## How to Measure Legislative District Compactness If You Only Know it When You See it<sup>1</sup>

### Gary King<sup>2</sup>

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<sup>1</sup>Based on joint work with Aaron Kaufman and Mayya Komisarchik <sup>2</sup>GaryKing.org

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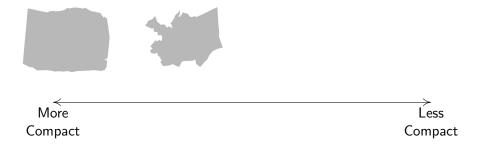
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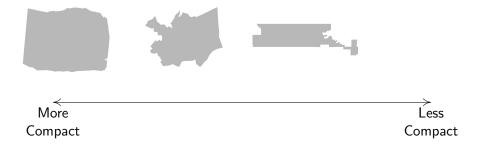
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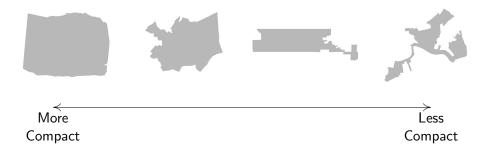
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    - Required in many other jurisdictions



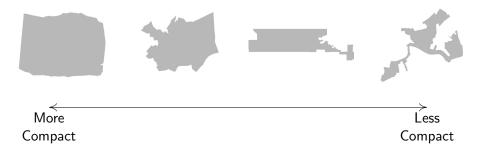




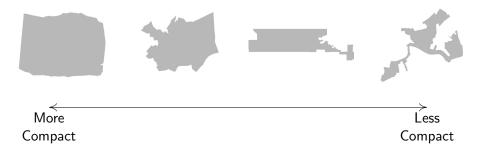




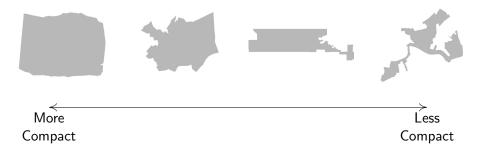
A simple single compactness dimension that you know when you see



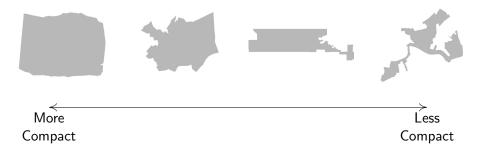
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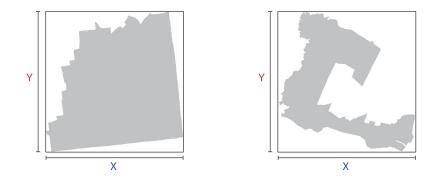
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- $\bullet \, \rightsquigarrow$  Let's start with existing measures by social scientists











Squarish districts more compact than long thin ones



In both districts:  $X/Y \approx 1.30$ 

Circular districts are most compact

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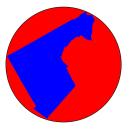




#### Measure 2: Reock, District / Bounding Circle Areas Circular districts are most compact

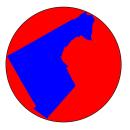


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In both cases,  $X/(Y + X) \approx 0.37$ 



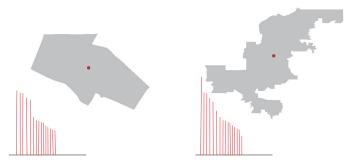






# Measure 3: Boyce-Clark, Variation in Centroid Deviations

All travel distances from center should be similar



In both cases,  $MAD(r)/\bar{r} \approx 0.31$ 

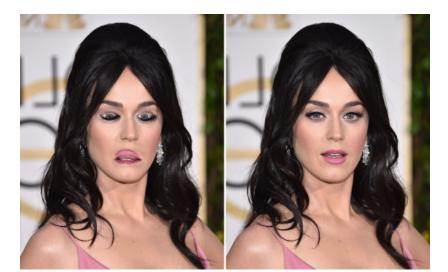
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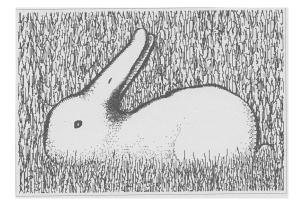
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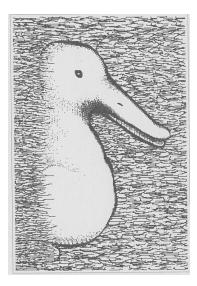
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# A Brief Interlude on Perception: See the Rabbit?



### A Brief Interlude on Perception: See the Rabbit Duck?



# A Brief Interlude on Perception: See the Frog?



# A Brief Interlude on Perception: See the Frog Horse?



#### • Existing measures of compactness:

• Nearly 100 proposed

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- Almost all are rotationally invariant

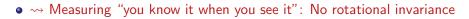
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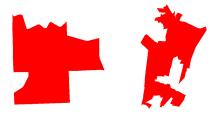
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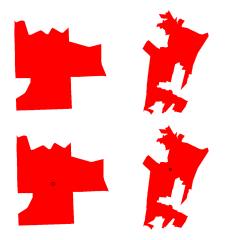


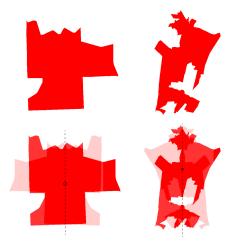
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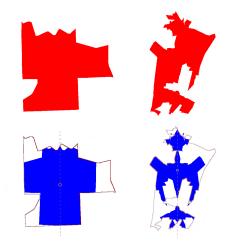


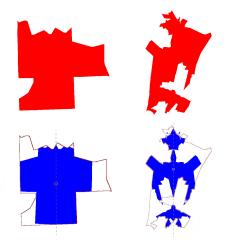
# New Measure: Y-Symmetry, area of symmetric reflection











In both cases, Overlap/Original Area  $\approx 0.34$ 

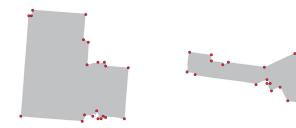
#### New Measure 2: Number of Visually Significant Corners Computer vision algorithm identifies "objects" in photos

Computer vision algorithm identifies "objects" in photos  $\rightsquigarrow$  Fewer corners is more compact

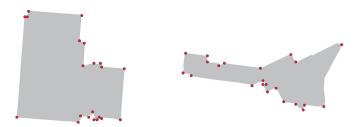
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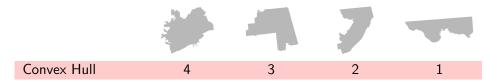


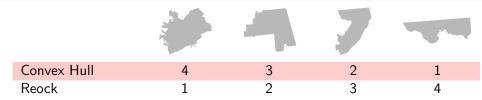
Both districts have 21 significant corners

# Which is more compact?

# \* - 7 -







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Reock	1	2	3	4
Polsby-Popper	4	1	2	3

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- Many more inconsistencies on individual districts

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Paired Comparisons (Fechner 1860; Thurstone 1912) v Ranking (very old, rarely used)

Paired Comparison



Paired Comparisons (Fechner 1860; Thurstone 1912) v Ranking (very old, rarely used)

#### Paired Comparison



Utterly fails on inter- and intra-coder reliability

Paired Comparisons (Fechner 1860; Thurstone 1912) v Ranking (very old, rarely used)

#### Full Ranking



Paired Comparisons (Fechner 1860; Thurstone 1912) v Ranking (very old, rarely used)

#### Full Ranking — on line

MOST Compact Here



**LEAST Compact Here** 

Paired Comparisons (Fechner 1860; Thurstone 1912) v Ranking (very old, rarely used)

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#### We show: very high reliability

Paired Comparisons (Fechner 1860; Thurstone 1912) v Ranking (very old, rarely used)

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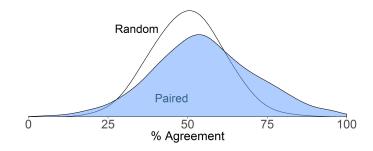
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# Intercoder Reliability of Pairs

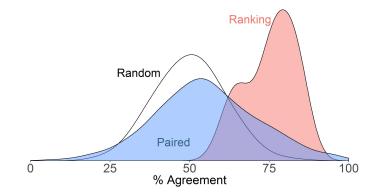
## Intercoder Reliability of Pairs

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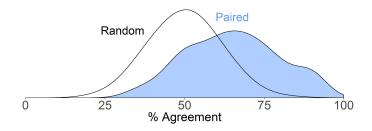
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# Intracoder Reliability of Pairs

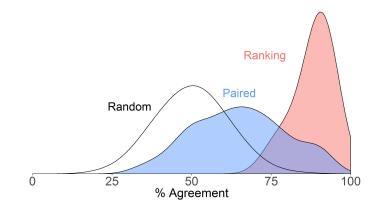
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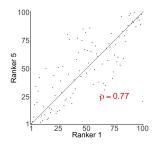
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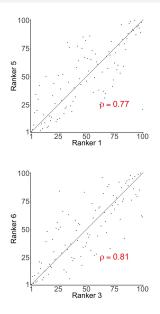


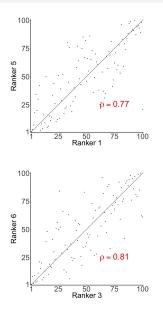
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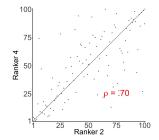
Paired Comparisons: better than chance; Pairs implied by ranks: much better

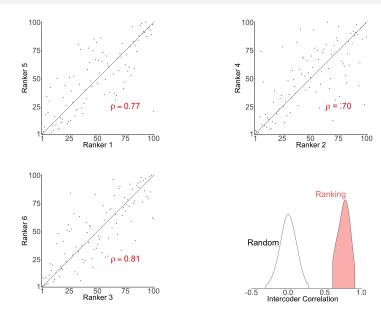


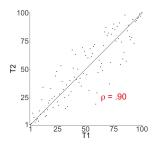


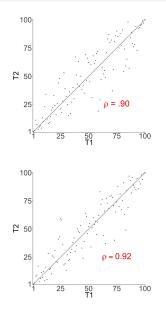


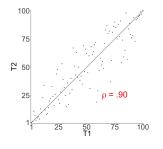


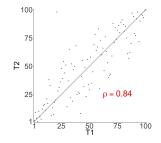


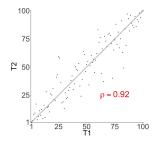


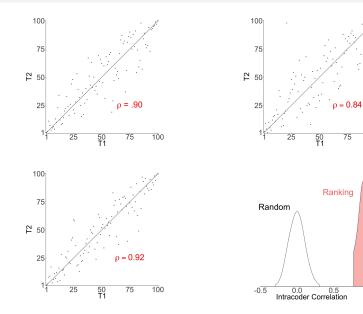












100

1.0

## So we can measure it. Can we model it?

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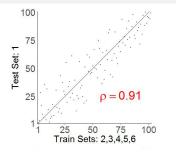
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#### So we can measure it. Can we model it? Goal: Compactness score = f(shape)

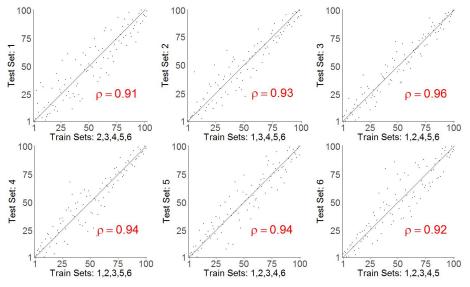
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Predict Test Set from 5 Training Sets

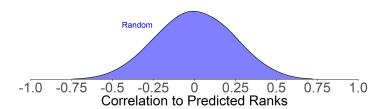
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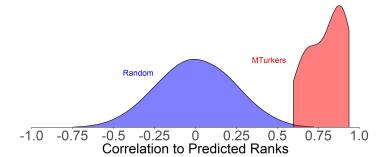


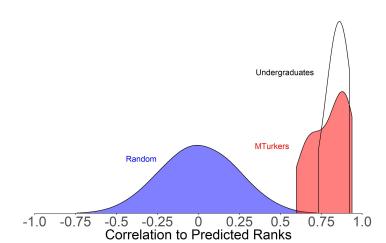
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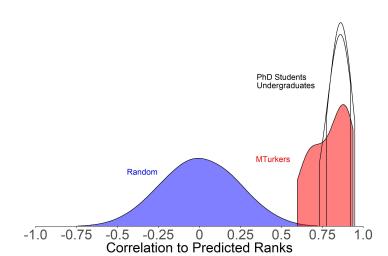


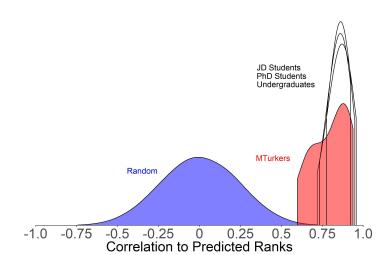
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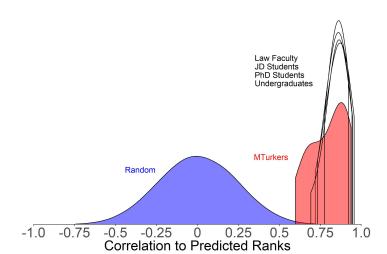


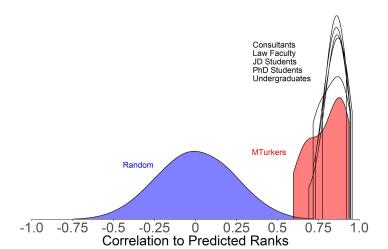


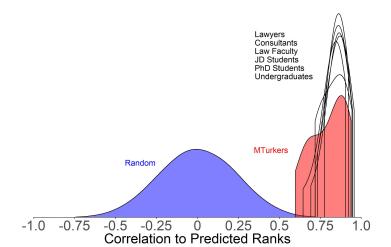


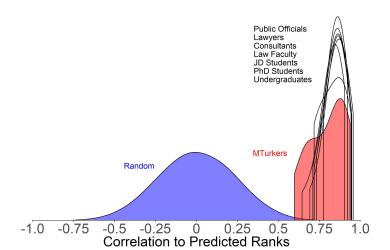


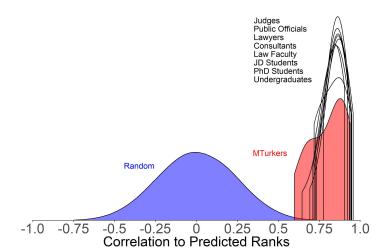




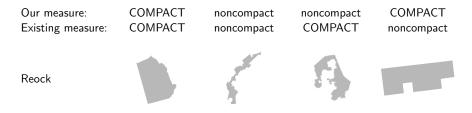


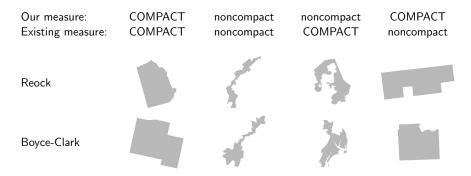


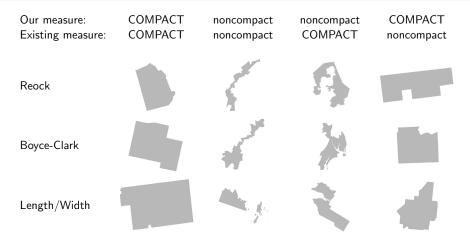


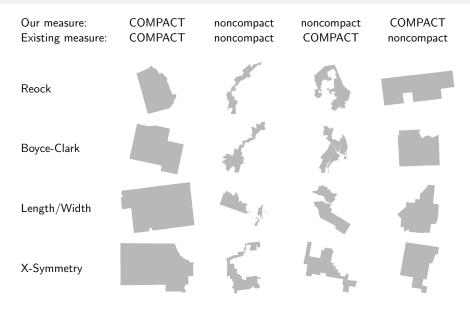


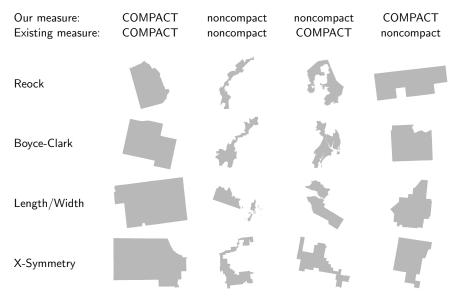
Our measure:	COMPACT	noncompact	noncompact	COMPACT
Existing measure:	COMPACT	noncompact	COMPACT	noncompact











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  - Very high intercoder & intracoder reliability
  - Very high predictive validity
  - Diverse people see it the same way
  - ~> Continue political science tradition of contributing to a fundamental part of representative democracy
- Accompanying this paper:
  - Measures: for 18,215 Congressional & State Legislative districts
  - Software to calculate compactness from any district shape
- Along the way:
  - New perspective on  $> 150 \mbox{ year consensus of ranking v paired comparisons}$
  - New directions for two venerable literatures

#### For more information





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Paper, data, software, slides: j.mp/Compactness