

How to Measure Legislative District Compactness If You Only Know it When You See it¹

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Society for Political Methodology, University of Wisconsin, 7/14/2017

¹Paper available at j.mp/Compactness

Redistricting Defines Democracy — & Needs Fixing

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 - Constrain redistricters.
 - E.g., Population equality, partisan fairness, racial fairness, respect for municipal boundaries . . . **compactness**

The Discipline & Redistricting

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- Political science contributions to the real world

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 - Required in many other jurisdictions

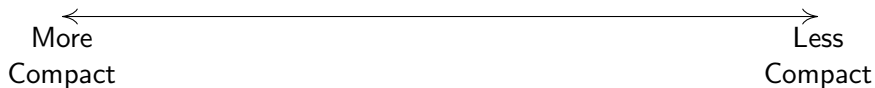
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A simple single compactness dimension that you know when you see

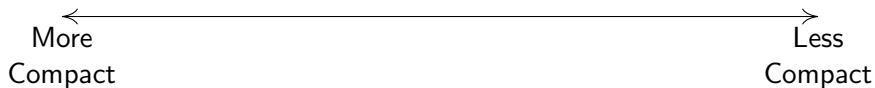
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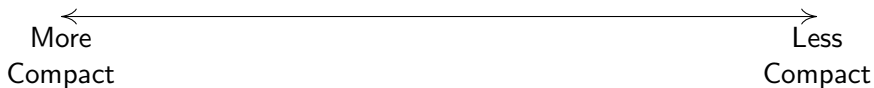
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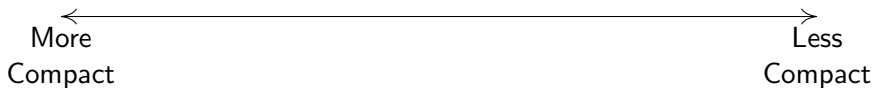
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←
More
Compact

→
Less
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- The dimension is intuitive

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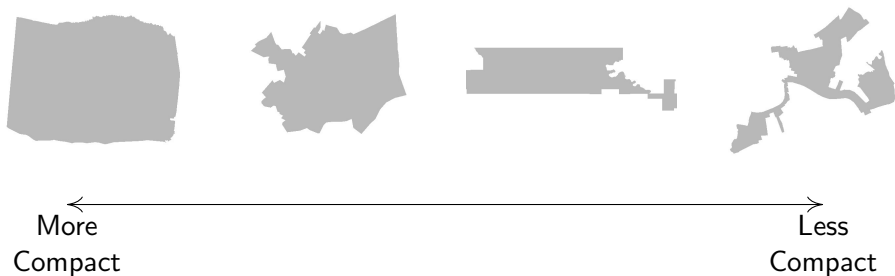
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- The dimension **is** intuitive
- How to **estimate** where a new district shape falls on this dimension?

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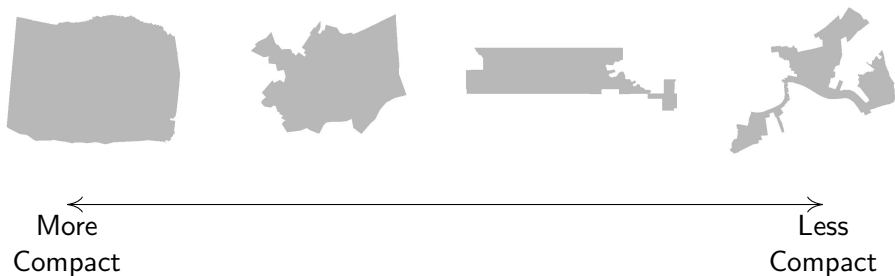
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- **↪ Let's start with existing measures by social scientists**

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Squarish districts more compact than long thin ones

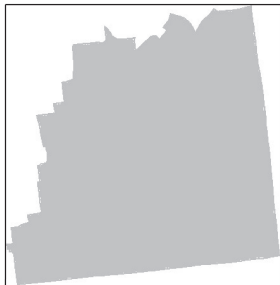
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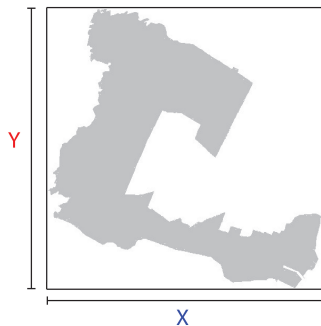
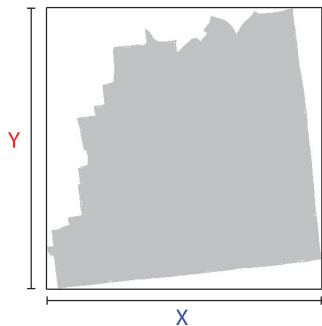
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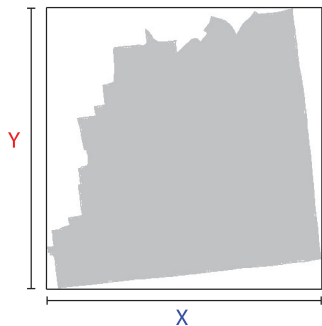
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In both districts: $X/Y \approx 1.30$

Measure 2: Reock, District / Bounding Circle Areas

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Circular districts are most compact

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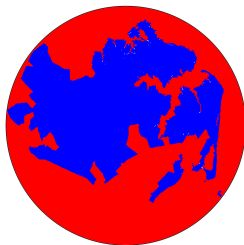
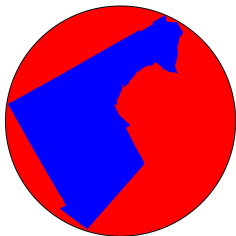
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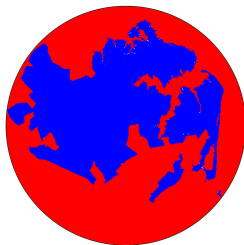
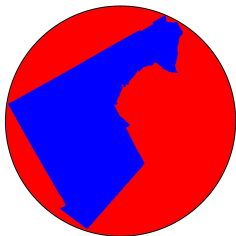
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In both cases, $X/(Y + X) \approx 0.37$

Measure 3: Boyce-Clark, Variation in Centroid Deviations

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All travel distances from center should be similar

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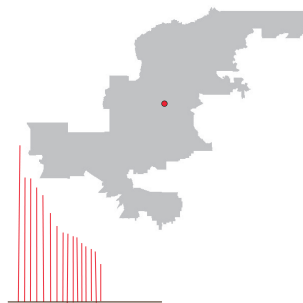
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In both cases, $\text{MAD}(r)/\bar{r} \approx 0.31$

A Brief Interlude: Can you Name this Celebrity?

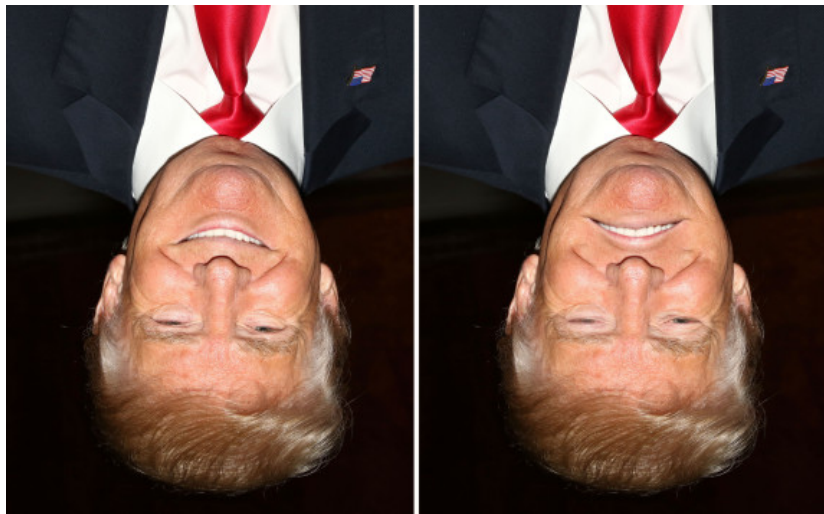
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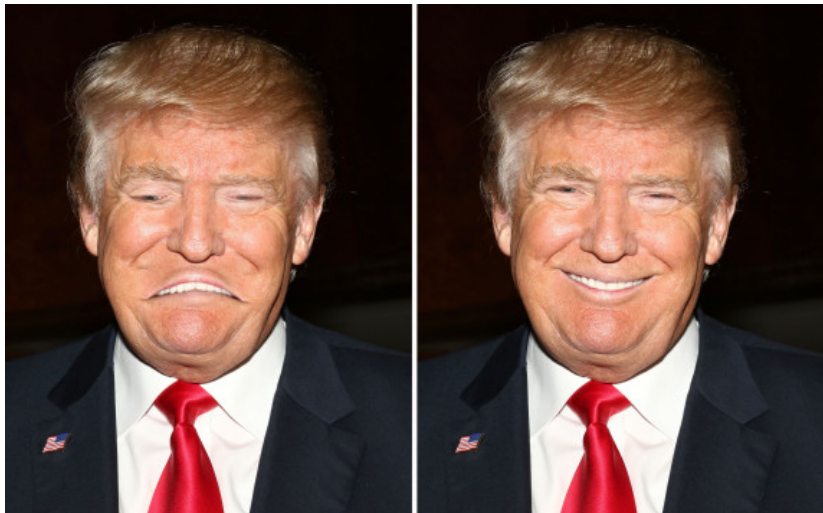
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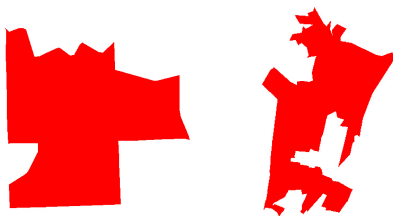
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Symmetric figures (circles, squares) are more compact

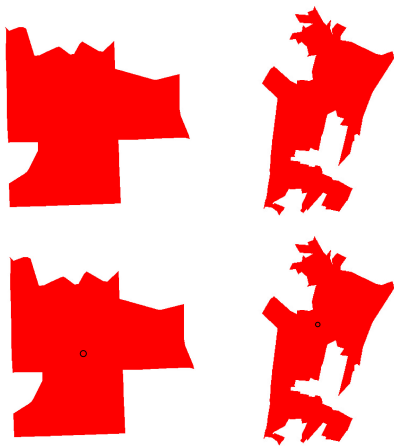
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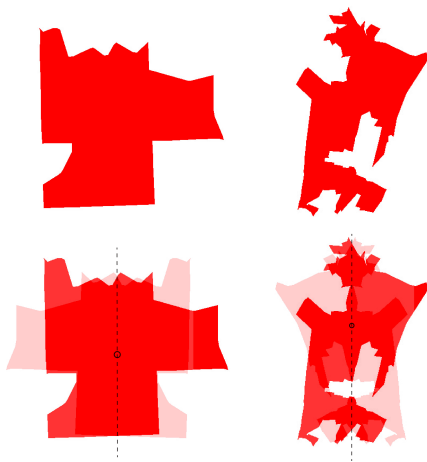
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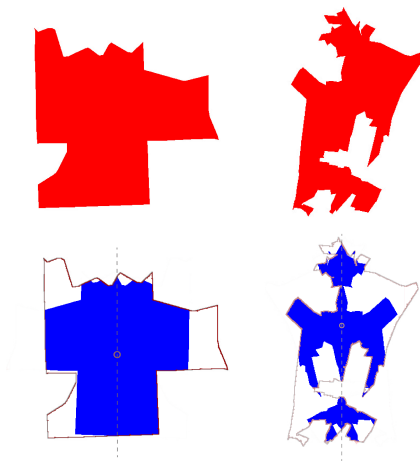
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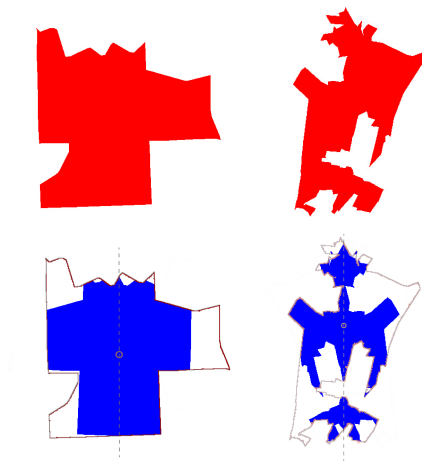
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In both cases, $\text{Overlap} / \text{Original Area} \approx 0.34$

New Measure 2: Number of Visually Significant Corners

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Computer vision algorithm identifies “objects” in photos

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~> Fewer corners is more compact

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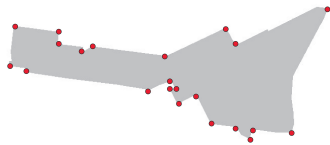
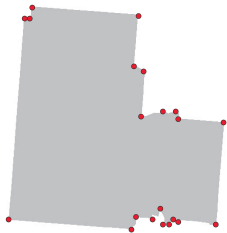
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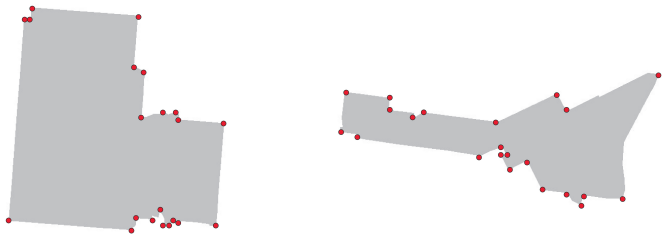
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New Measure 2: Number of Visually Significant Corners

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Both districts have 21 significant corners

Which is more compact?



Which is more compact? Depends on the standard!



Which is more compact? Depends on the standard!



Convex Hull

4

3

2

1

Which is more compact? Depends on the standard!



Convex Hull

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3

2

1

Reock

1

2

3

4

Which is more compact? Depends on the standard!



Convex Hull	4	3	2	1
Reock	1	2	3	4
Polsby-Popper	4	1	2	3

Which is more compact? Depends on the standard!



Convex Hull	4	3	2	1
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Boyce-Clark	2	3	1	4

Which is more compact? Depends on the standard!



Convex Hull	4	3	2	1
Reock	1	2	3	4
Polsby-Popper	4	1	2	3
Boyce-Clark	2	3	1	4
Length/Width	3	2	1	4

Which is more compact? Depends on the standard!



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Reock	1	2	3	4
Polsby-Popper	4	1	2	3
Boyce-Clark	2	3	1	4
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X-Axis Symmetry	1	4	3	2

Which is more compact? Depends on the standard!



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Significant Corners	4	1	3	2

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- 7 measures;

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Length/Width	3	2	1	4
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- 7 measures; 7 unique rankings

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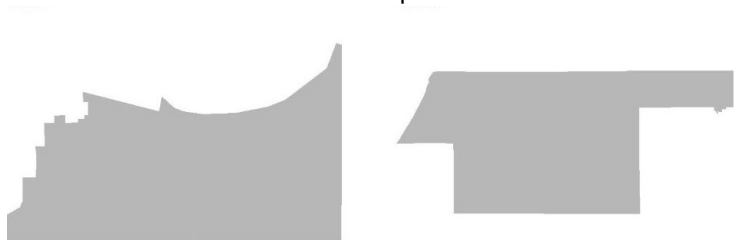
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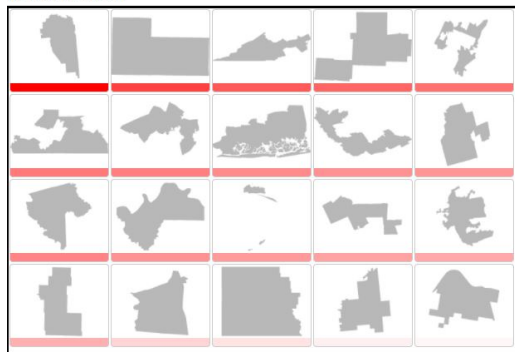
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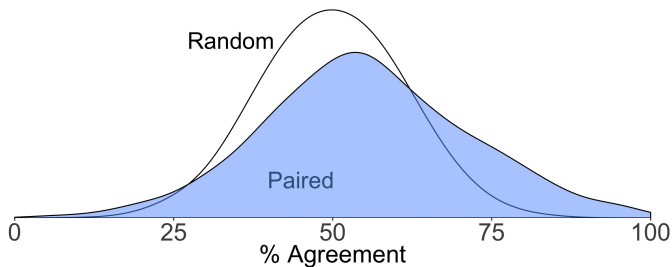
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Ranking: all evaluations on **one dimension** of user's choice

Intercoder Reliability of Pairs

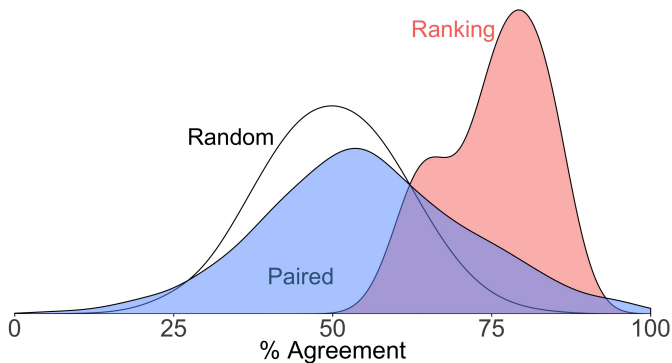
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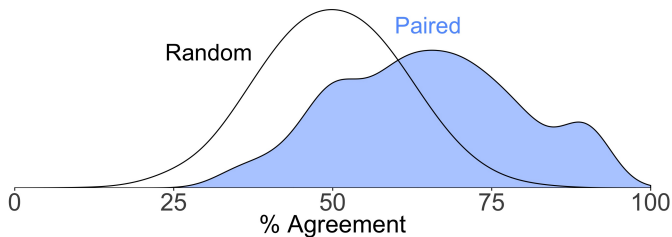
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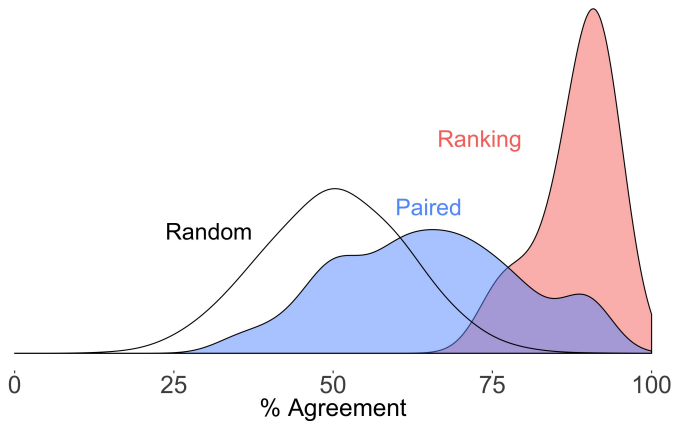
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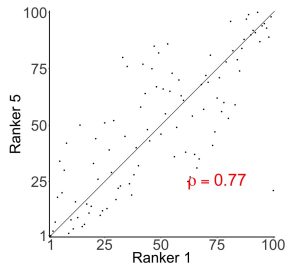


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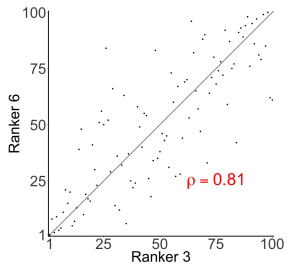
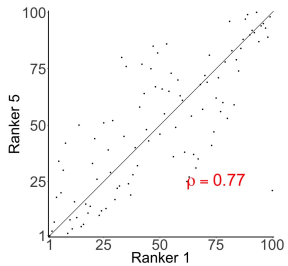
Paired Comparisons: better than chance; Ranking: much better



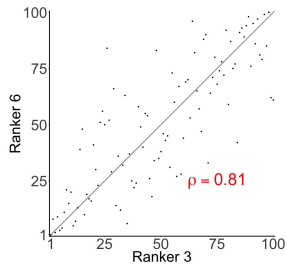
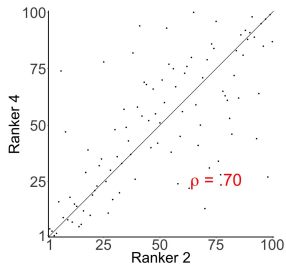
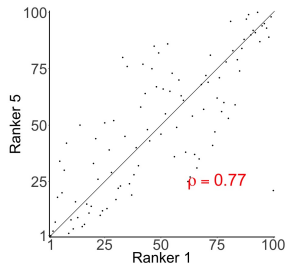
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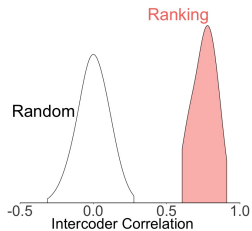
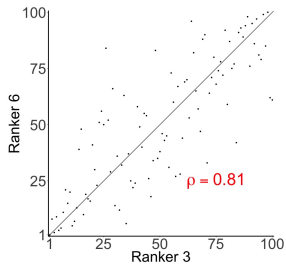
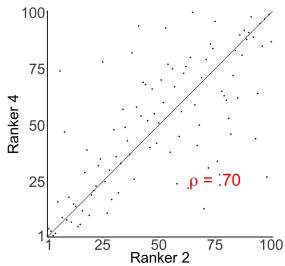
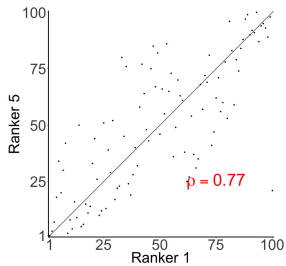
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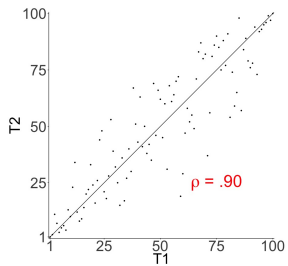
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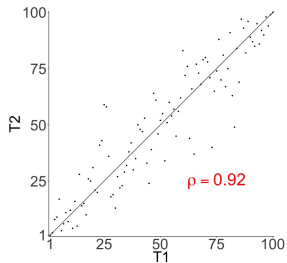
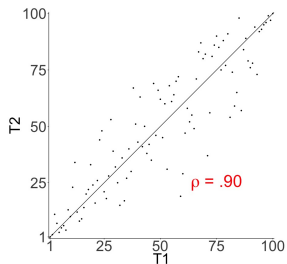
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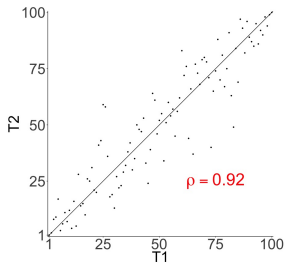
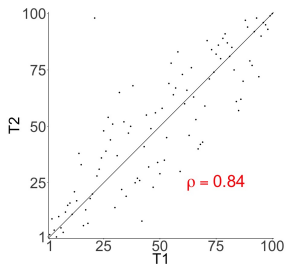
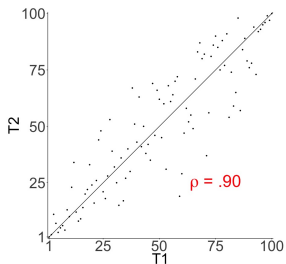
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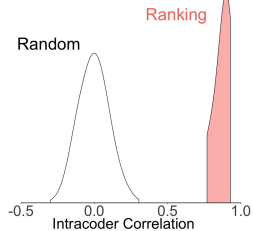
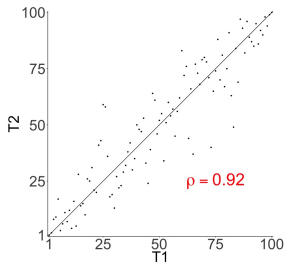
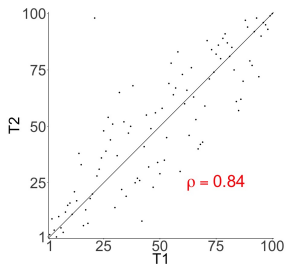
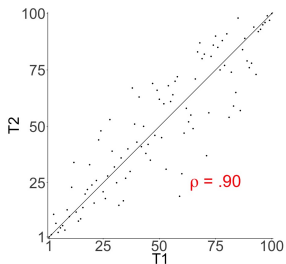
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Goal: Compactness score = $f(\text{shape})$

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- **Ensemble of predictive methods:** least squares, AdaBoosted decision trees, SVM, random forests. . .

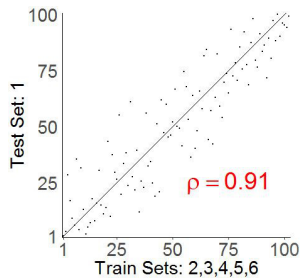
Model Validation: 6-Fold Cross-validation

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Predict Test Set from 5 Training Sets

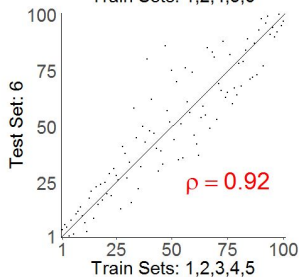
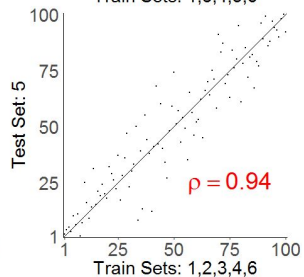
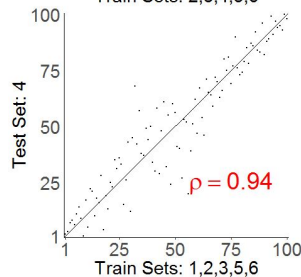
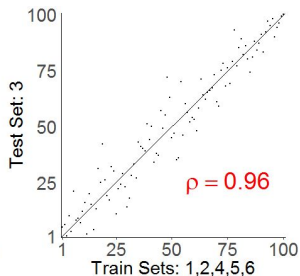
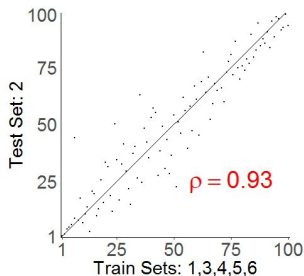
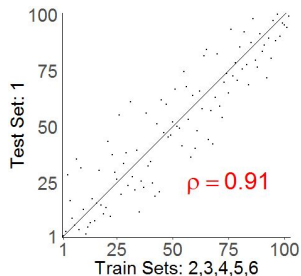
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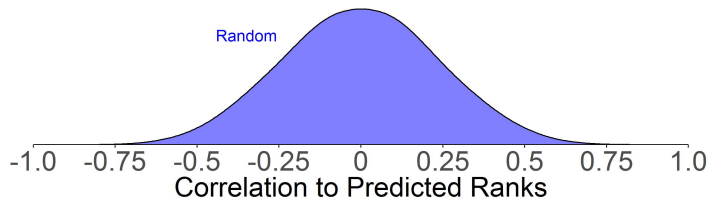
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Respondents ranging from ordinary citizens to those responsible for redistricting

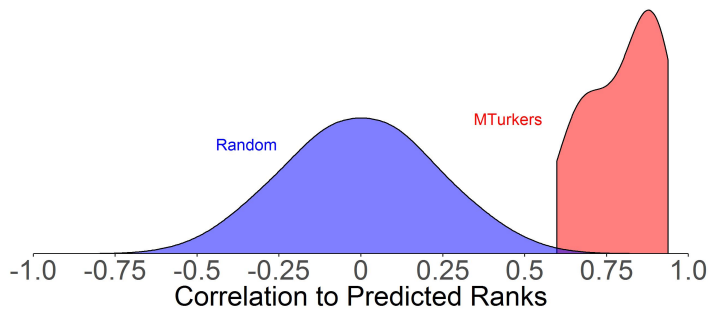
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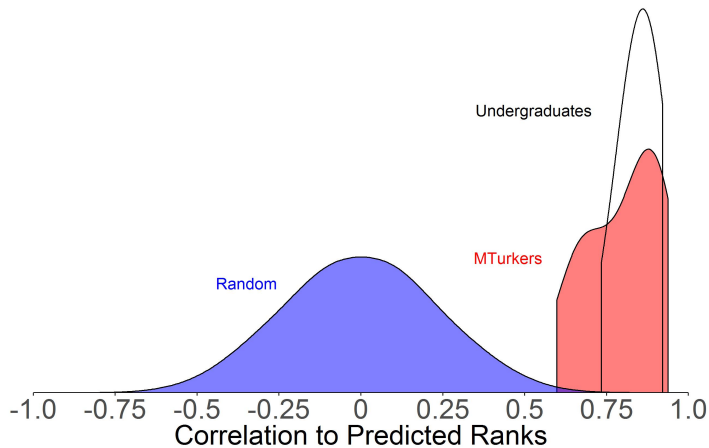
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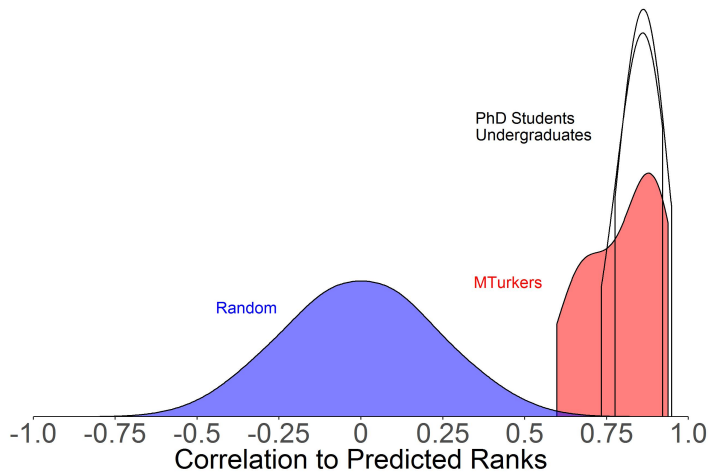
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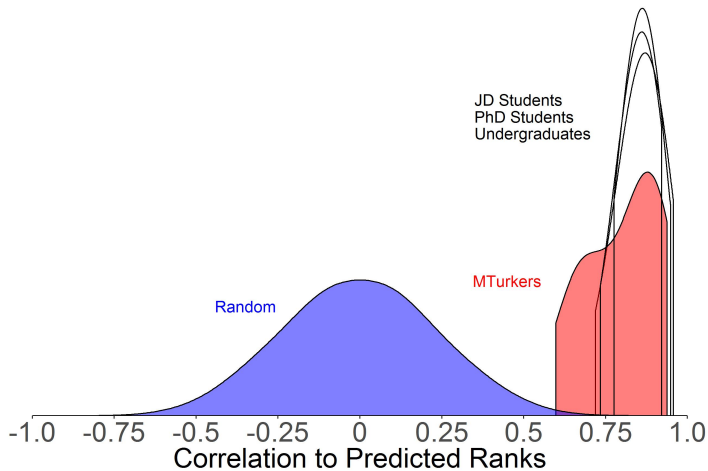
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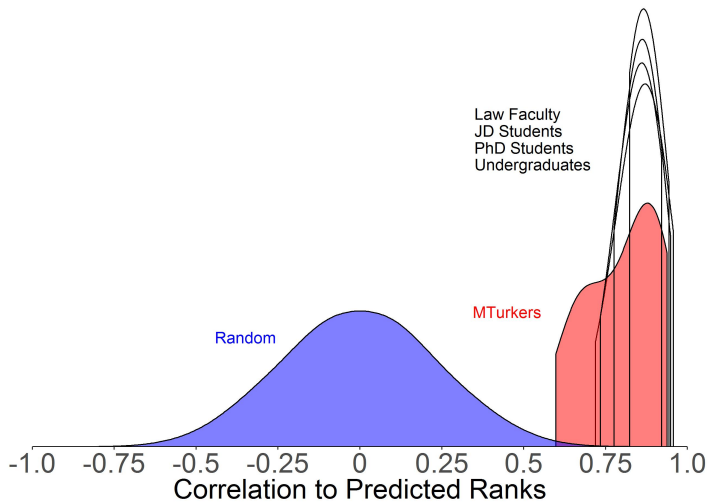
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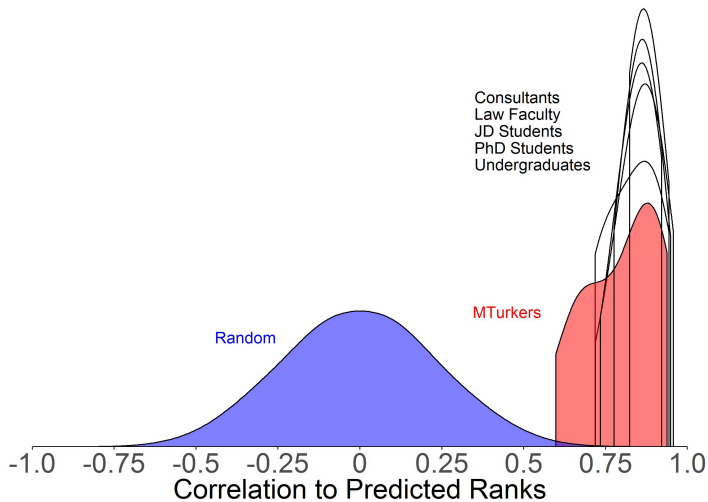
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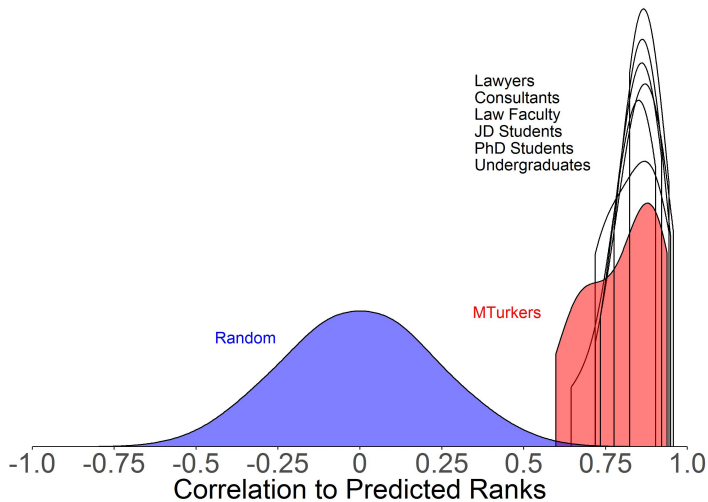
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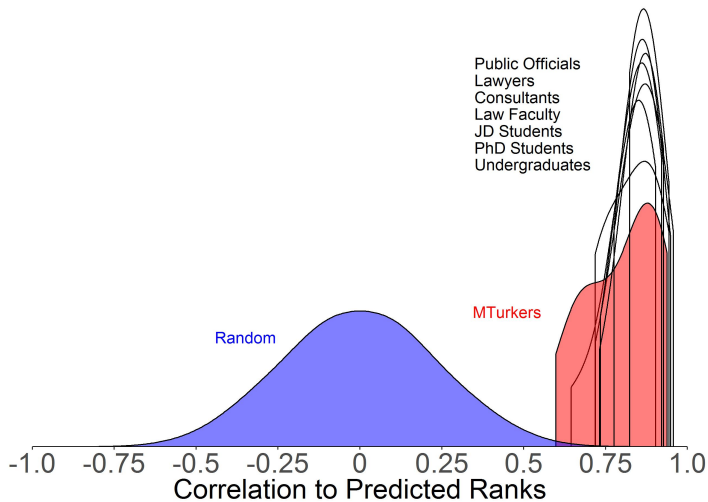
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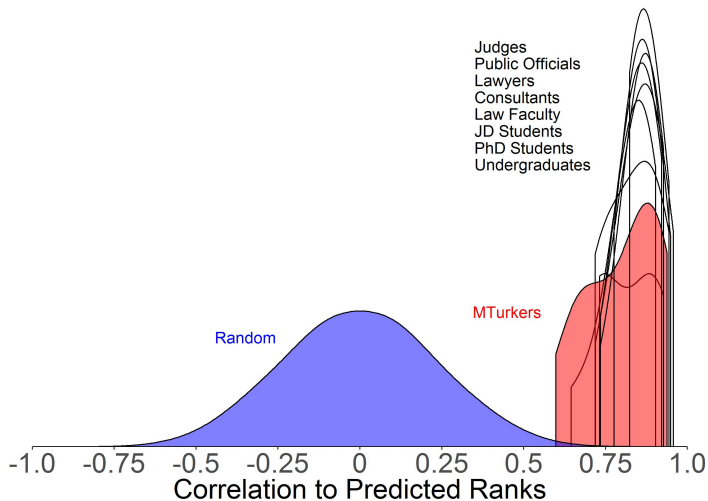
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