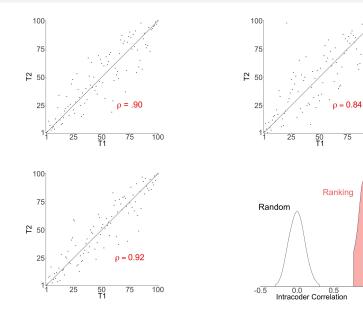












100

1.0

So we can measure it. Can we model it?

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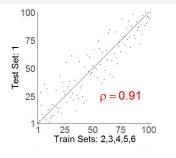
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So we can measure it. Can we model it? Goal: Compactness score = f(shape)

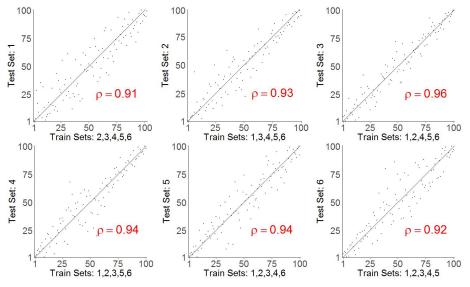
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Predict Test Set from 5 Training Sets

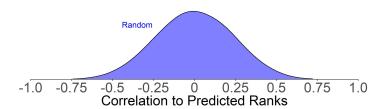
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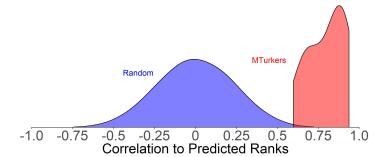


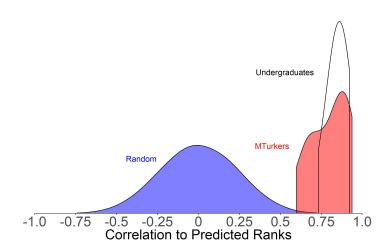
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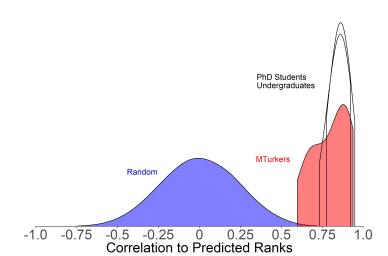


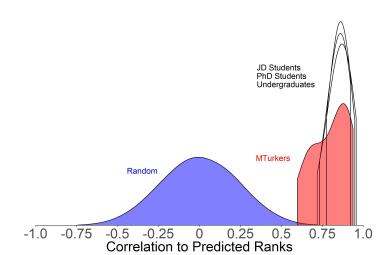
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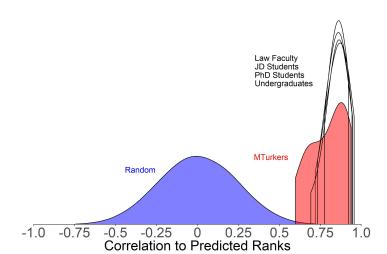


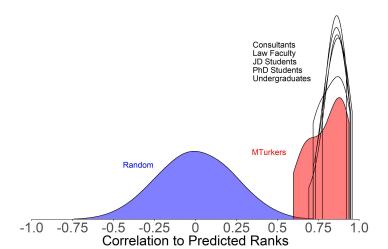


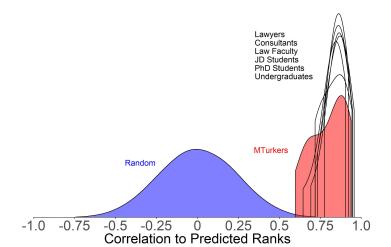


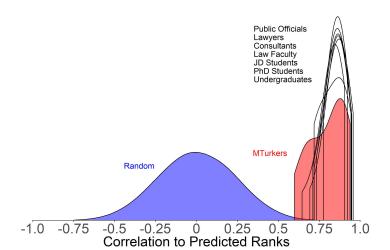


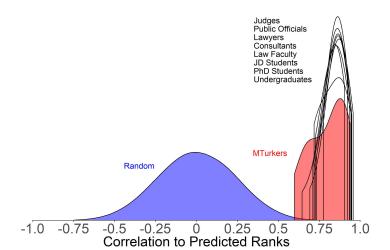




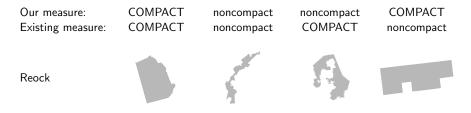


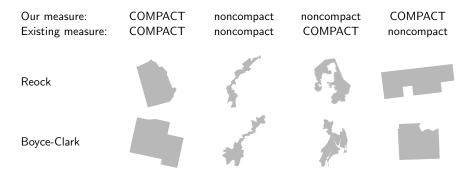


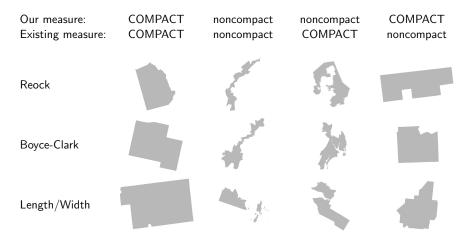


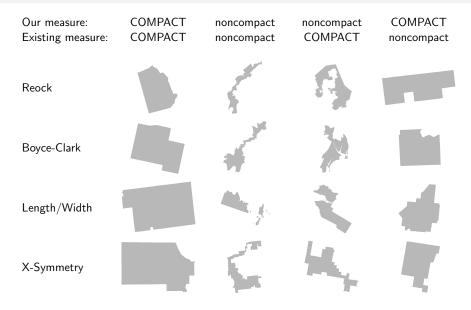


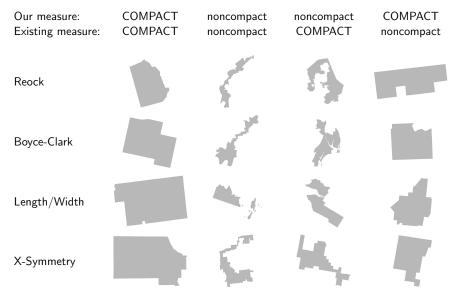
Our measure:	COMPACT	noncompact	noncompact	COMPACT
Existing measure:	COMPACT	noncompact	COMPACT	noncompact











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For more information





AaronRKaufman.com

GaryKing.org



MayyaKomisarchik.com

Paper, data, software, slides: j.mp/Compactness