How to Measure Legislative District Compactness If You Only Know it When You See it¹

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¹Based on joint work with Aaron Kaufman and Mayya Komisarchik

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 - Laptops beat supercomputers!

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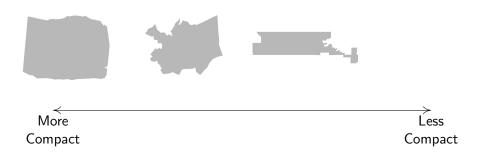
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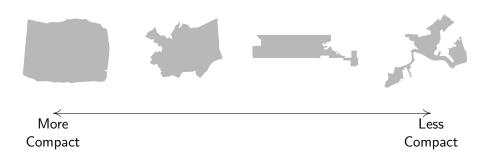
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 - Required in many other jurisdictions



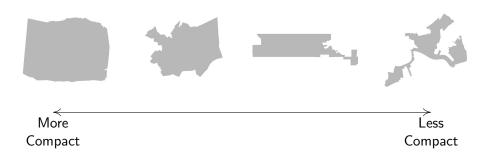




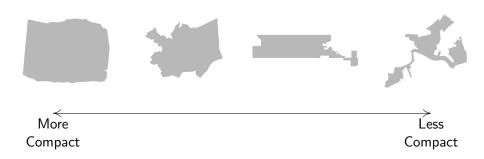




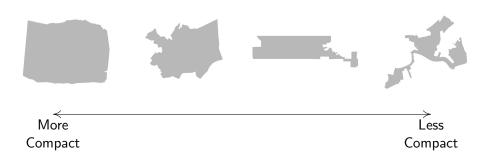
A simple single compactness dimension that you know when you see



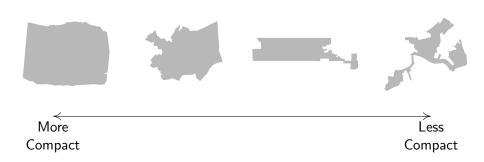
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- What is the start with existing measures by social scientists

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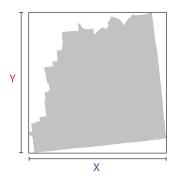
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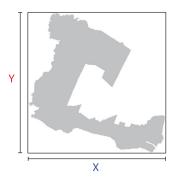




Measure 1: Length/Width Ratio of Min Bounding Box

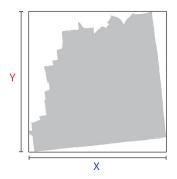
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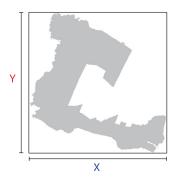




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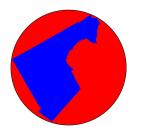
In both districts: $X/Y \approx 1.30$





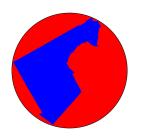






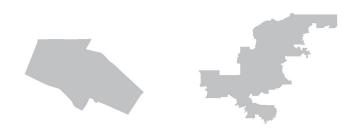


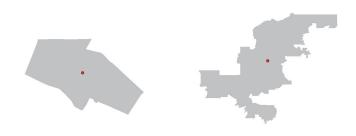
Circular districts are most compact

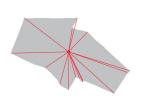




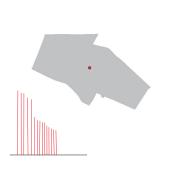
In both cases, $X/(Y + X) \approx 0.37$















In both cases, $MAD(r)/\bar{r} \approx 0.31$

A Brief Rotational Invariance Interlude:

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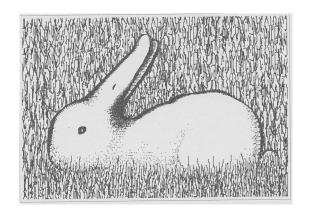




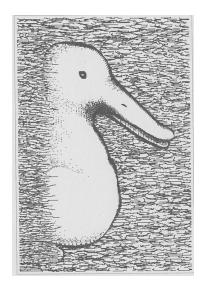
A Brief Rotational Invariance Interlude: Can you Name this Celebrity?



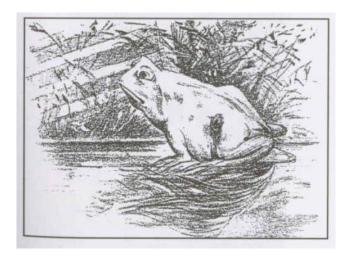
A Brief Interlude on Perception: See the Rabbit?



A Brief Interlude on Perception: See the Rabbit Duck?



A Brief Interlude on Perception: See the Frog?



A Brief Interlude on Perception: See the Frog Horse?



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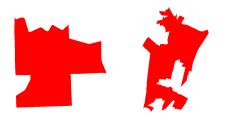
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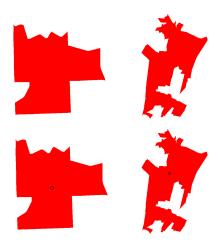


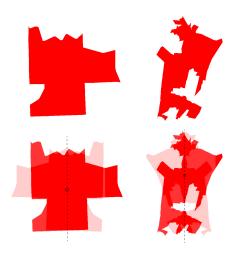
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→ Measuring "you know it when you see it": No rotational invariance

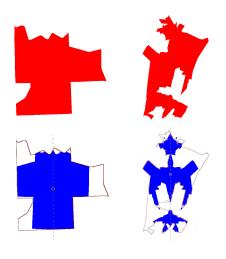






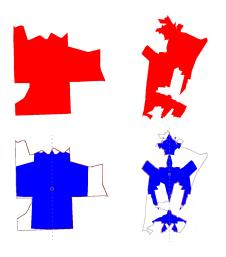
New Measure: Y-Symmetry, area of symmetric reflection

Symmetric figures (circles, squares) are more compact



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In both cases, Overlap/Original Area ≈ 0.34

Computer vision algorithm identifies "objects" in photos

Computer vision algorithm identifies "objects" in photos → Fewer corners is more compact

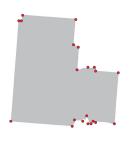
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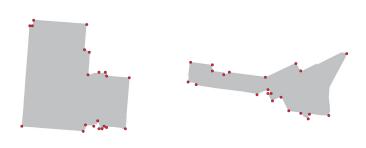
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Computer vision algorithm identifies "objects" in photos

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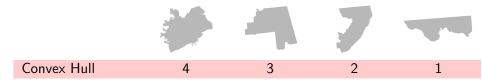


Both districts have 21 significant corners

Which is more compact?







	3			
Convex Hull	4	3	2	1
Reock	1	2	3	4

Convex Hull	4	3	2	1
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		-	7	
Convex Hull	4	3	2	1
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X-Axis Symmetry	1	4	3	2

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• 7 measures; 7 unique rankings

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3	2	1	4
1	4	3	2
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	4 1 4 2 3 1 4	4 3 1 2 4 1 2 3 3 2 1 4 4 1	4 3 2 1 2 3 4 1 2 2 3 1 3 2 1 1 4 3 4 1 3

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- 7 measures; 7 unique rankings
- Unusual? From 18,215 Congressional and State Legislative Districts, we found 162 trillion others (about 0.15%)
- Many more inconsistencies on individual districts

• (Recall) The concept of compactness

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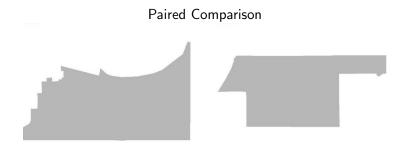
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 - I.e., estimate the one dimension of legal interest; show it has:

Spanning the Academic-Legal Divide

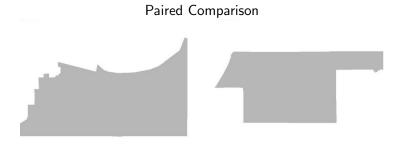
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 - high predictive accuracy



Paired Comparisons (Fechner 1860; Thurstone 1912) v Ranking (very old, rarely used)



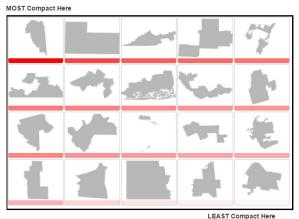
Utterly fails on inter- and intra-coder reliability





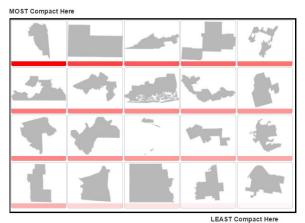
Paired Comparisons (Fechner 1860; Thurstone 1912) v Ranking (very old, rarely used)

Full Ranking — on line



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We show: very high reliability

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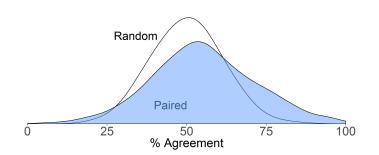
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 - Paired Comparisons can be answered on different dimensions
 Ranking: all evaluations on one dimension of user's choice

Intercoder Reliability of Pairs

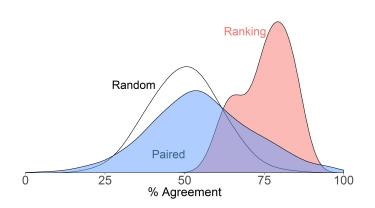
Intercoder Reliability of Pairs

Paired Comparisons: only slightly better than chance;



Intercoder Reliability of Pairs

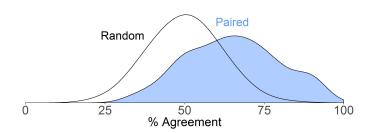
Paired Comparisons: only slightly better than chance; Pairs implied by ranks: better



Intracoder Reliability of Pairs

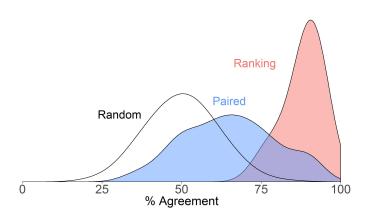
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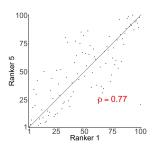
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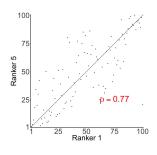


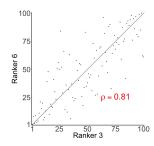
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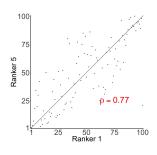
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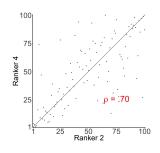


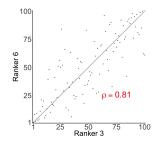


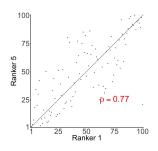


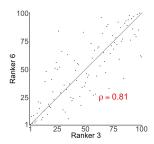


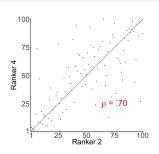


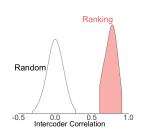


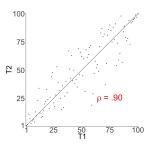


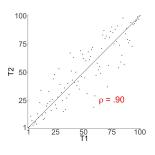


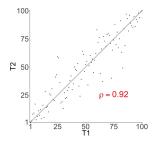


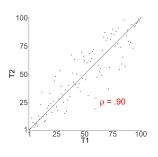


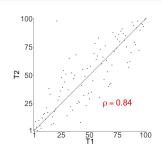


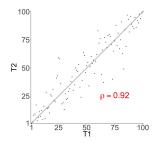


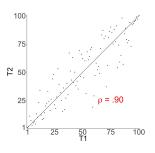


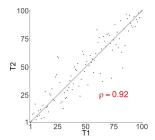


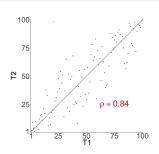


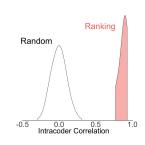












Goal: Compactness score = f(shape)

• Training data: Outcome variable from human rankings

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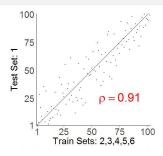
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 - Tell! squarish, with minimal arms, pockets, islands, or jagged edges

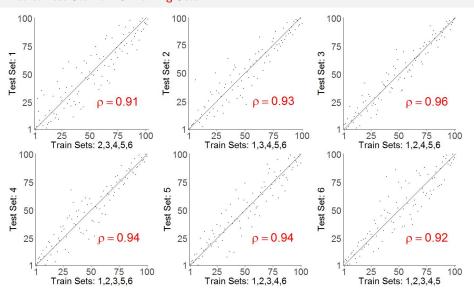
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 - (Not a description of any one existing measure)

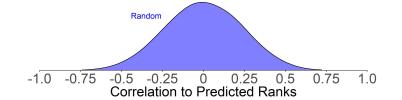
Predict Test Set from 5 Training Sets

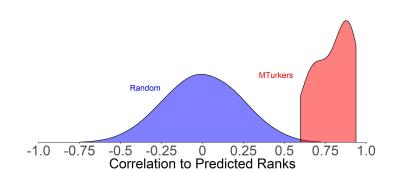
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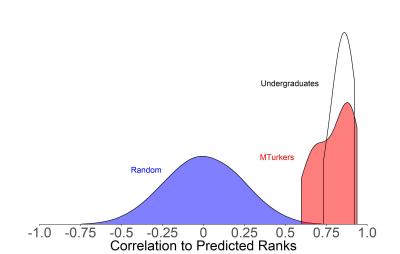


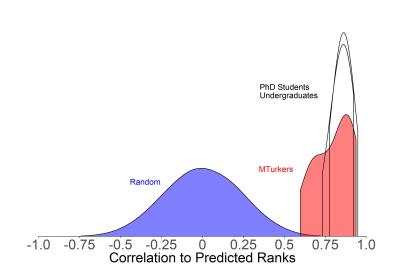
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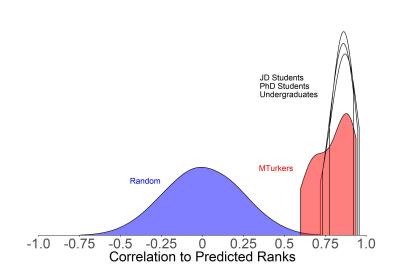


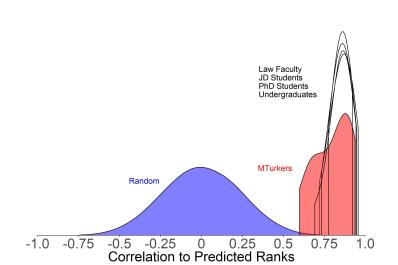


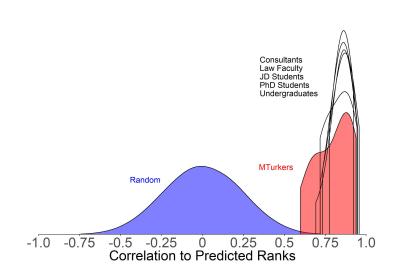


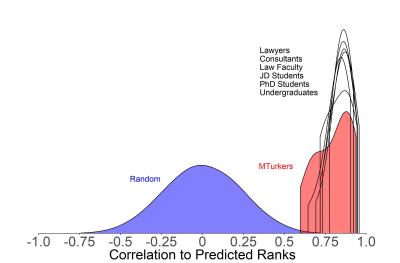


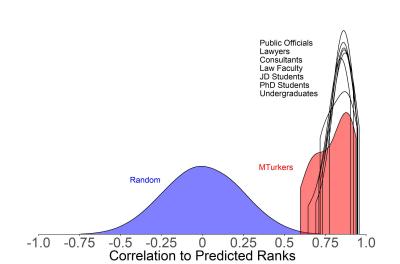


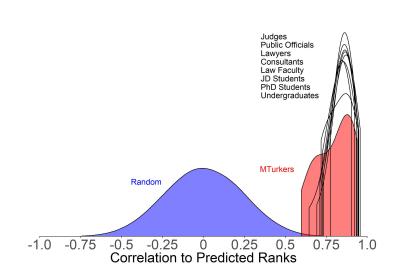












Our measure: Existing measure:

COMPACT COMPACT noncompact noncompact

noncompact COMPACT COMPACT noncompact

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COMPACT COMPACT

noncompact noncompact

 $\begin{array}{c} \mathsf{noncompact} \\ \mathsf{COMPACT} \end{array}$

COMPACT noncompact

Reock









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COMPACT COMPACT

noncompact noncompact

noncompact COMPACT

COMPACT noncompact

Reock



September 1





Boyce-Clark









COMPACT COMPACT Our measure: noncompact noncompact Existing measure: **COMPACT COMPACT** noncompact noncompact Reock Boyce-Clark Length/Width

Our measure: Existing measure:	COMPACT COMPACT	noncompact noncompact	noncompact COMPACT	COMPACT noncompact
Reock		S. Carrier		
Boyce-Clark		3 P. J. W.		
Length/Width		A. A.	1	4
X-Symmetry			H	

What do you think?

Our measure: Existing measure:	COMPACT COMPACT	noncompact noncompact	noncompact COMPACT	COMPACT noncompact
Reock		S. Carlotte		
Boyce-Clark		affirmation.		
Length/Width				4
X-Symmetry			HA.	

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For more information



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Paper, data, software, slides: j.mp/Compactness