How to Measure Legislative District Compactness If You Only Know it When You See it¹

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¹Based on joint work with Aaron Kaufman and Mayya Komisarchik

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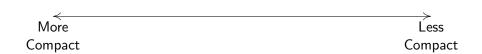
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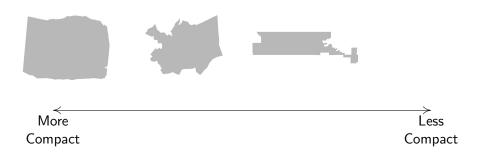
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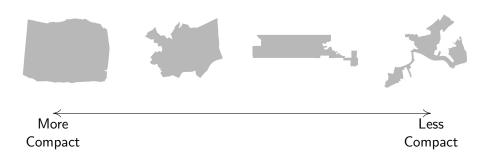
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 - Required in many other jurisdictions



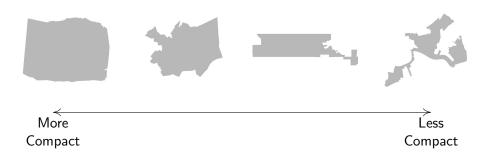




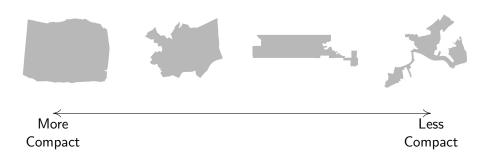




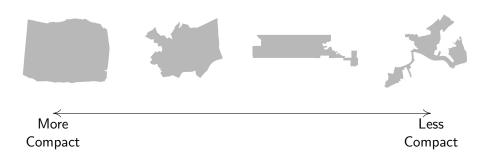
A simple single compactness dimension that you know when you see



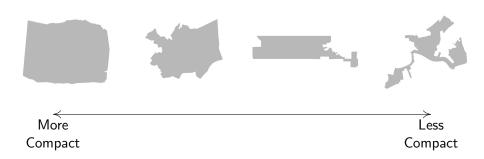
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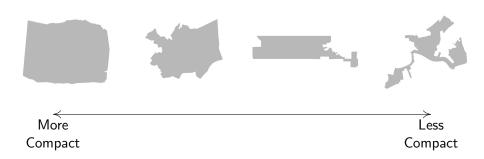
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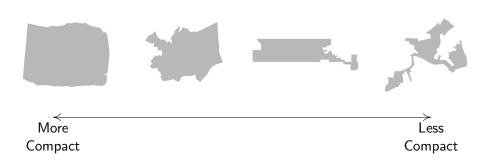
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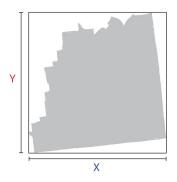
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- \longrightarrow Let's start with existing measures by social scientists

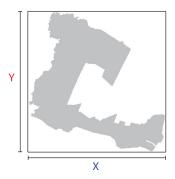




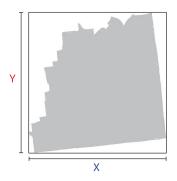


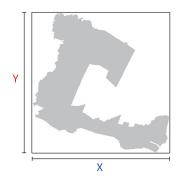






Squarish districts more compact than long thin ones





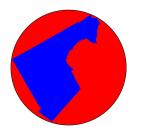
In both districts: $X/Y \approx 1.30$





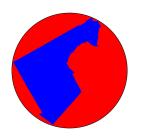








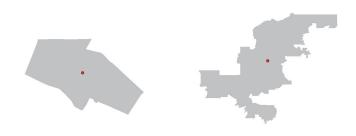
Circular districts are most compact





In both cases, $X/(Y + X) \approx 0.37$













Measure 3: Boyce-Clark, Variation in Centroid Deviations

All travel distances from center should be similar



In both cases, $MAD(r)/\bar{r} \approx 0.31$

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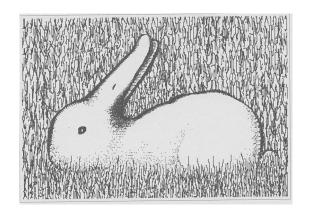




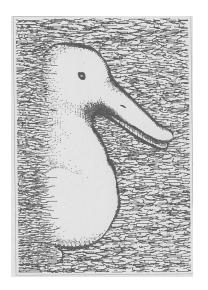
A Brief Rotational Invariance Interlude: Can you Name this Celebrity?



A Brief Interlude on Perception: See the Rabbit?



A Brief Interlude on Perception: See the Rabbit Duck?



A Brief Interlude on Perception: See the Frog?



A Brief Interlude on Perception: See the Frog Horse?



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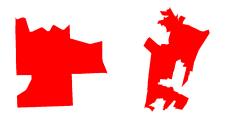
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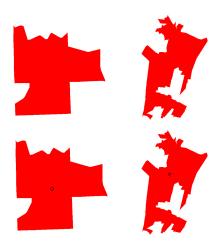


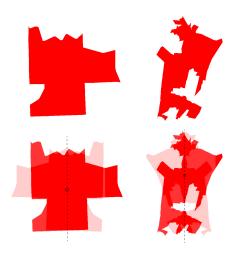
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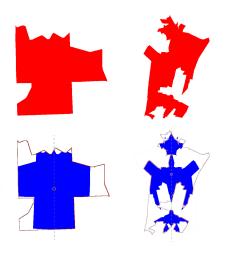


→ Measuring "you know it when you see it": No rotational invariance

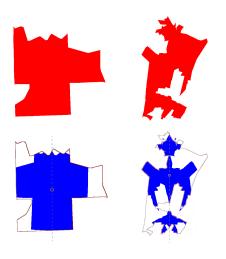








Symmetric figures (circles, squares) are more compact



In both cases, Overlap/Original Area ≈ 0.34

Computer vision algorithm identifies "objects" in photos

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→ Fewer corners is more compact

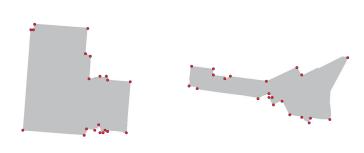
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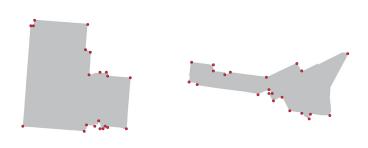
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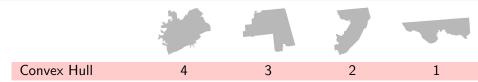


Both districts have 21 significant corners

Which is more compact?







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- Many more inconsistencies on individual districts

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- Our Hypothesis: both are right

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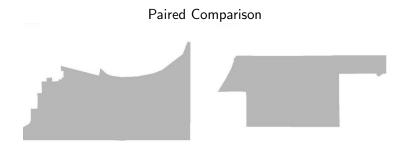
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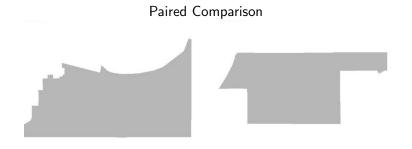
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Paired Comparisons (Fechner 1860; Thurstone 1912) v Ranking (very old, rarely used)



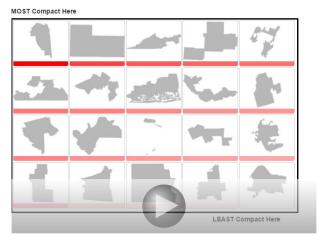
Utterly fails on inter- and intra-coder reliability





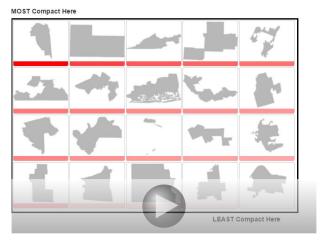
Paired Comparisons (Fechner 1860; Thurstone 1912) v Ranking (very old, rarely used)

Full Ranking — on line



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We show: very high reliability

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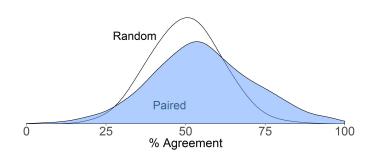
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 Ranking: all evaluations on one dimension of user's choice

Intercoder Reliability of Pairs

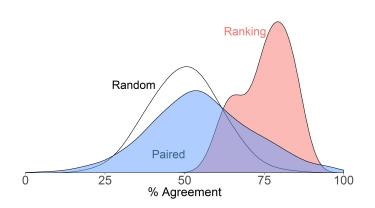
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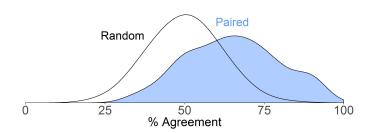
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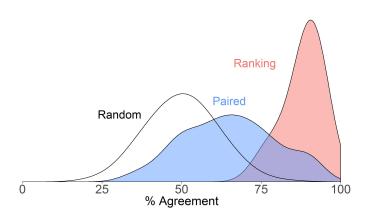
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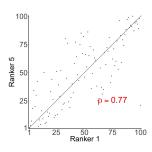
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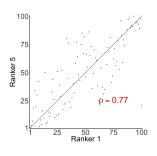


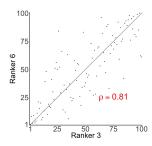
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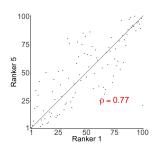
Paired Comparisons: better than chance; Pairs implied by ranks: much better

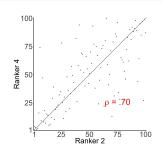


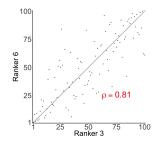


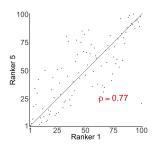


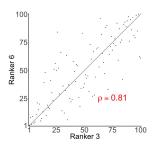


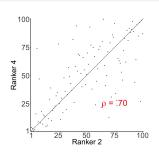


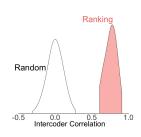


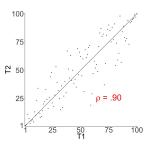


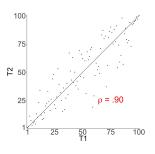


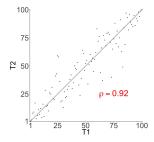


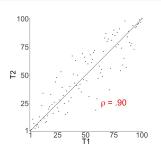


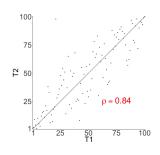


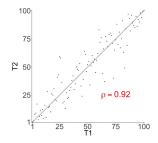


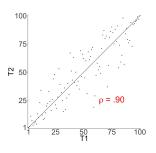


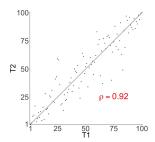


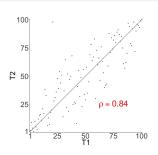


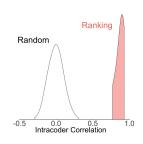












Goal: Compactness score = f(shape)

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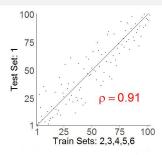
So we can measure it. Can we model it?

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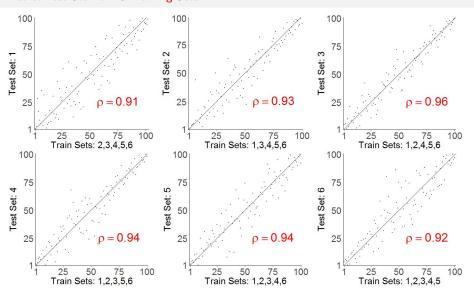
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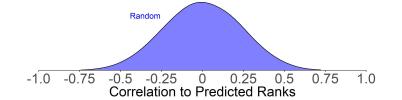
Predict Test Set from 5 Training Sets

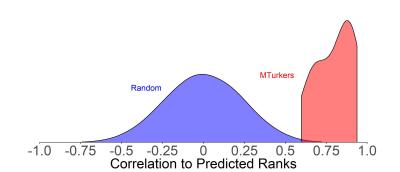
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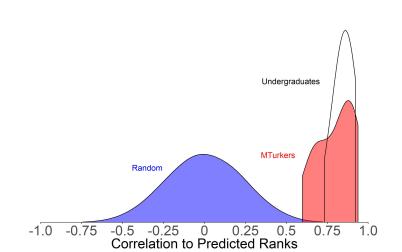


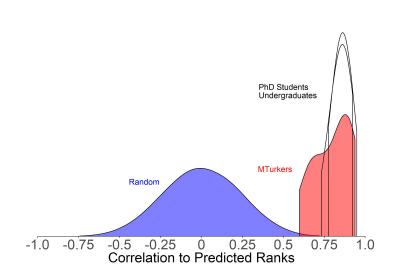
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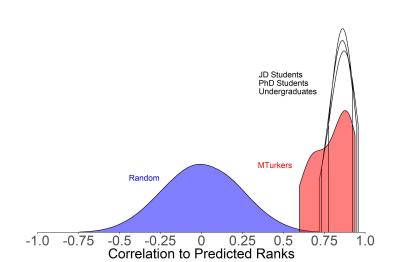


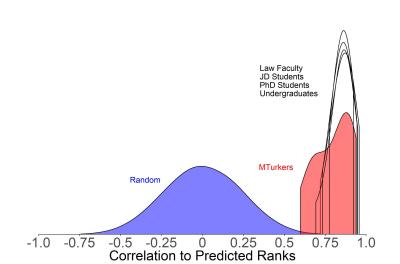


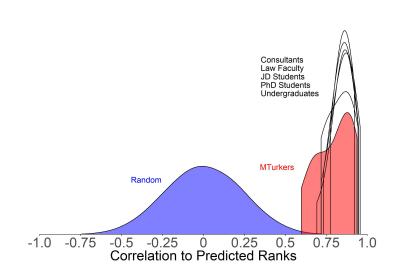


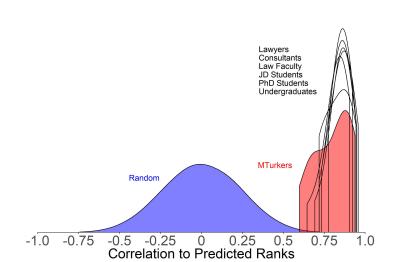


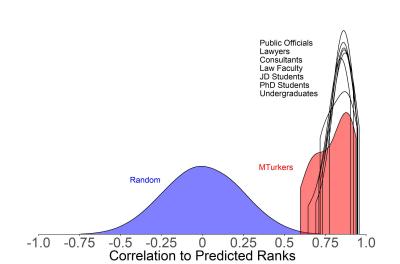


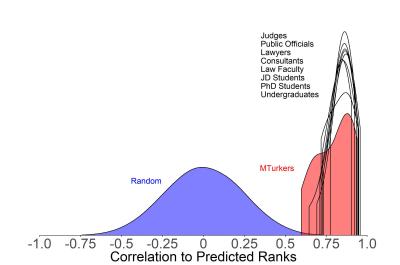












Our measure: Existing measure:

COMPACT COMPACT

noncompact noncompact

noncompact COMPACT COMPACT noncompact

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 $\begin{array}{c} \mathsf{noncompact} \\ \mathsf{COMPACT} \end{array}$

COMPACT noncompact

Reock









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COMPACT noncompact

Reock



September 1





Boyce-Clark









COMPACT COMPACT Our measure: noncompact noncompact Existing measure: **COMPACT COMPACT** noncompact noncompact Reock Boyce-Clark Length/Width

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 - The Legal Concept: one dimensional and simple
- A proposed resolution: measure the one dimension everyone sees
 - Calculated solely from district geometry
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 - Diverse people see it the same way
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 - Measures: for 18,215 Congressional & State Legislative districts
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For more information







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Paper, data, software, slides: j.mp/Compactness