How to Measure Legislative District Compactness If You Only Know it When You See it¹

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¹Based on joint work with Aaron Kaufman and Mayya Komisarchik

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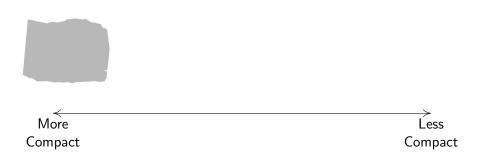
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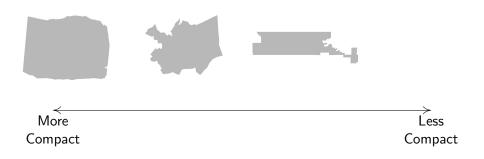
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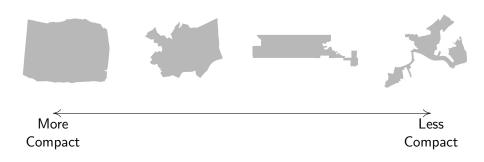
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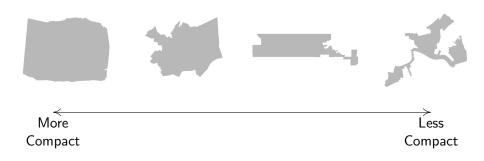




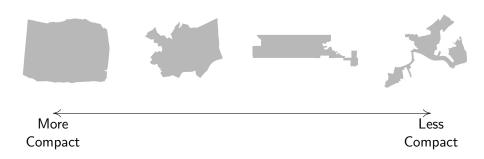




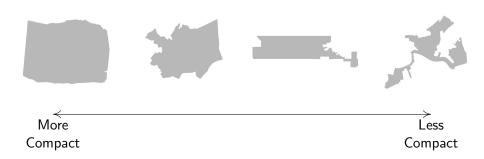
A simple single compactness dimension that you know when you see



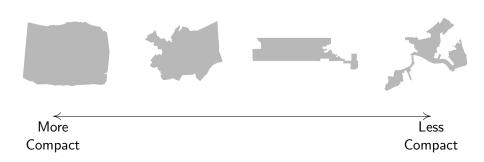
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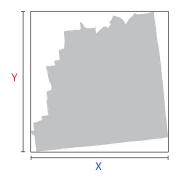
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- What is the start with existing measures by social scientists

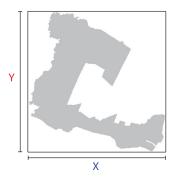




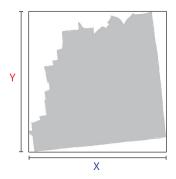


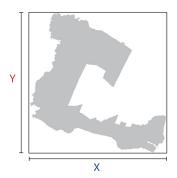






Squarish districts more compact than long thin ones





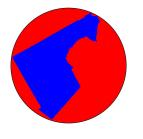
In both districts: $X/Y \approx 1.30$





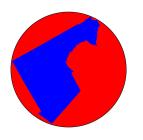






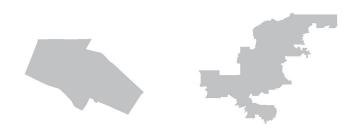


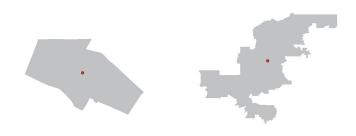
Circular districts are most compact





In both cases, $X/(Y + X) \approx 0.37$















In both cases, $MAD(r)/\bar{r} \approx 0.31$

A Brief Rotational Invariance Interlude:

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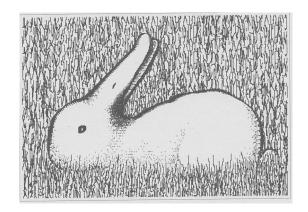




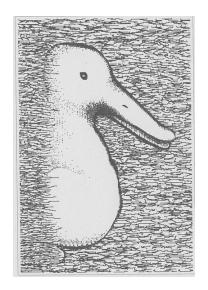
A Brief Rotational Invariance Interlude: Can you Name this Celebrity?



A Brief Interlude on Perception: See the Rabbit?



A Brief Interlude on Perception: See the Rabbit Duck?



A Brief Interlude on Perception: See the Frog?



A Brief Interlude on Perception: See the Frog Horse?



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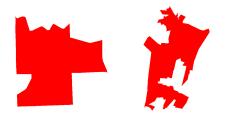
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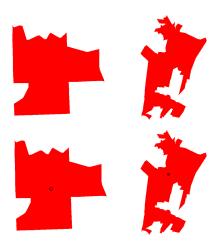


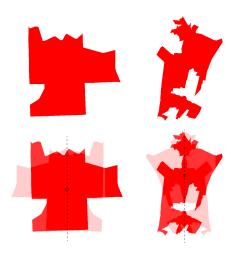
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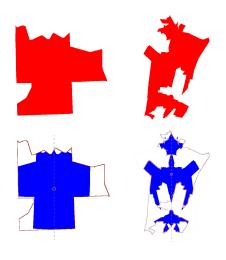


→ Measuring "you know it when you see it": No rotational invariance

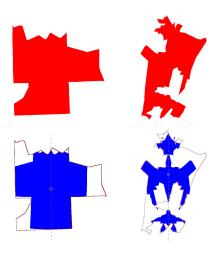








Symmetric figures (circles, squares) are more compact



In both cases, Overlap/Original Area ≈ 0.34

Computer vision algorithm identifies "objects" in photos

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→ Fewer corners is more compact

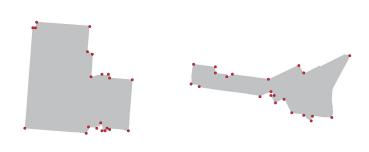
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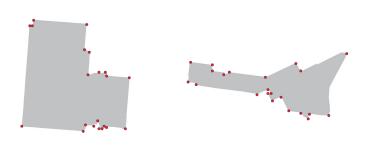
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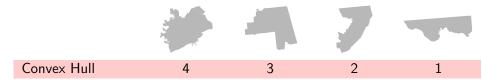


Both districts have 21 significant corners

Which is more compact?







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Reock	1	2	3	4

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- Many more inconsistencies on individual districts

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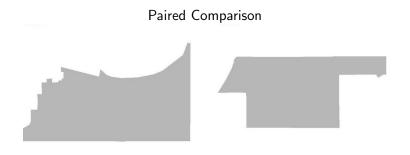
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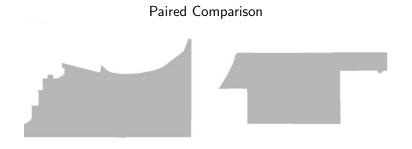
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Paired Comparisons (Fechner 1860; Thurstone 1912) v Ranking (very old, rarely used)



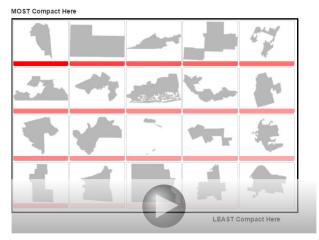
Utterly fails on inter- and intra-coder reliability





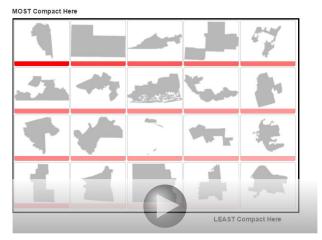
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Full Ranking — on line



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We show: very high reliability

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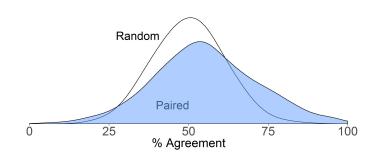
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 Ranking: all evaluations on one dimension of user's choice

Intercoder Reliability of Pairs

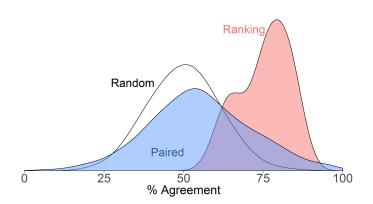
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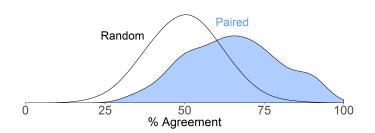
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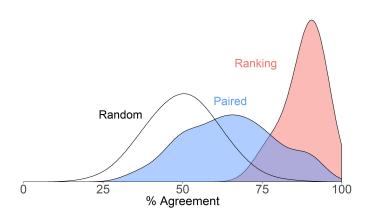
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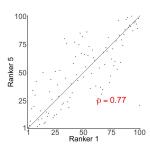
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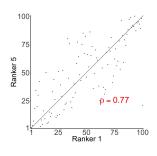


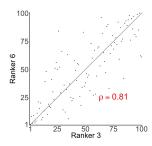
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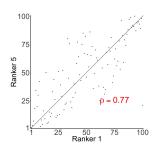
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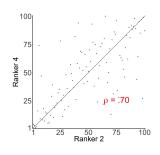


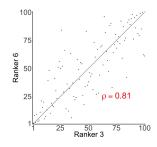


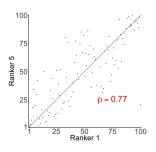


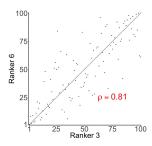


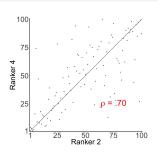


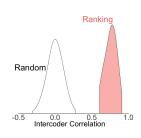


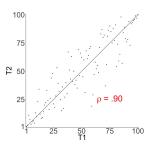


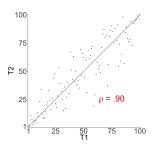


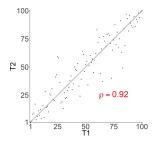


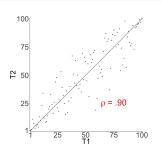


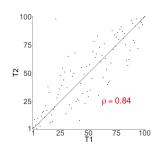


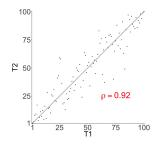


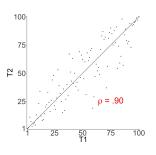


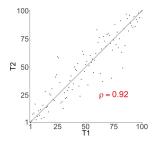


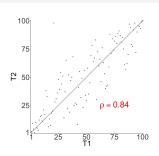


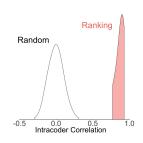












Goal: Compactness score = f(shape)

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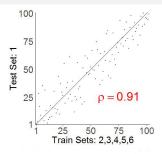
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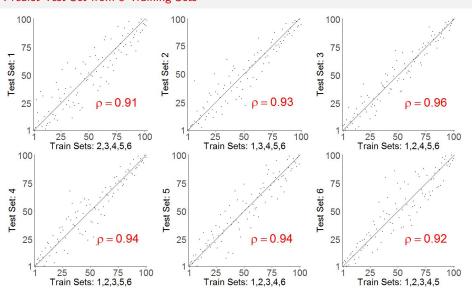
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Predict Test Set from 5 Training Sets

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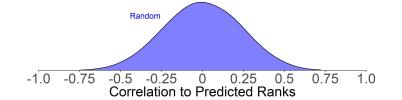


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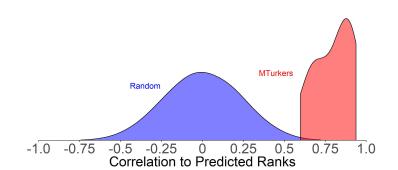


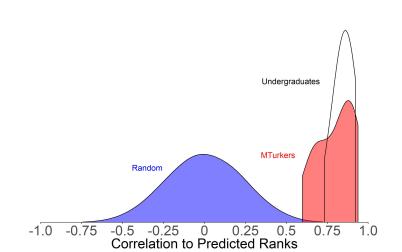
Respondents ranging from ordinary citizens to those responsible for redistricting

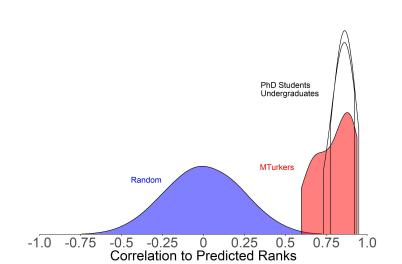
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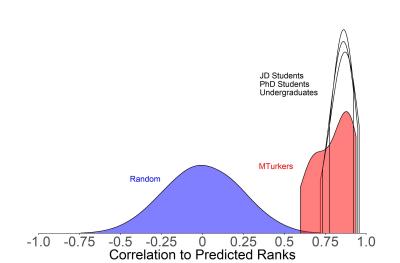


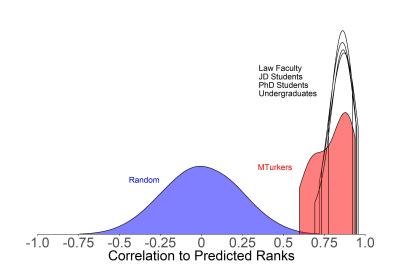
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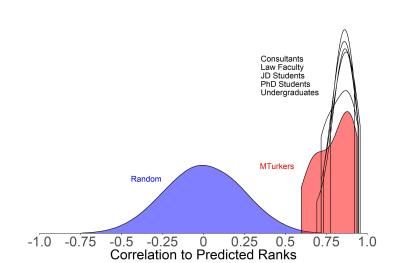


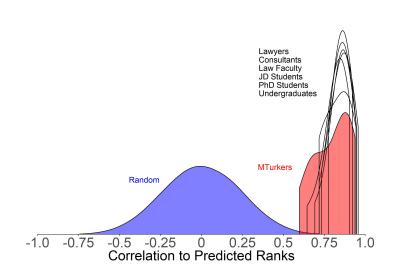


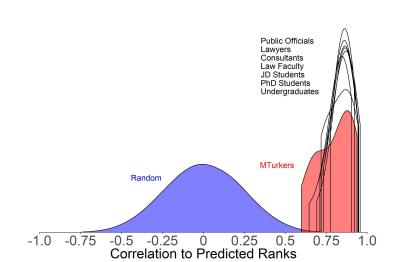


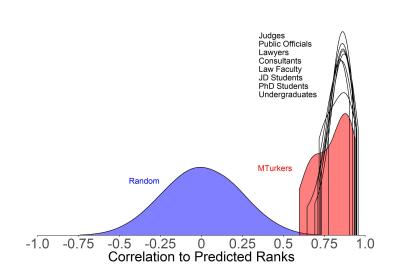












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COMPACT COMPACT

noncompact noncompact

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 $\begin{array}{c} \mathsf{noncompact} \\ \mathsf{COMPACT} \end{array}$

COMPACT noncompact

Reock









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S. Carrier





Boyce-Clark









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For more information







AaronRKaufman.com

GaryKing.org

j.mp/MayyaKomisarchik

Paper, data, software, slides: j.mp/Compactness