How to Measure Legislative District Compactness If You Only Know it When You See it¹

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¹Based on joint work with Aaron Kaufman and Mayya Komisarchik

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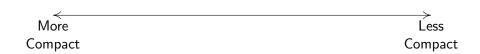
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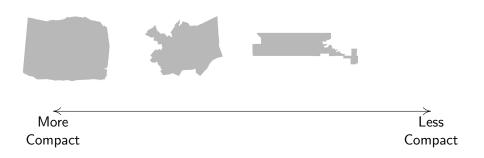
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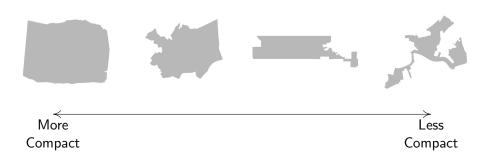
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 - Required in many other jurisdictions



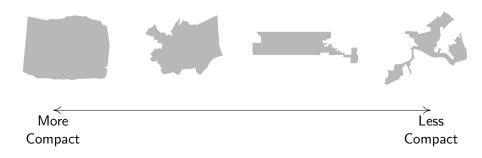




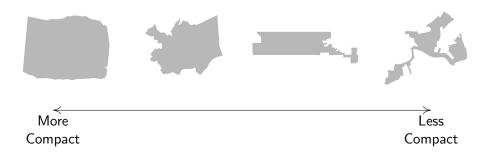




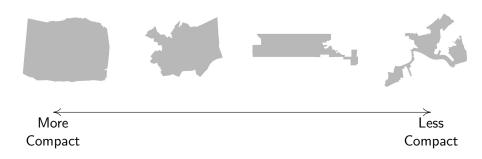
A simple single compactness dimension that you know when you see



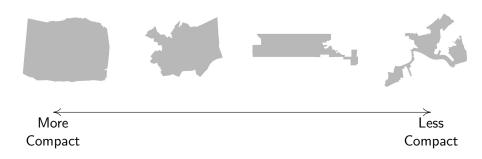
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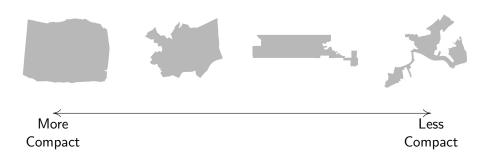
- The dimension is intuitive
- How to estimate where a new district shape falls on this dimension?



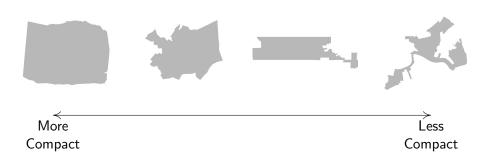
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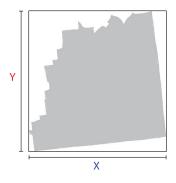
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- \(\sim \) Let's start with existing measures by social scientists

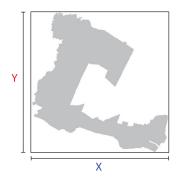




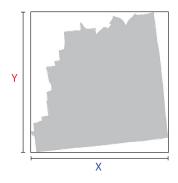


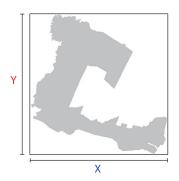






Squarish districts more compact than long thin ones





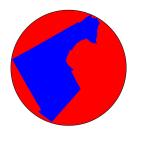
In both districts: $X/Y \approx 1.30$





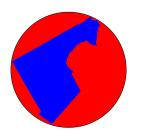






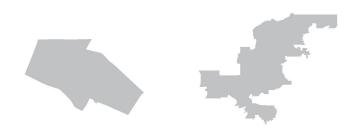


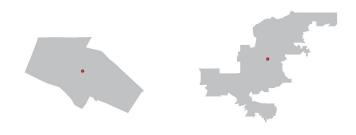
Circular districts are most compact





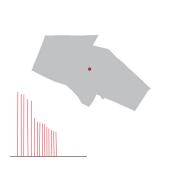
In both cases, $X/(Y + X) \approx 0.37$













Measure 3: Boyce-Clark, Variation in Centroid Deviations

All travel distances from center should be similar

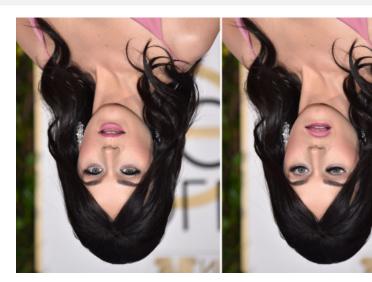


In both cases, $MAD(r)/\bar{r} \approx 0.31$

A Brief Rotational Invariance Interlude:

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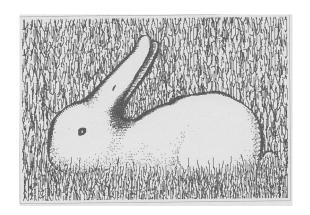


A Brief Rotational Invariance Interlude: Can you Name this Celebrity?

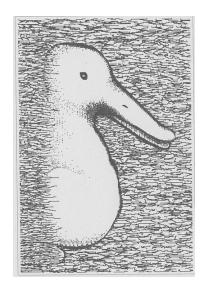




A Brief Interlude on Perception: See the Rabbit?



A Brief Interlude on Perception: See the Rabbit Duck?



A Brief Interlude on Perception: See the Frog?



A Brief Interlude on Perception: See the Frog Horse?



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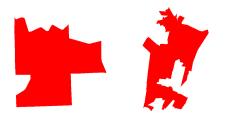
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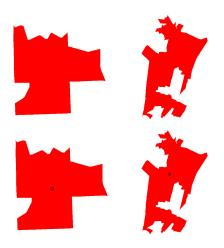


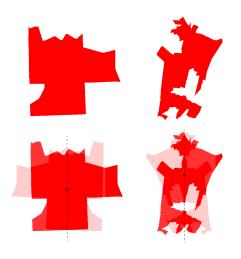
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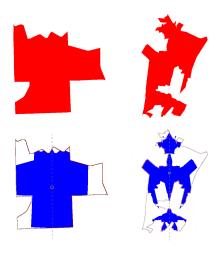


→ Measuring "you know it when you see it": No rotational invariance

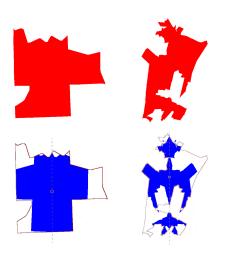








Symmetric figures (circles, squares) are more compact



In both cases, Overlap/Original Area ≈ 0.34

Computer vision algorithm identifies "objects" in photos

Computer vision algorithm identifies "objects" in photos

→ Fewer corners is more compact

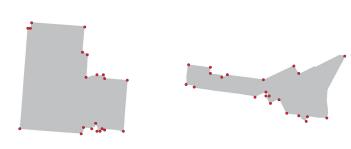
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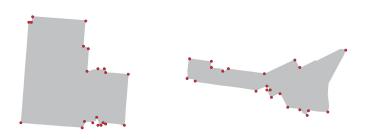
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Both districts have 21 significant corners

Which is more compact?







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Reock	1	2	3	4

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• 7 measures; 7 unique rankings

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- Many more inconsistencies on individual districts

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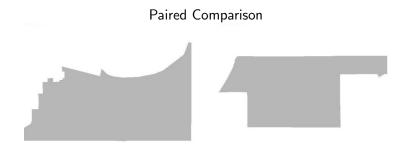
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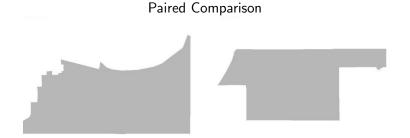
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Paired Comparisons (Fechner 1860; Thurstone 1912) v Ranking (very old, rarely used)



Utterly fails on inter- and intra-coder reliability





Paired Comparisons (Fechner 1860; Thurstone 1912) v Ranking (very old, rarely used)

Full Ranking — on line





LEAST Compact Here

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We show: very high reliability

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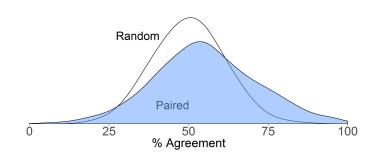
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 Ranking: all evaluations on one dimension of user's choice

Intercoder Reliability of Pairs

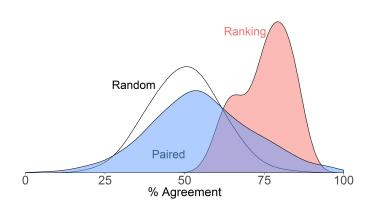
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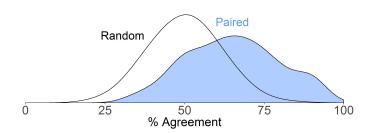
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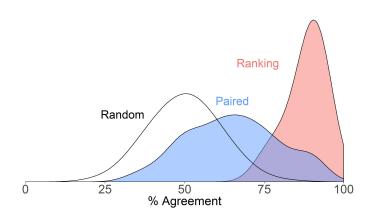
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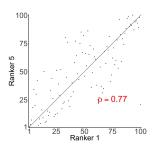
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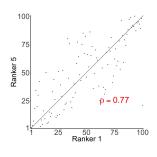


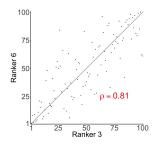
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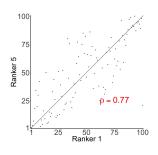
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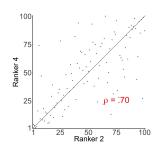


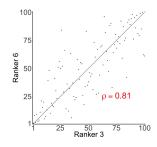


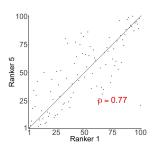


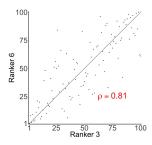


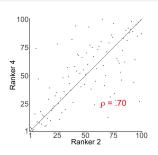


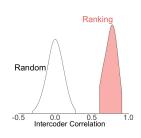


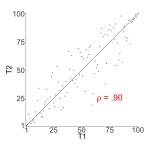


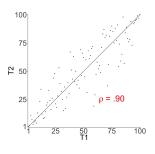


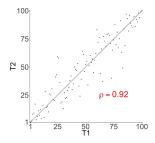


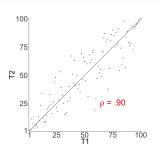


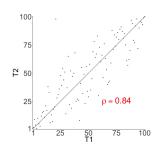


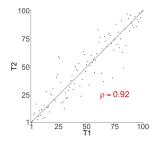


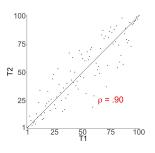


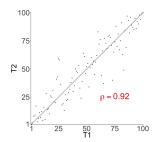


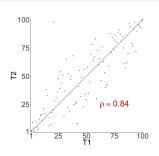


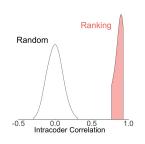












Goal: Compactness score = f(shape)

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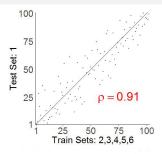
So we can measure it. Can we model it?

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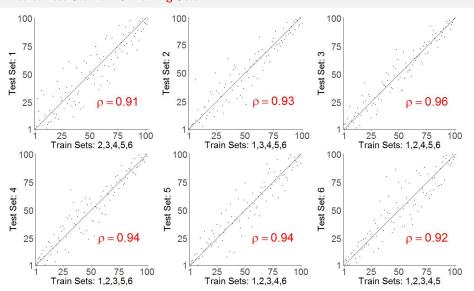
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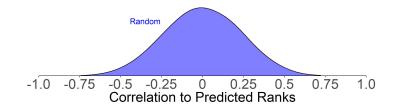
Predict Test Set from 5 Training Sets

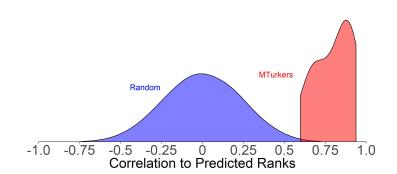
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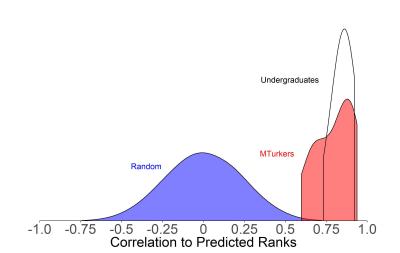


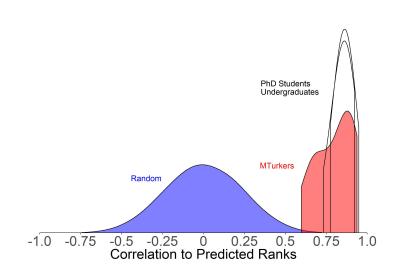
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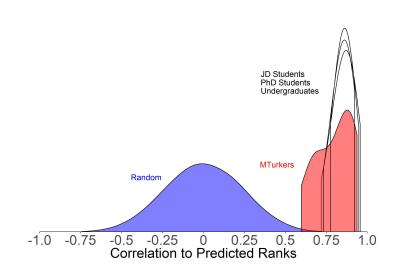


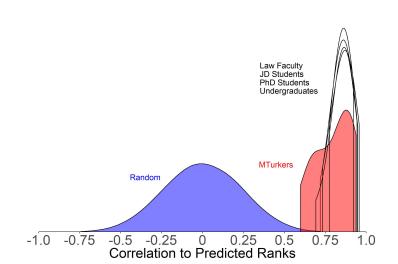


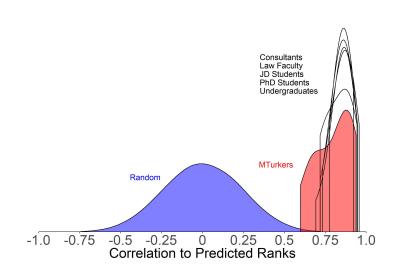


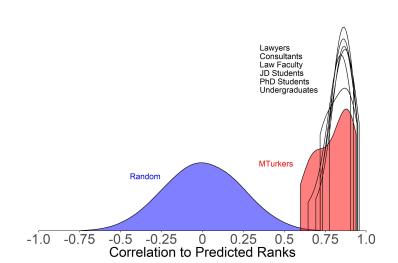


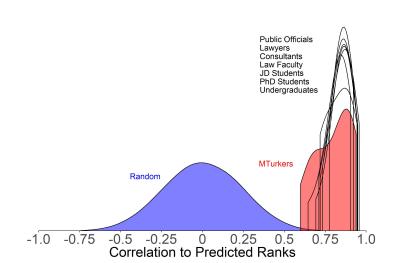


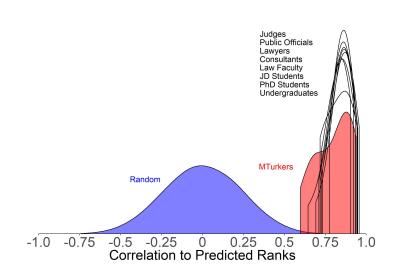












Our measure: Existing measure:

COMPACT COMPACT

noncompact noncompact noncompact COMPACT COMPACT noncompact

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 $\begin{array}{c} \text{noncompact} \\ \text{COMPACT} \end{array}$

COMPACT noncompact

Reock









Our measure: Existing measure:

COMPACT COMPACT

noncompact noncompact

noncompact COMPACT

COMPACT noncompact

Reock

Sparker .





Boyce-Clark









COMPACT Our measure: noncompact **COMPACT** noncompact **COMPACT** COMPACT Existing measure: noncompact noncompact Reock Boyce-Clark Length/Width

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Reock		S. Carlotte		
Boyce-Clark		aff when		
Length/Width			1	4
X-Symmetry			H	

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 - Software to calculate compactness from any district shape

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For more information







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Paper, data, software, slides: j.mp/Compactness