Demographic Forecasting

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Joint work with Federico Girosi (RAND)
with contributions from Kevin Quinn and Gregory Wawro

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  - demography & sociology
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  - economics & social security and retirement planning
  - actuarial science & insurance companies
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- forecasts and farcasts based on much more information

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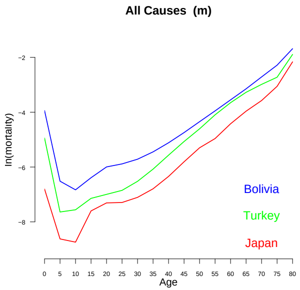
- Most time series are very short. A majority of countries have only a few isolated annual observations; only 54 countries have at least 20 observations; Africa, AIDS, & Malaria are real problems

Existing Method 1: Parameterize the Age Profile



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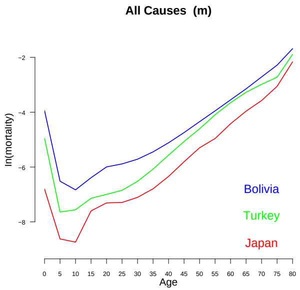

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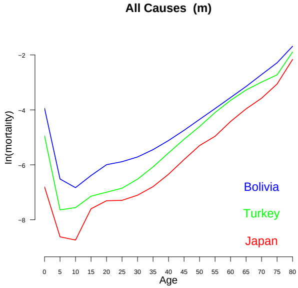

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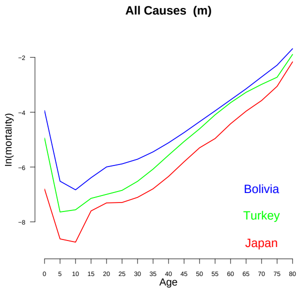

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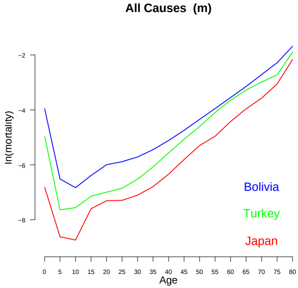

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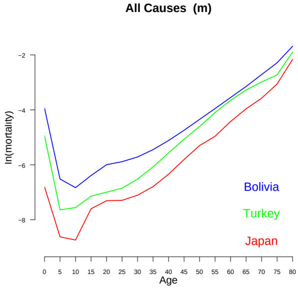

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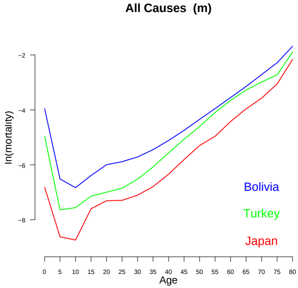

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- But does it fit anything else?

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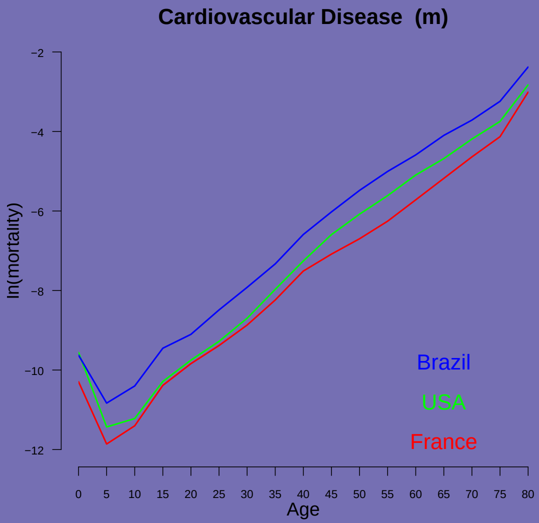

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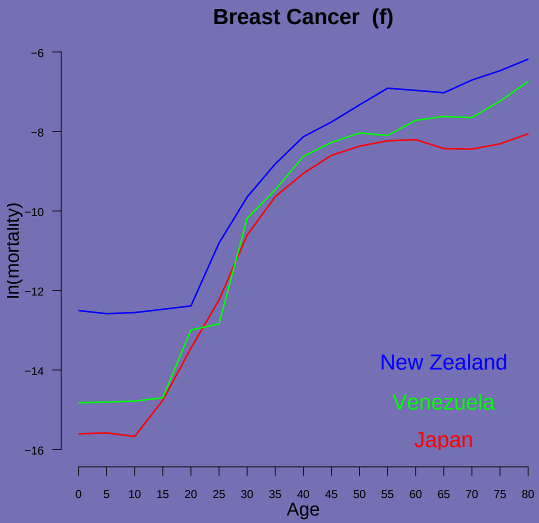

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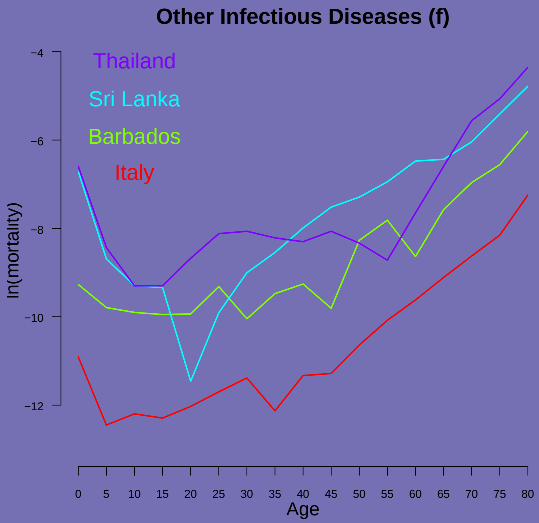

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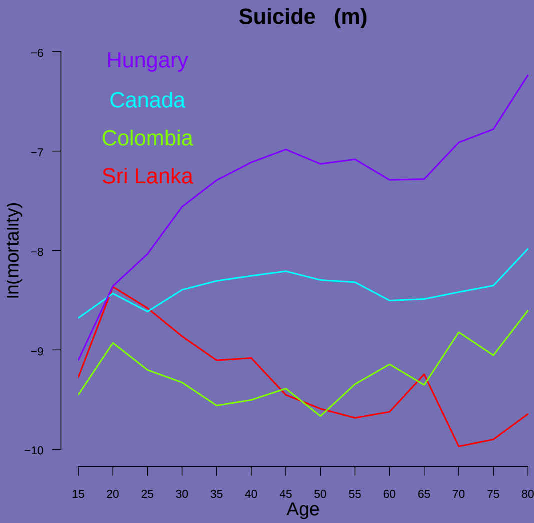

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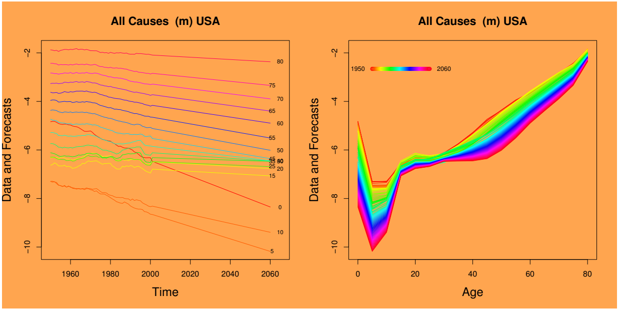

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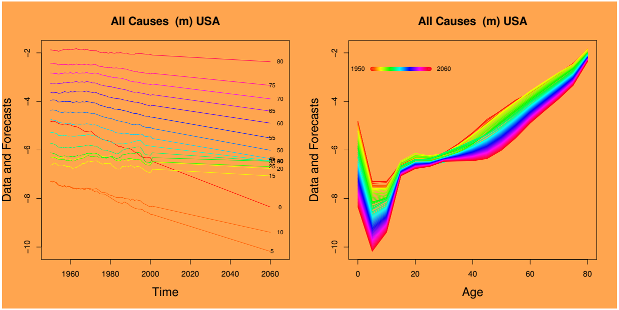

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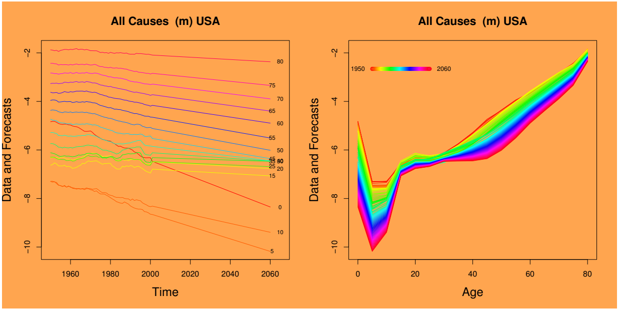

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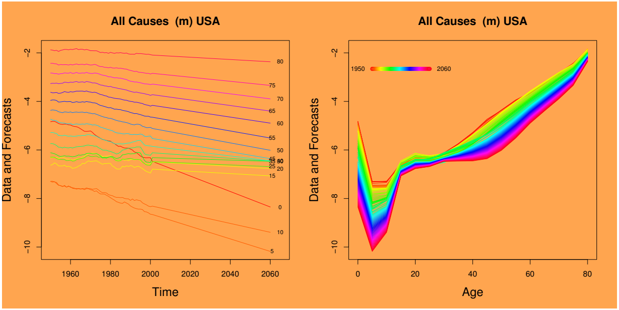

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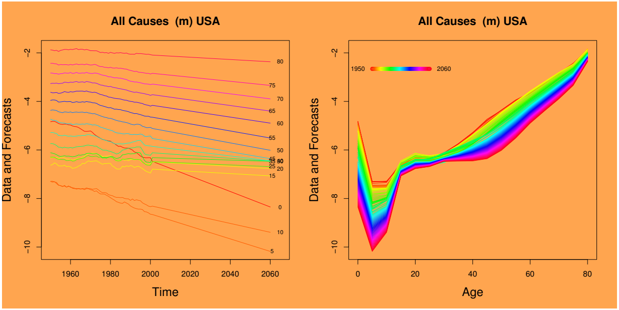

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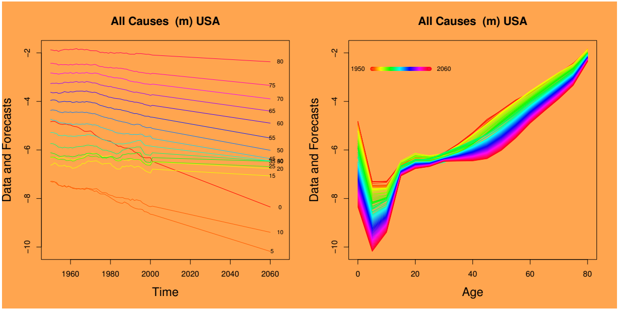

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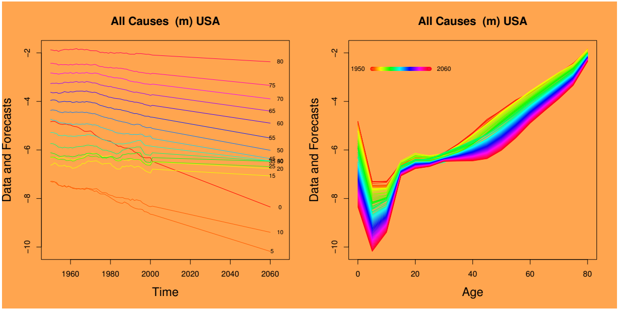

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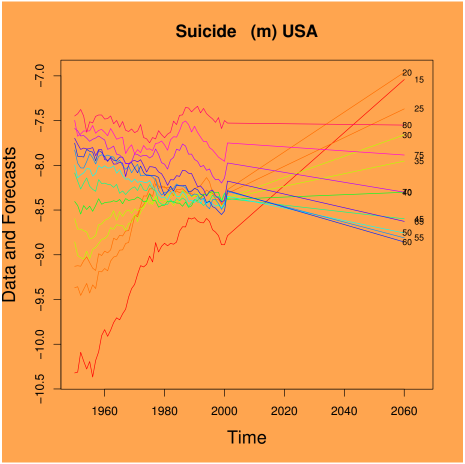

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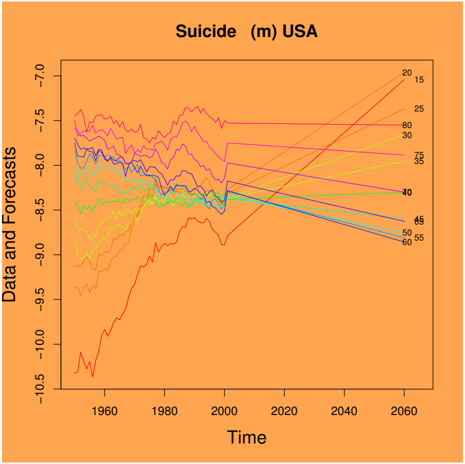

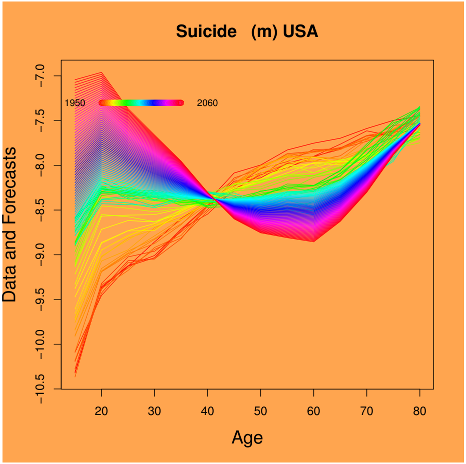

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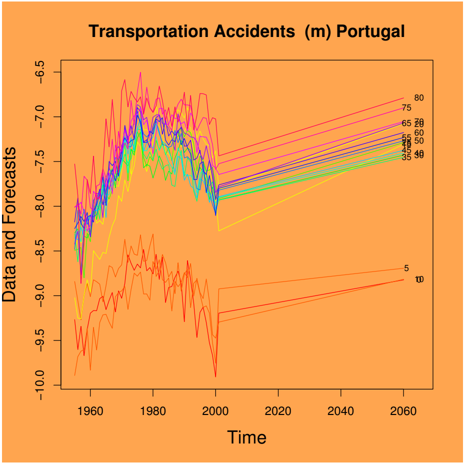

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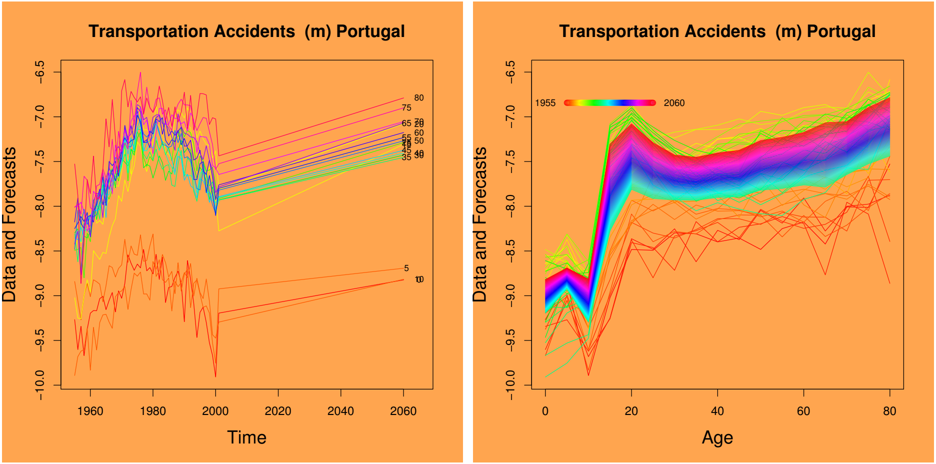

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  - same covariates required in all cross-sections

Partial Pooling via a Bayesian Hierarchical Approach

- Likelihood for equation-by-equation least squares:

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Partial Pooling via a Bayesian Hierarchical Approach

- Likelihood for equation-by-equation least squares:

$$\mathcal{P}(m \mid \boldsymbol{\beta}_i, \sigma_i) = \prod_t \mathcal{N}\left(m_{it} \mid \mathbf{Z}_{it}\boldsymbol{\beta}_i, \sigma_i^2\right)$$

- Add priors and form a posterior

$$\begin{aligned}\mathcal{P}(\boldsymbol{\beta}, \sigma, \theta \mid m) &\propto \mathcal{P}(m \mid \boldsymbol{\beta}, \sigma) \times \mathcal{P}(\boldsymbol{\beta} \mid \theta) \times \mathcal{P}(\theta)\mathcal{P}(\sigma) \\ &= (\text{Likelihood}) \times (\text{Key Prior}) \times (\text{Other priors})\end{aligned}$$

- Calculate point estimate for $\boldsymbol{\beta}$ (for $\hat{y}$) as the mean posterior:

$$\boldsymbol{\beta}^{\text{Bayes}} \equiv \int \boldsymbol{\beta}\mathcal{P}(\boldsymbol{\beta}, \sigma, \theta \mid m)\, d\boldsymbol{\beta} d\theta d\sigma$$

- The hard part: specifying the prior for $\boldsymbol{\beta}$ and, as always, **Z**
- The easy part: *easy-to-use software* to implement everything we discuss today.

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Natural choice for the prior:

$$\mathcal{P}(\boldsymbol{\beta} \mid \Phi) \propto \exp\left(-\frac{1}{2}\sum_{ij} s_{ij}\|\boldsymbol{\beta}_i - \boldsymbol{\beta}_j\|^2_\Phi\right)$$

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- Extensive trial-and-error runs: no useful parameter values.

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3. In the subspace, we can invert $\mu = \mathbf{Z}\boldsymbol{\beta}$ as $\boldsymbol{\beta} = (\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\mu$, giving:

$$\mathcal{P}(\boldsymbol{\beta} \mid \theta) \propto \exp\left(-\frac{1}{2}H[\mu,\theta]\right) = \exp\left(-\frac{1}{2}H[\mathbf{Z}\boldsymbol{\beta},\theta]\right)$$

   the same prior on μ, expressed as a function of $\boldsymbol{\beta}$ (with constant Jacobian).

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- We start with one-dimensional $P(\mu_{cat})$, and treat it as the multidimensional $P(\beta_{ca})$, constant in all directions but $Z_{cat}\beta_{ca}$.

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- Fully Bayesian: The same theory of inference applies
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- The normalization matrix Φ is unnecessary (normalization is performed by $\mathbf{Z}$, which is known)

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- where W^n is a matrix uniquely determined by n and θ

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where we have defined:

$$\mathbf{C}_{aa'} \equiv \frac{1}{T}\mathbf{Z}'_a\mathbf{Z}_{a'} \qquad \mathbf{Z}_a \text{ is a } T \times d_a \text{ data matrix for age group } a$$

The Prior on the Coefficients β

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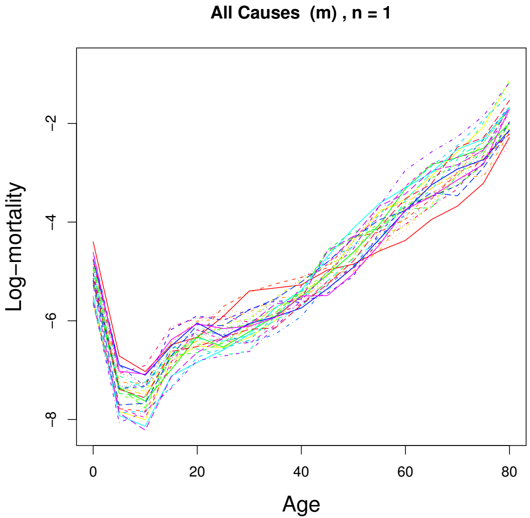
- Different age groups can have different covariates: the matrices $\mathbf{C}_{aa'}$ are rectangular ($d_a \times d_{a'}$).

Samples From Age Prior



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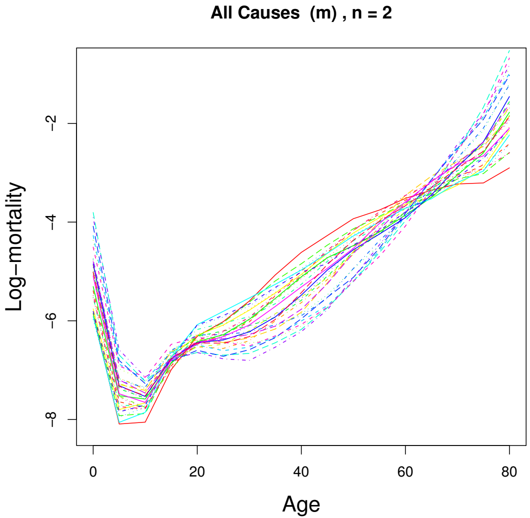

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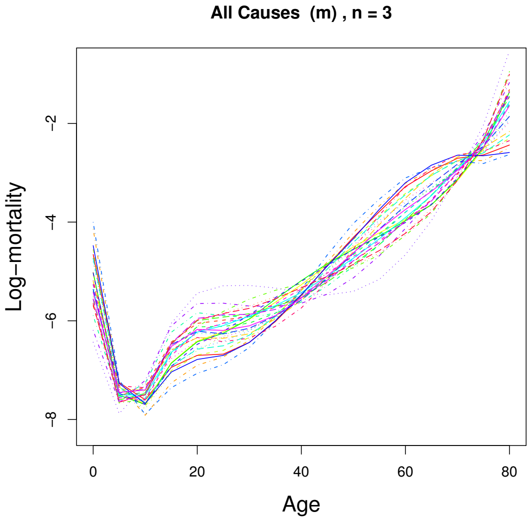

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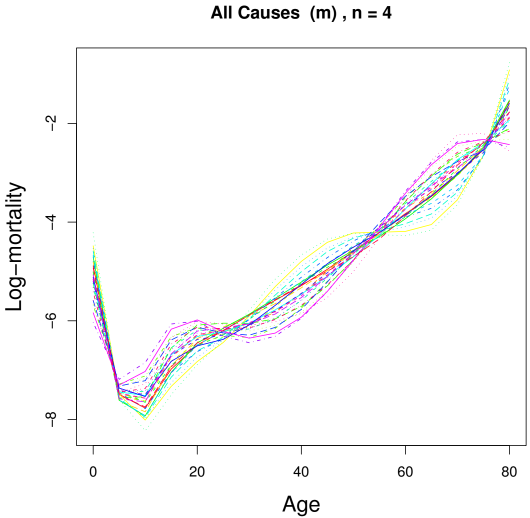

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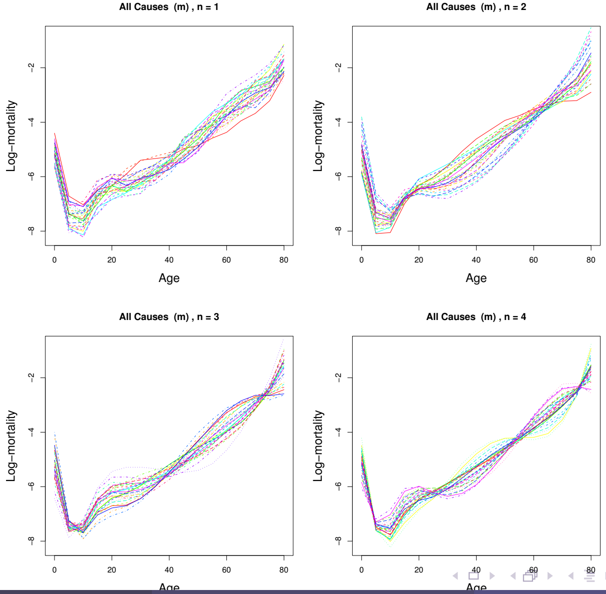

Formalizing (Prior) Indifference

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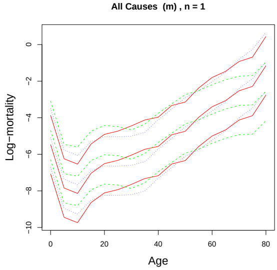

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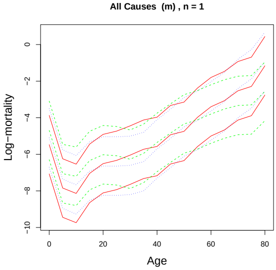
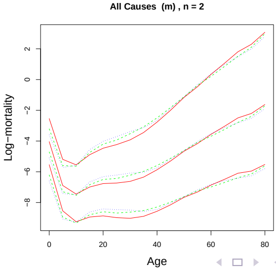


Level and slope indifference



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- θ controls the prior standard deviation

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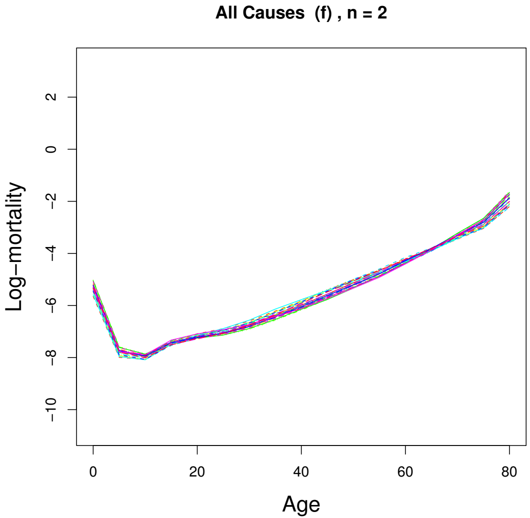

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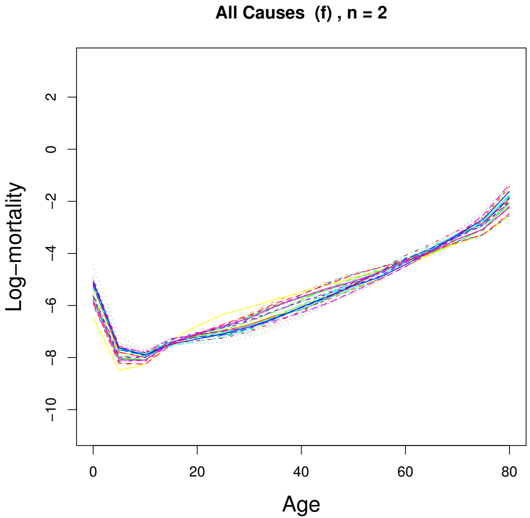

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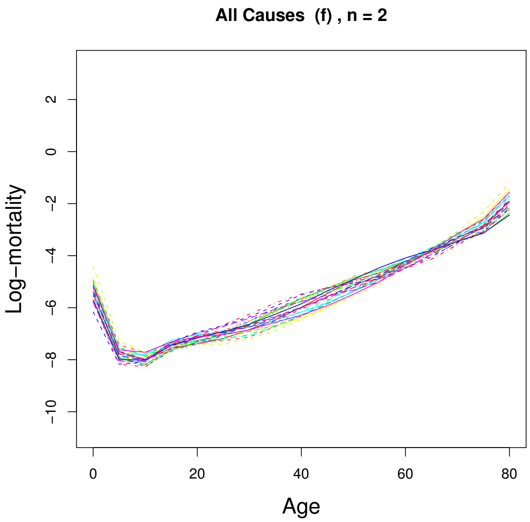

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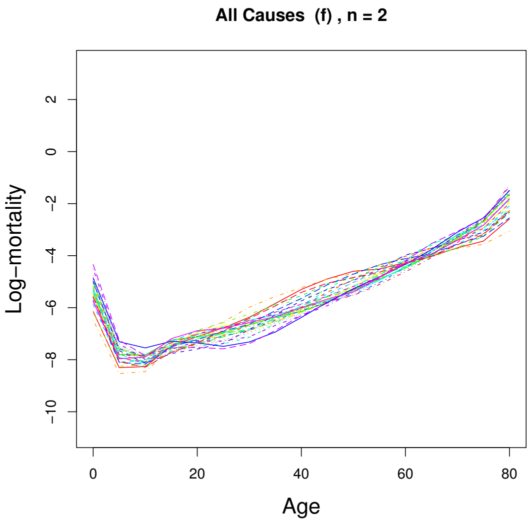

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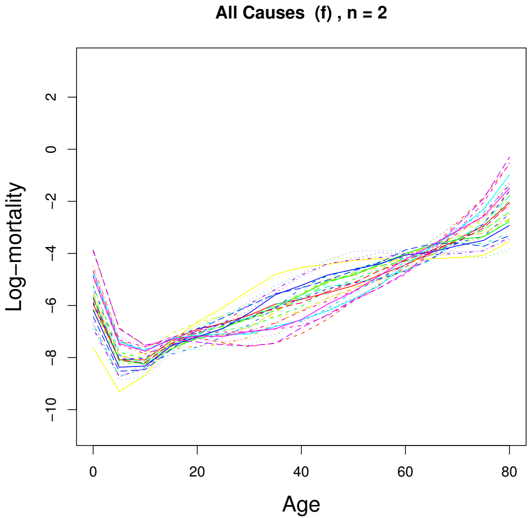

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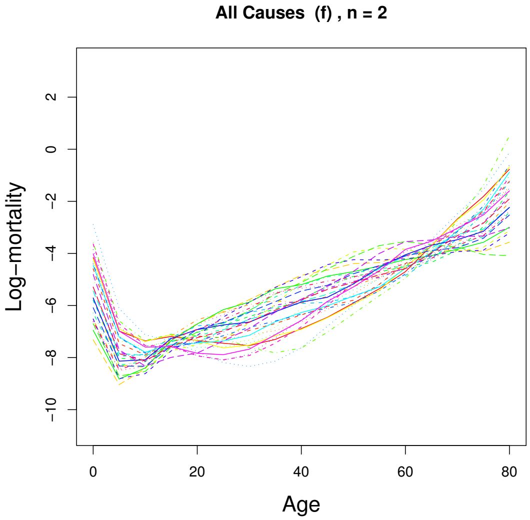

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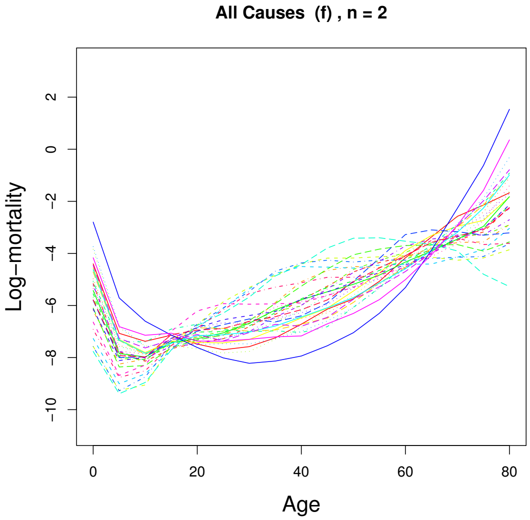

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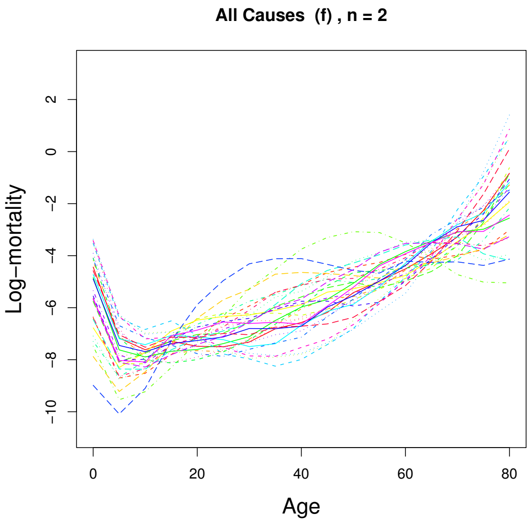

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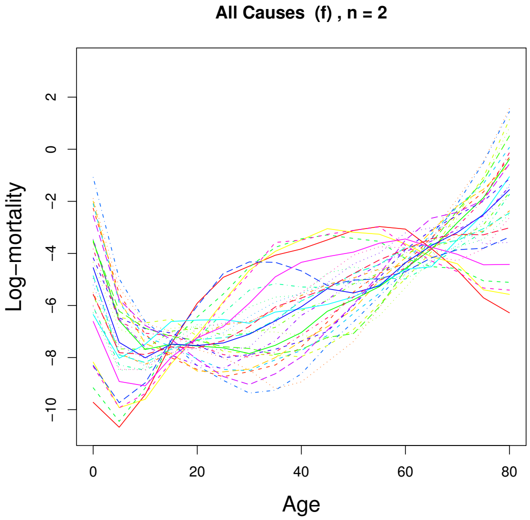

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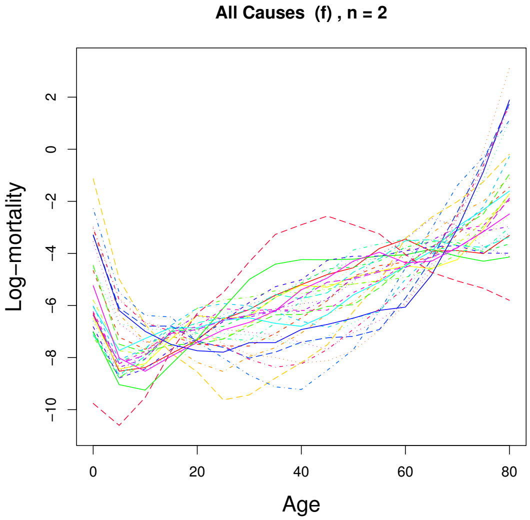

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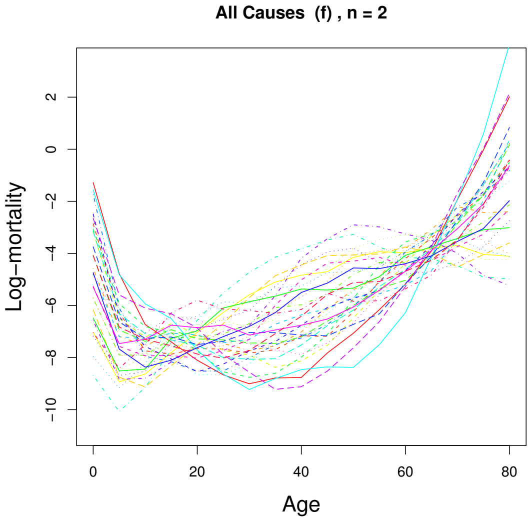

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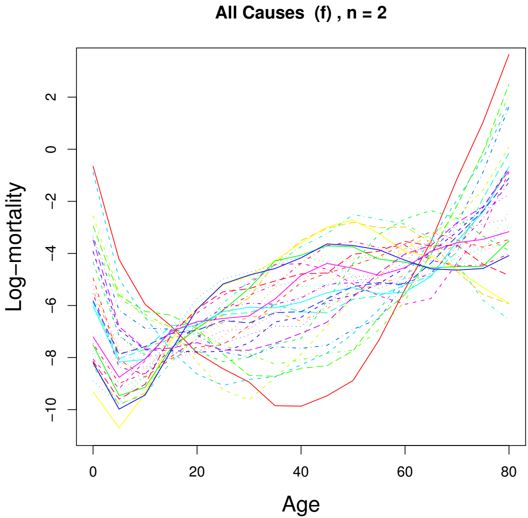

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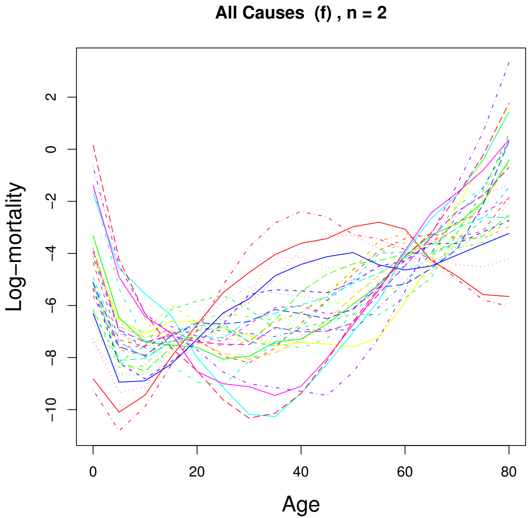

Generalizations

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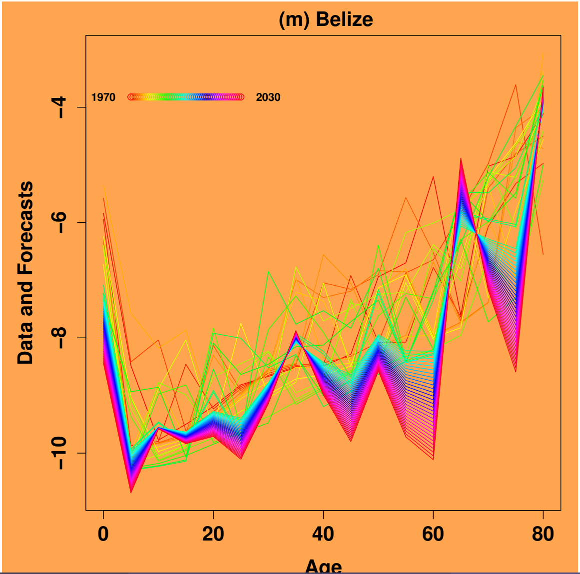
- The mathematical form for *all* these (separately or together) turns out to be the same:

$$\mathcal{P}(\beta \mid \theta) \propto \exp\left(-\frac{\theta}{2}\sum_{ij} W_{ij}\beta_i' \mathbf{C}_{ij}\beta_j\right), \qquad \mathbf{C}_{aa'} \equiv \frac{1}{T}\mathbf{Z}_a\mathbf{Z}_{a'}$$

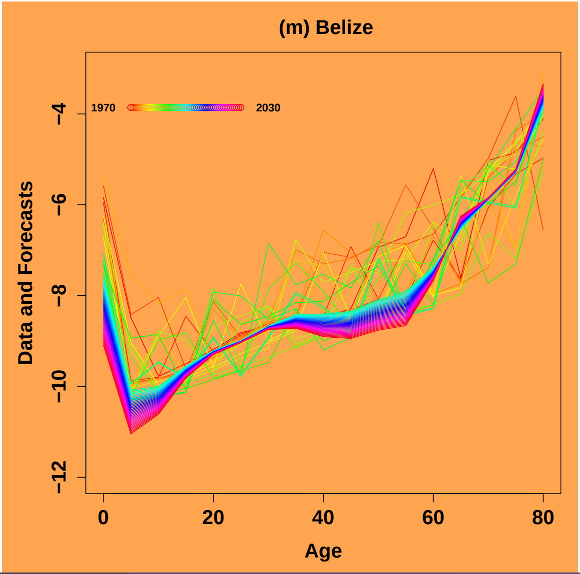
Mortality from Respiratory Infections, Males

Least Squares



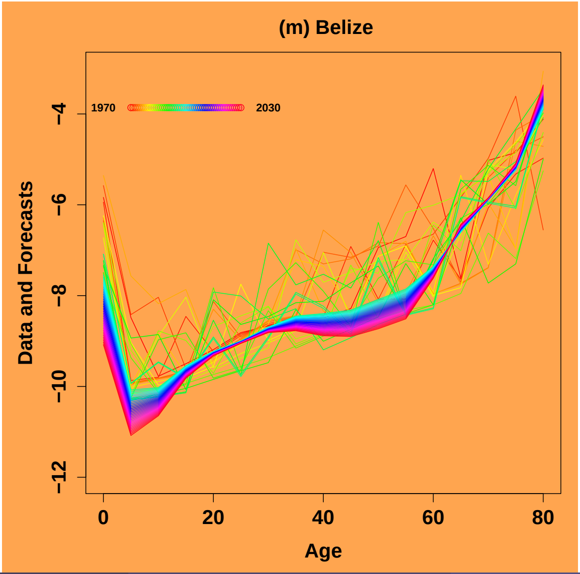
Mortality from Respiratory Infections, males, $\sigma = 2.00$

Smoothing over Age Groups



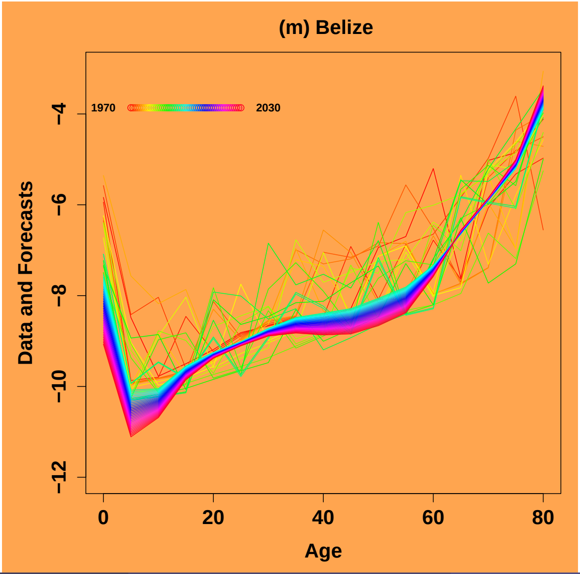
Mortality from Respiratory Infections, males, $\sigma = 1.51$

Smoothing over Age Groups



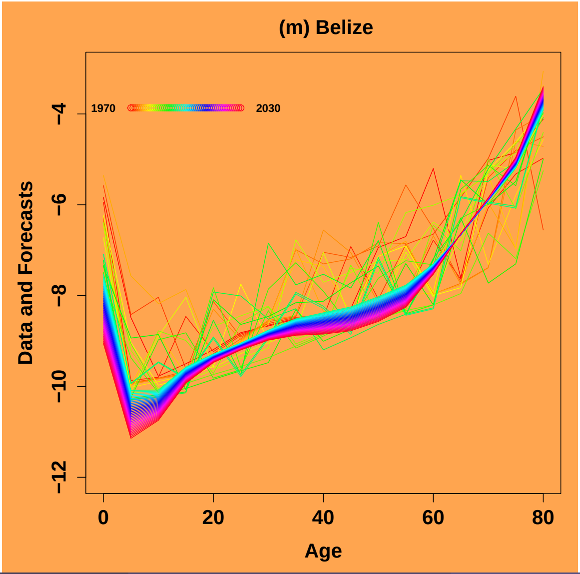
Mortality from Respiratory Infections, males, $\sigma = 1.15$

Smoothing over Age Groups



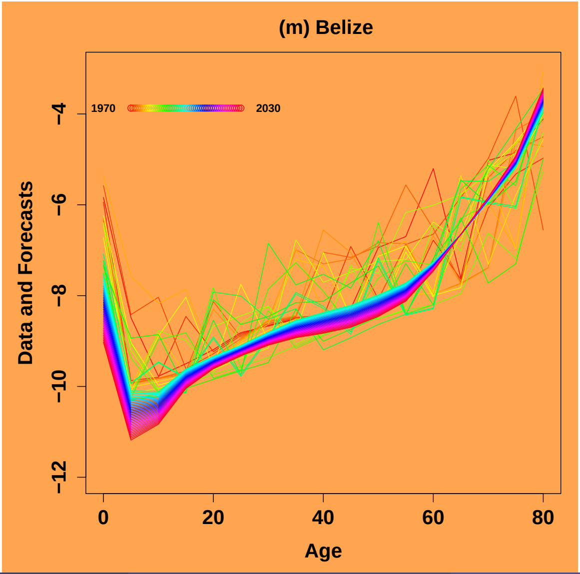
Mortality from Respiratory Infections, males, $\sigma = 0.87$

Smoothing over Age Groups



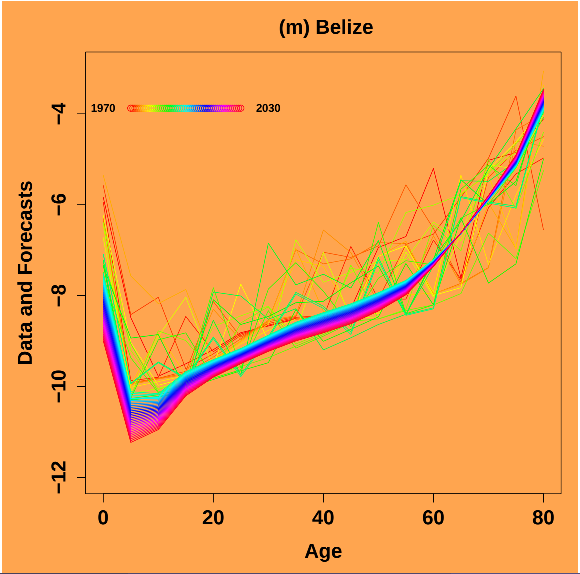
Mortality from Respiratory Infections, males, $\sigma = 0.66$

Smoothing over Age Groups



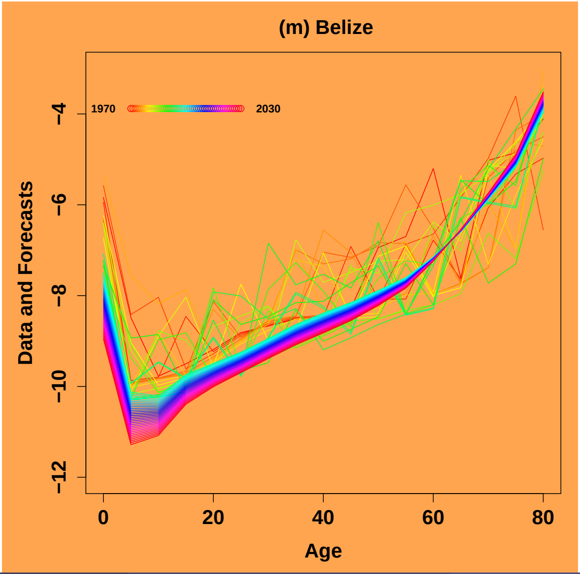
Mortality from Respiratory Infections, males, $\sigma = 0.50$

Smoothing over Age Groups



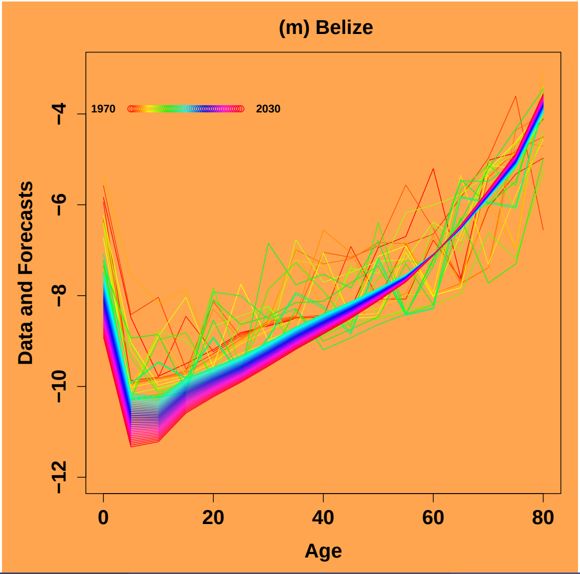
Mortality from Respiratory Infections, males, $\sigma = 0.38$

Smoothing over Age Groups



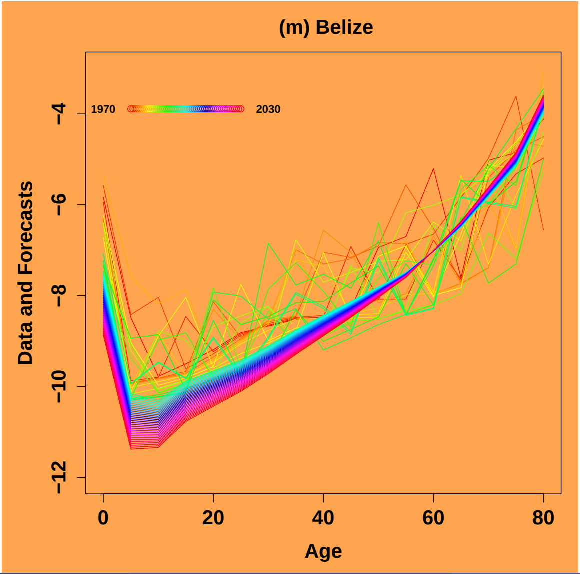
Mortality from Respiratory Infections, males, $\sigma = 0.28$

Smoothing over Age Groups



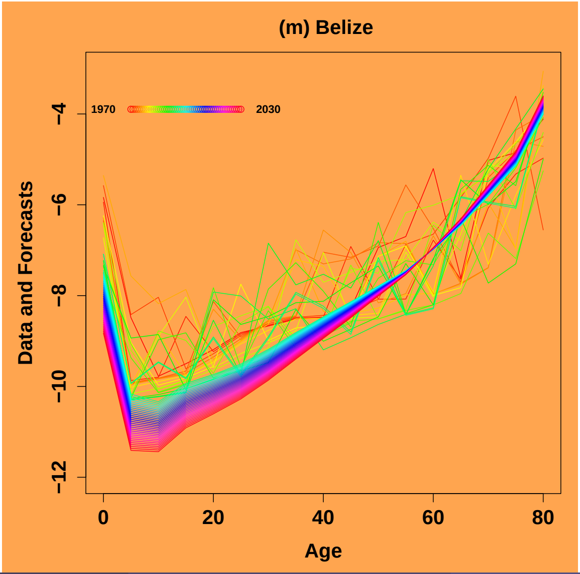
Mortality from Respiratory Infections, males, $\sigma = 0.21$

Smoothing over Age Groups



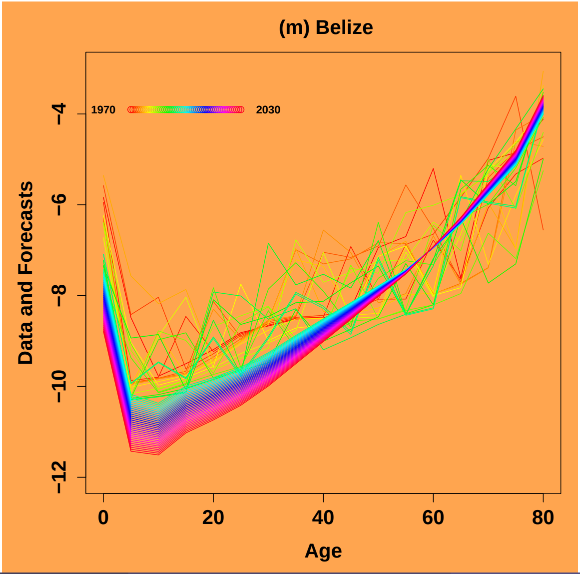
Mortality from Respiratory Infections, males, $\sigma = 0.16$

Smoothing over Age Groups



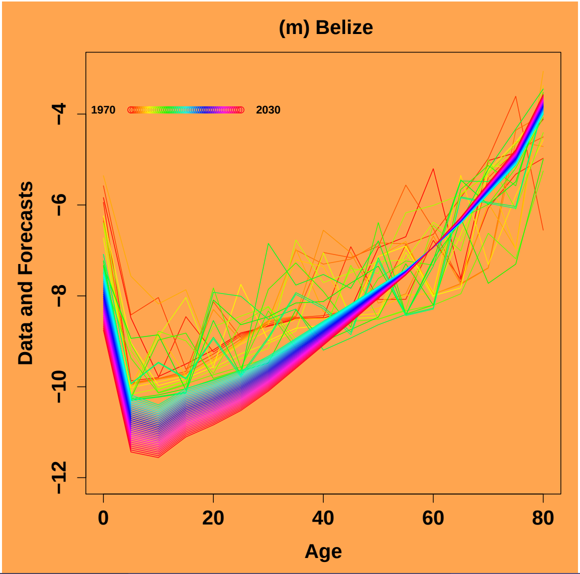
Mortality from Respiratory Infections, males, $\sigma = 0.12$

Smoothing over Age Groups



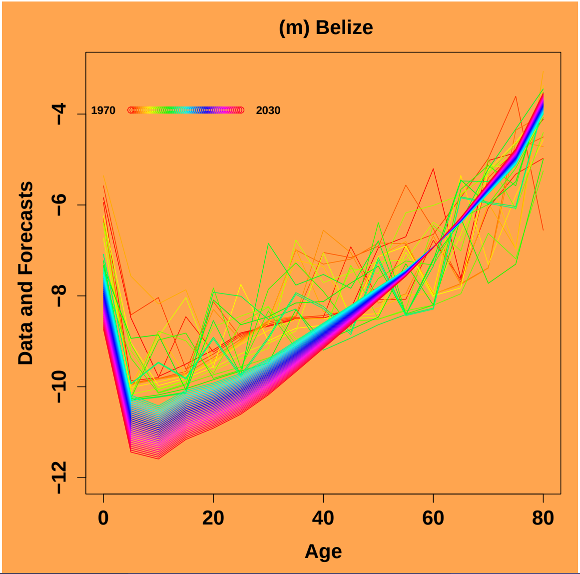
Mortality from Respiratory Infections, males, $\sigma = 0.09$

Smoothing over Age Groups



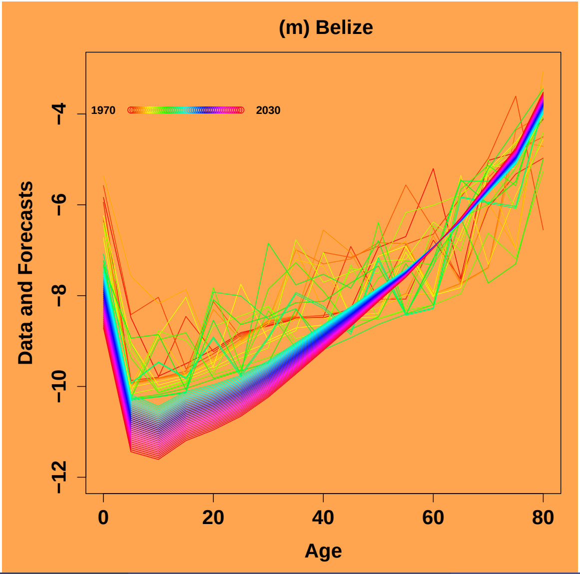
Mortality from Respiratory Infections, males, $\sigma = 0.07$

Smoothing over Age Groups



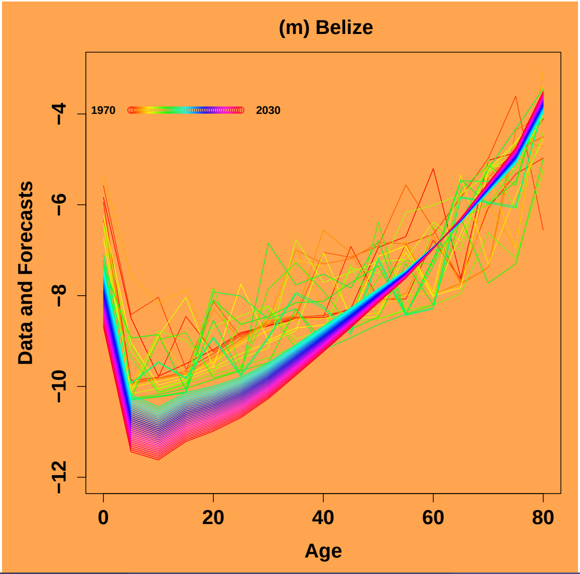
Mortality from Respiratory Infections, males, $\sigma = 0.05$

Smoothing over Age Groups



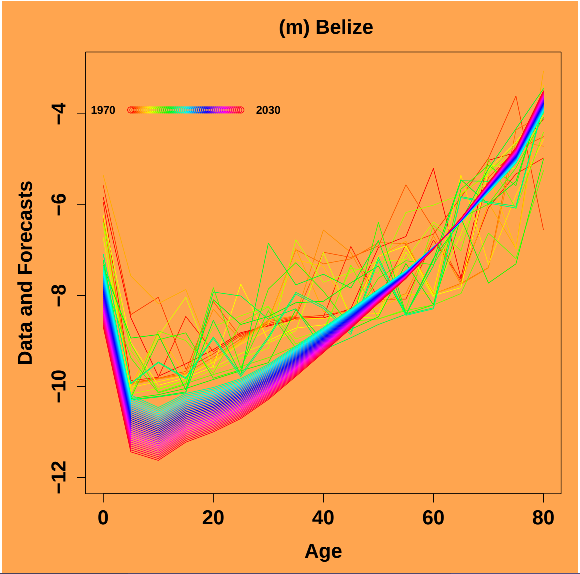
Mortality from Respiratory Infections, males, $\sigma = 0.04$

Smoothing over Age Groups



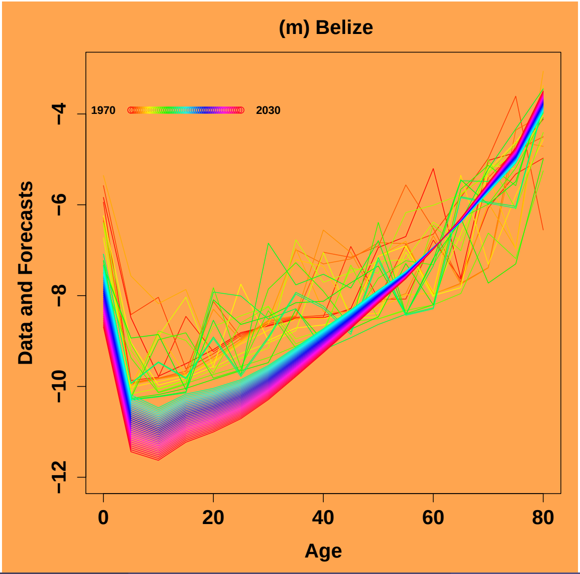
Mortality from Respiratory Infections, males, $\sigma = 0.03$

Smoothing over Age Groups



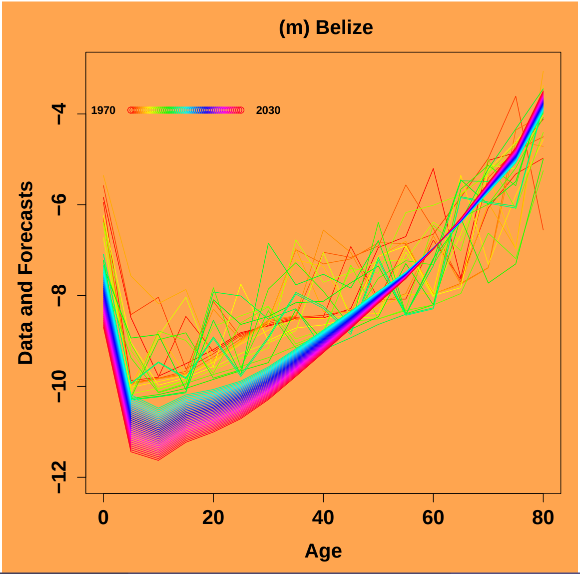
Mortality from Respiratory Infections, males, $\sigma = 0.02$

Smoothing over Age Groups



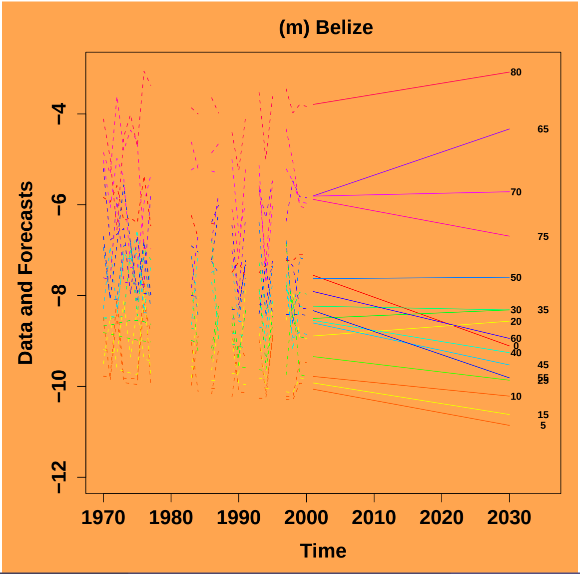
Mortality from Respiratory Infections, males, $\sigma = 0.01$

Smoothing over Age Groups



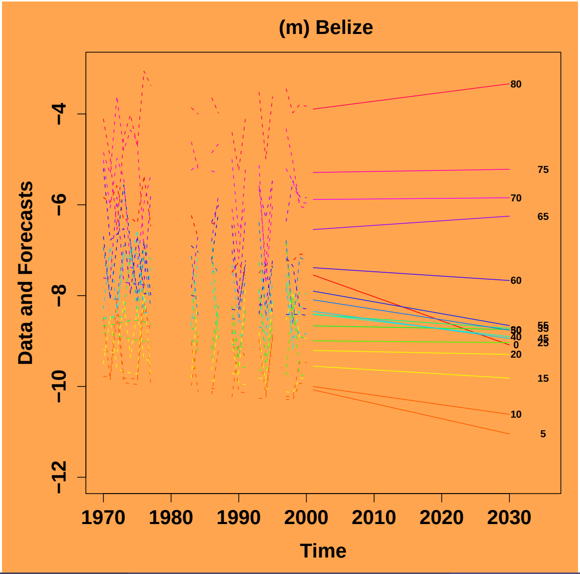
Mortality from Respiratory Infections, males

Least Squares



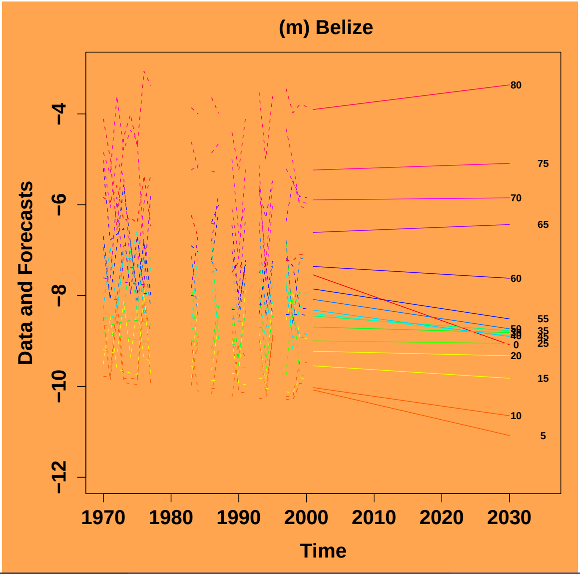
Mortality from Respiratory Infections, males, $\sigma = 2.00$

Smoothing over Age Groups



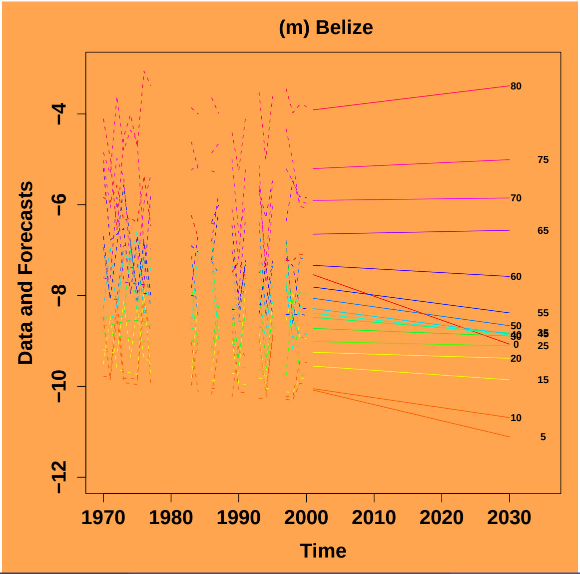
Mortality from Respiratory Infections, males, $\sigma = 1.51$

Smoothing over Age Groups



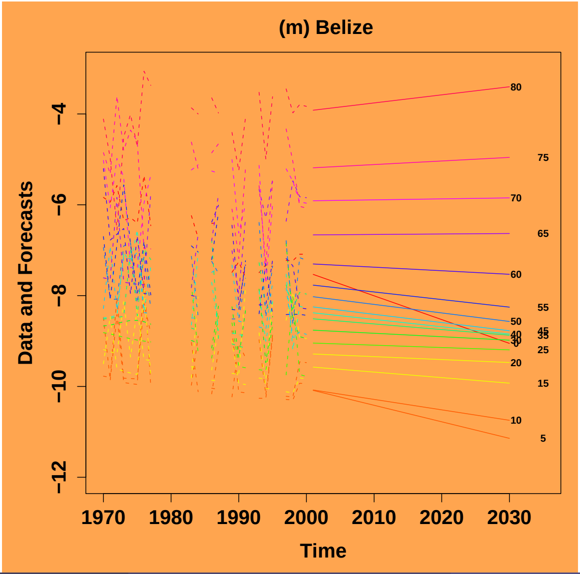
Mortality from Respiratory Infections, males, $\sigma = 1.15$

Smoothing over Age Groups



Mortality from Respiratory Infections, males, $\sigma = 0.87$

Smoothing over Age Groups



Mortality from Respiratory Infections, males, $\sigma = 0.66$

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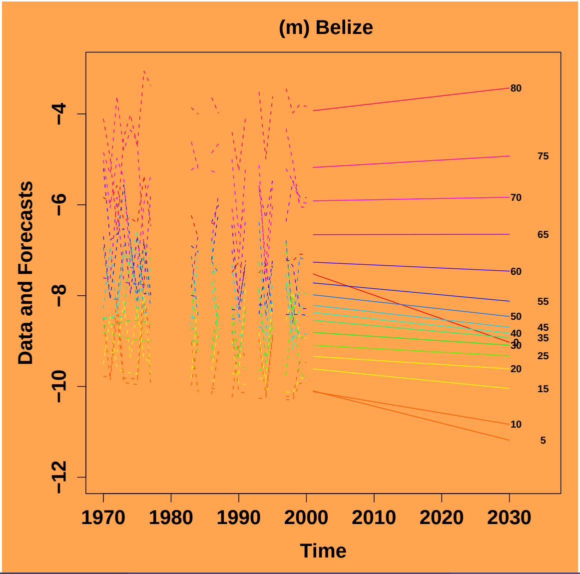

Mortality from Respiratory Infections, males, $\sigma = 0.50$

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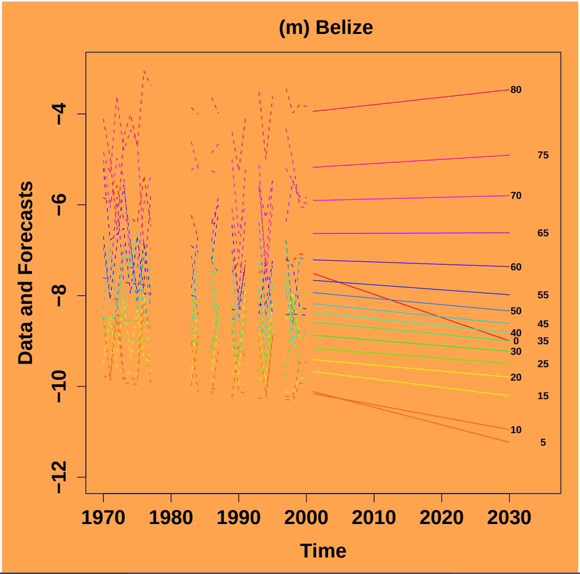

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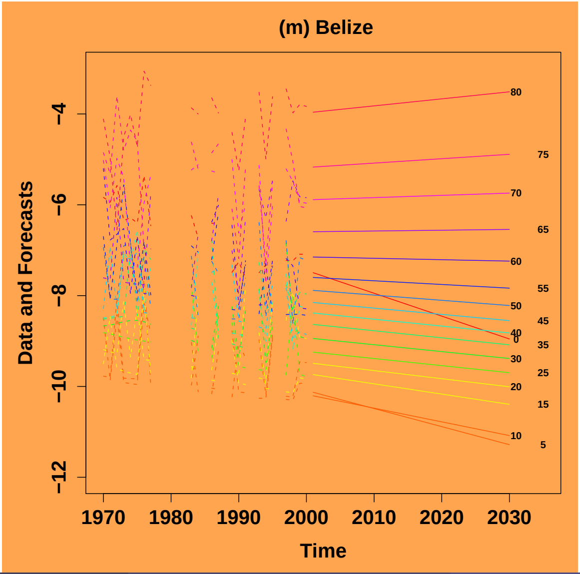
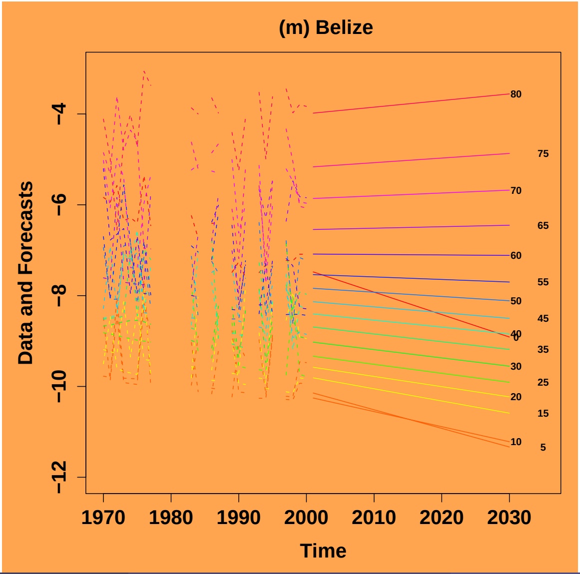


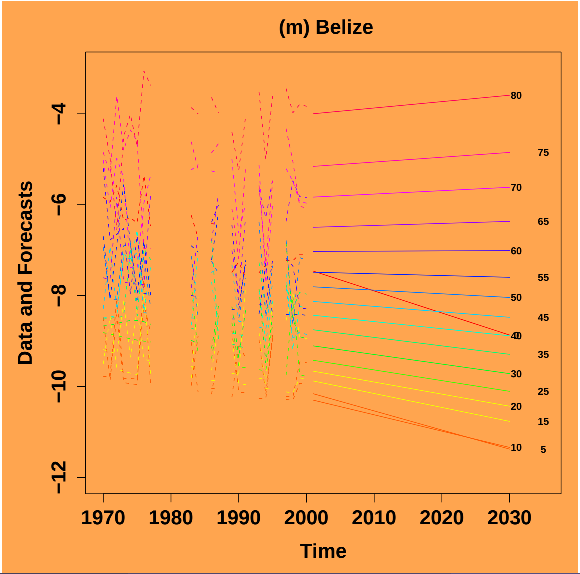
Mortality from Respiratory Infections, males, $\sigma = 0.28$

Smoothing over Age Groups



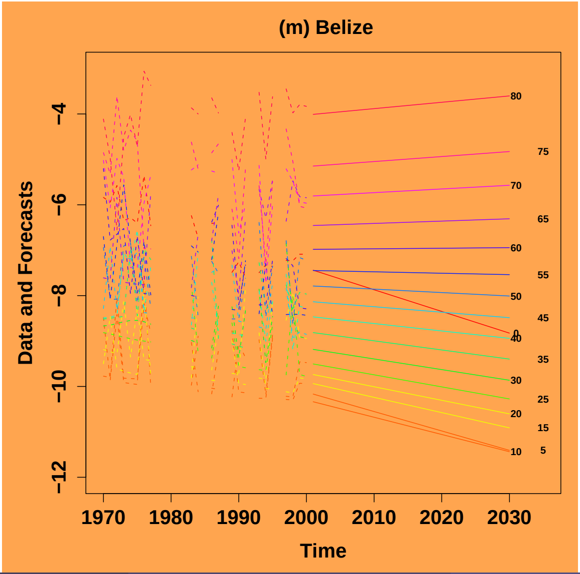
Mortality from Respiratory Infections, males, $\sigma = 0.21$

Smoothing over Age Groups



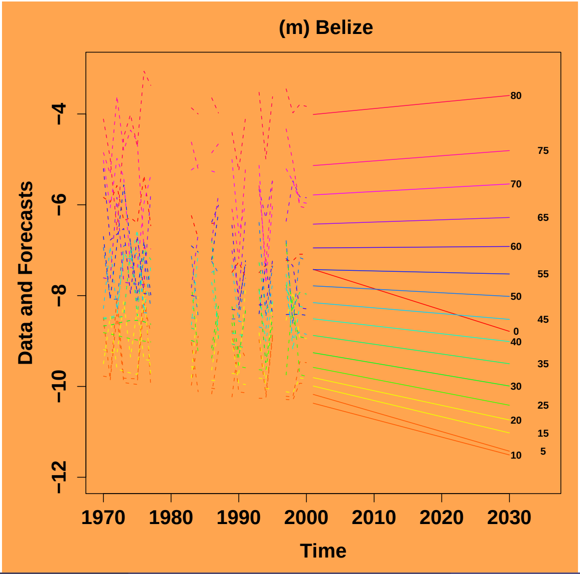
Mortality from Respiratory Infections, males, $\sigma = 0.16$

Smoothing over Age Groups



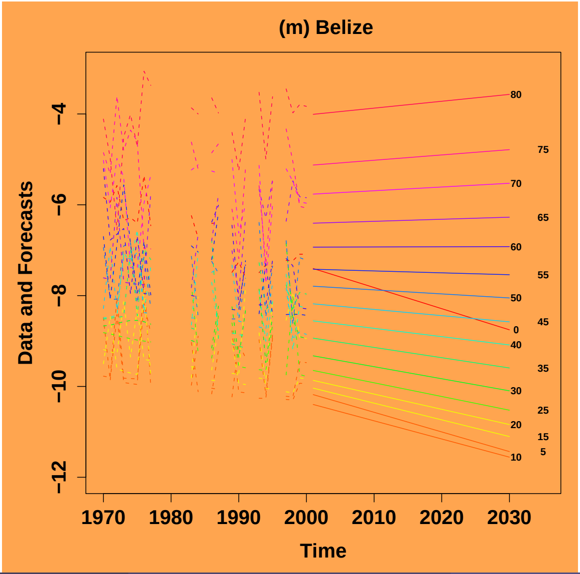
Mortality from Respiratory Infections, males, $\sigma = 0.12$

Smoothing over Age Groups



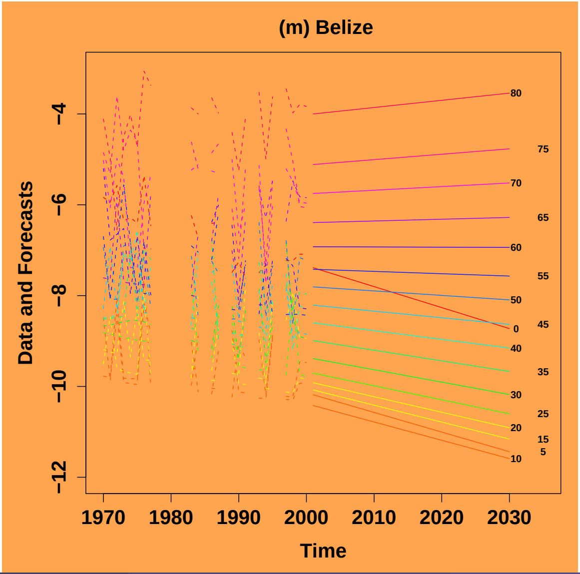
Mortality from Respiratory Infections, males, $\sigma = 0.09$

Smoothing over Age Groups



Mortality from Respiratory Infections, males, $\sigma = 0.07$

Smoothing over Age Groups



Mortality from Respiratory Infections, males, $\sigma = 0.05$

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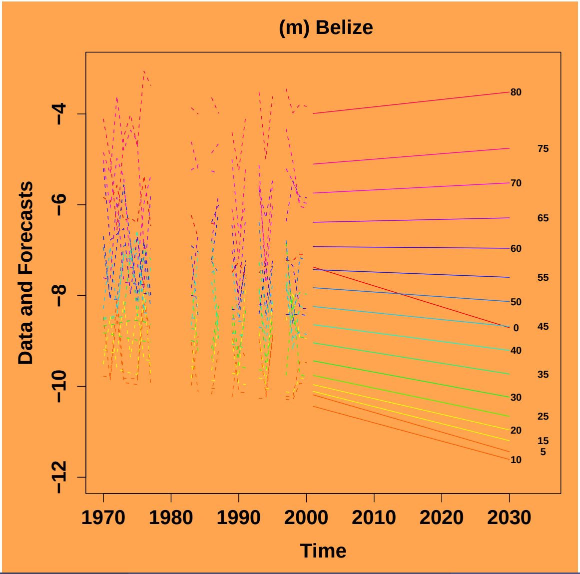

Mortality from Respiratory Infections, males, $\sigma = 0.04$

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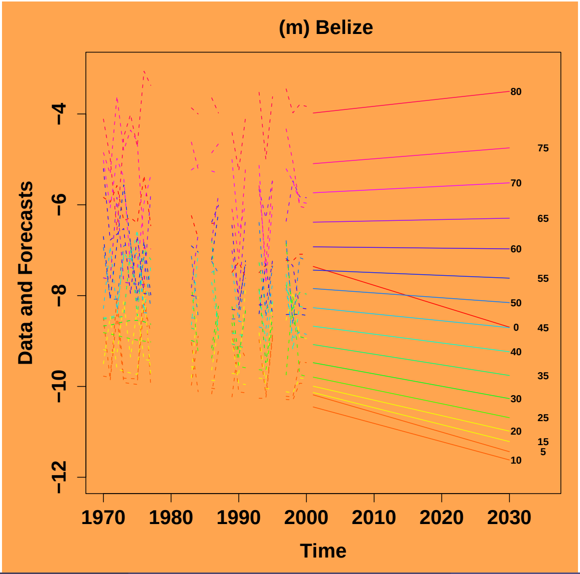
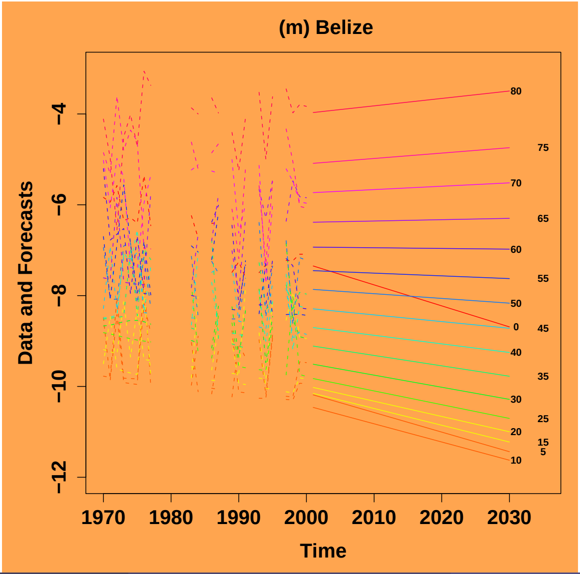


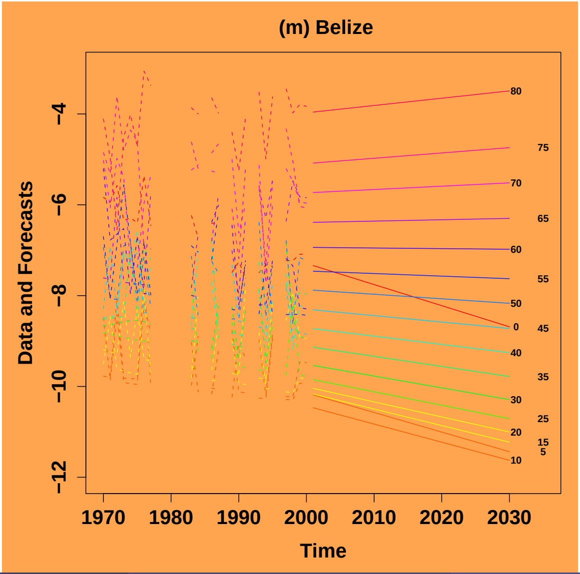
Mortality from Respiratory Infections, males, $\sigma = 0.03$

Smoothing over Age Groups



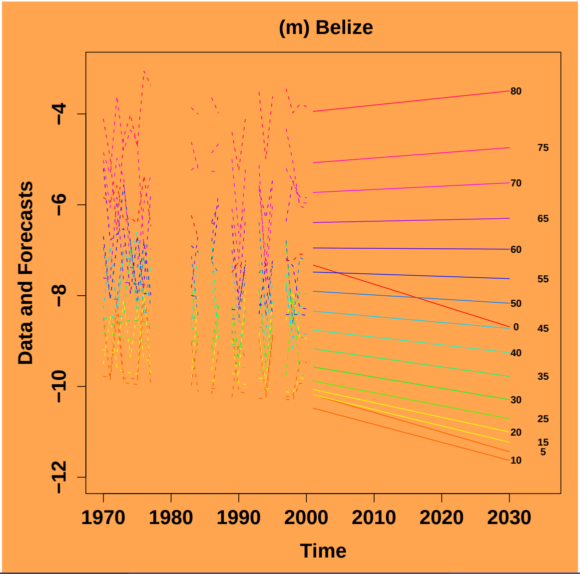
Mortality from Respiratory Infections, males, $\sigma = 0.02$

Smoothing over Age Groups



Mortality from Respiratory Infections, males, $\sigma = 0.01$

Smoothing over Age Groups



Smoothing Trends over Age Groups

Smoothing Trends over Age Groups

Log-mortality in Belize males from respiratory infections

Smoothing Trends over Age Groups

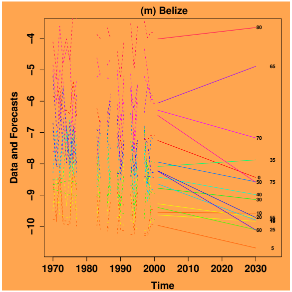
Log-mortality in Belize males from respiratory infections

Least Squares

Smoothing Trends over Age Groups

Log-mortality in Belize males from respiratory infections

Least Squares



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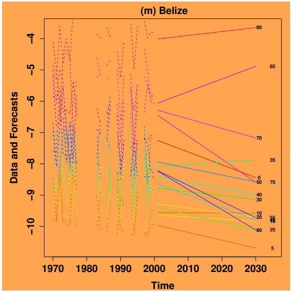

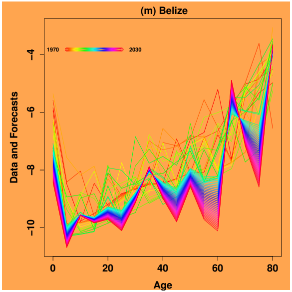

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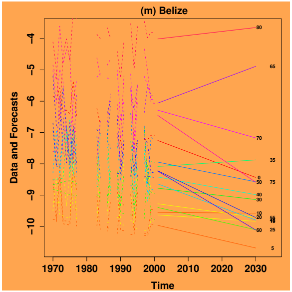

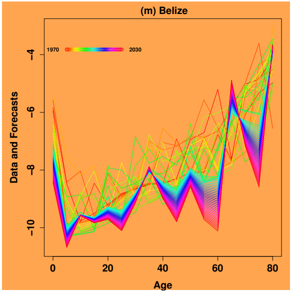

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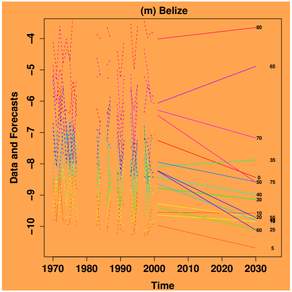

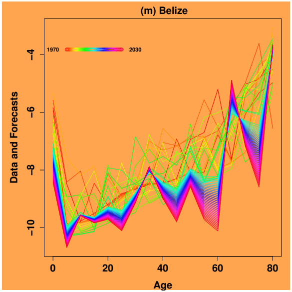

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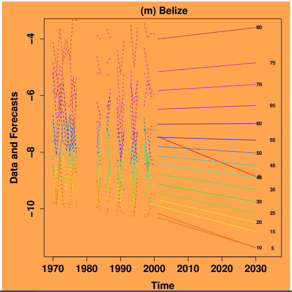

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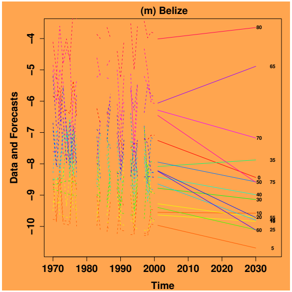

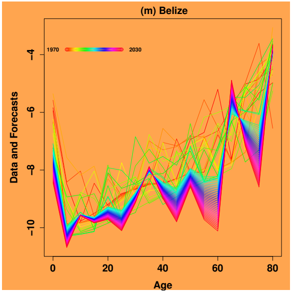

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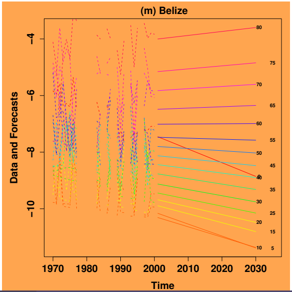

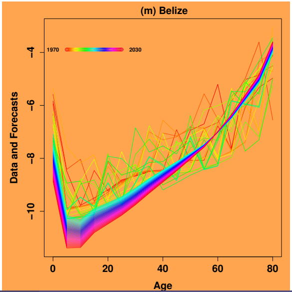

Smoothing Trends over Age Groups and Time

Smoothing Trends over Age Groups and Time

Log-Mortality in Bulgarian males from respiratory infections

Least Squares

Smoothing Trends over Age Groups and Time

Log-Mortality in Bulgarian males from respiratory infections

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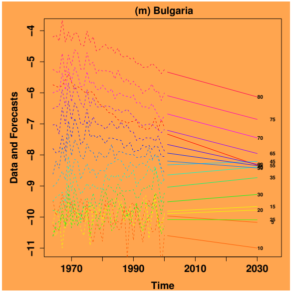
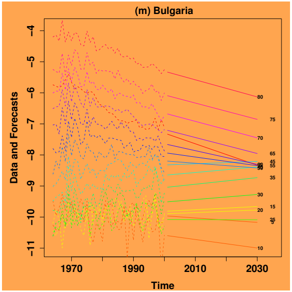
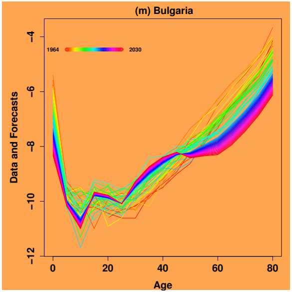


Smoothing Trends over Age Groups and Time

Log-Mortality in Bulgarian males from respiratory infections

Least Squares





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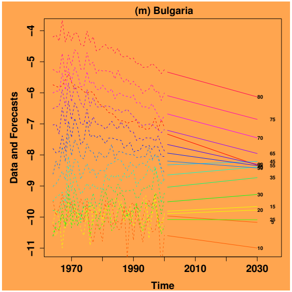

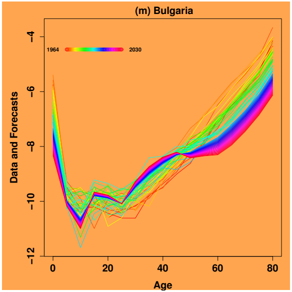

Smoothing
Age and Time

Smoothing Trends over Age Groups and Time

Log-Mortality in Bulgarian males from respiratory infections

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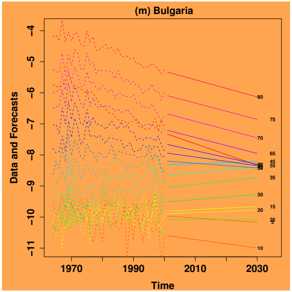

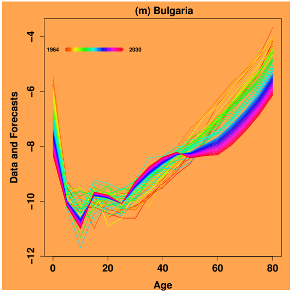

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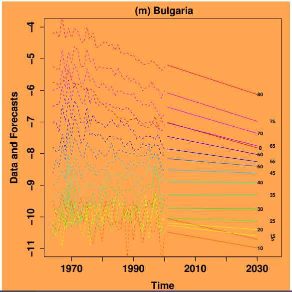

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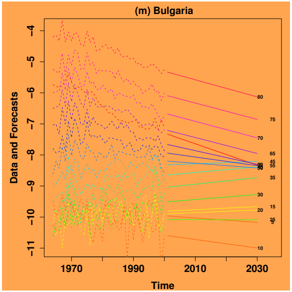

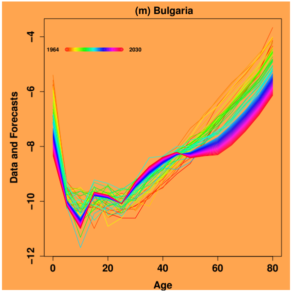

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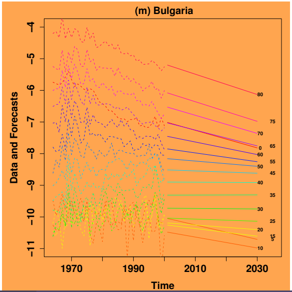

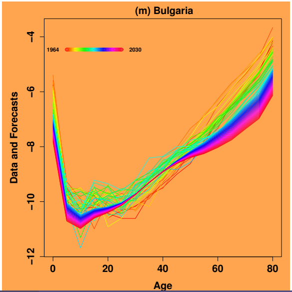

Using Covariates (GDP, tobacco, trend, log trend)

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Lung cancer in Korean Males

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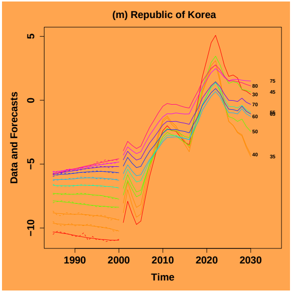

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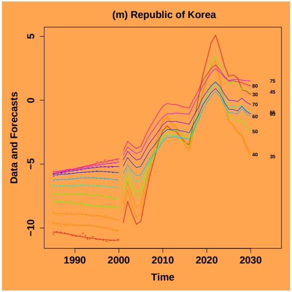

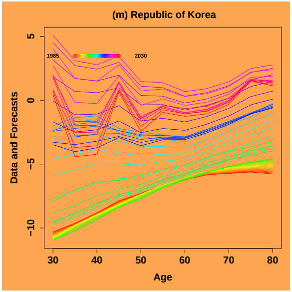

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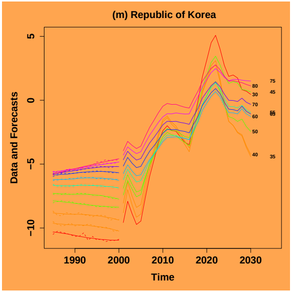

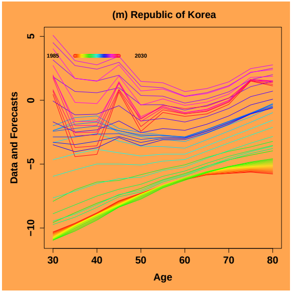

Smooth over age, time, age/time

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Lung cancer in Korean Males

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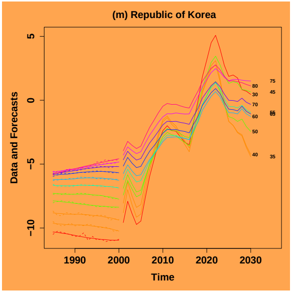

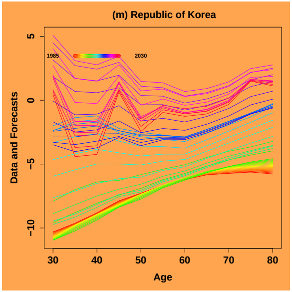

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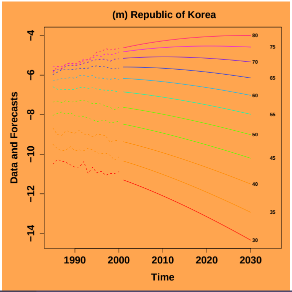

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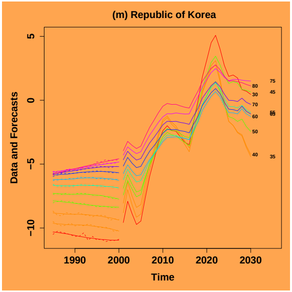

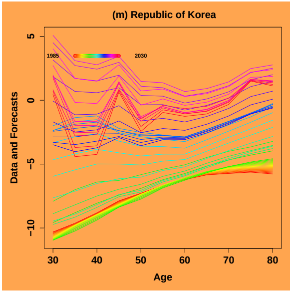

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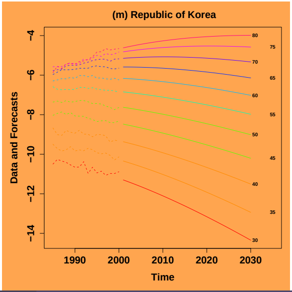

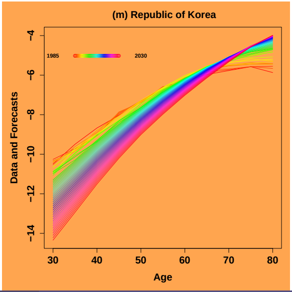

Using Covariates (GDP, tobacco, trend, log trend)

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Lung cancer in Males, Singapore

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Least Squares

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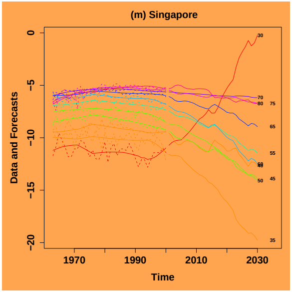

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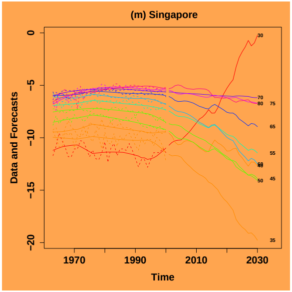
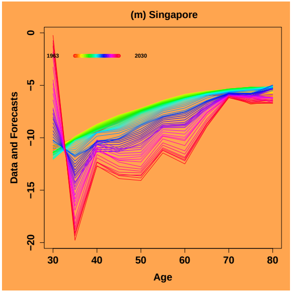
Least Squares





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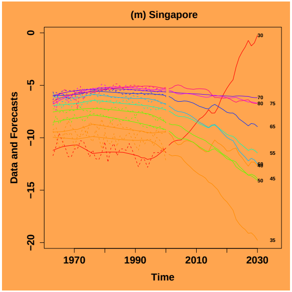
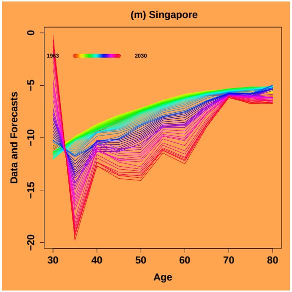
Least Squares





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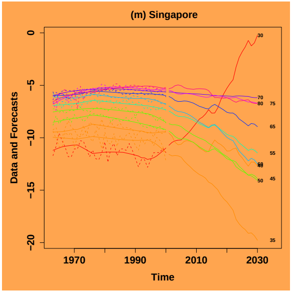

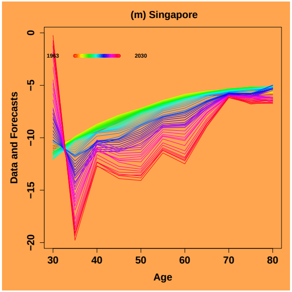

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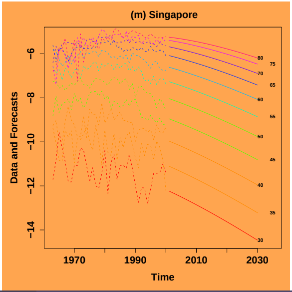

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Lung cancer in Males, Singapore

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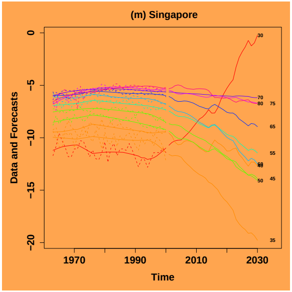

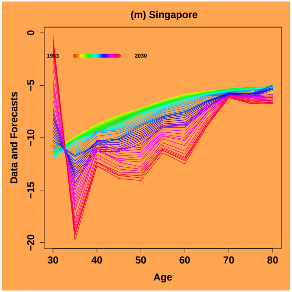

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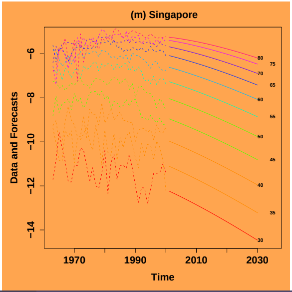
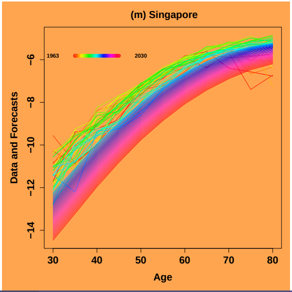
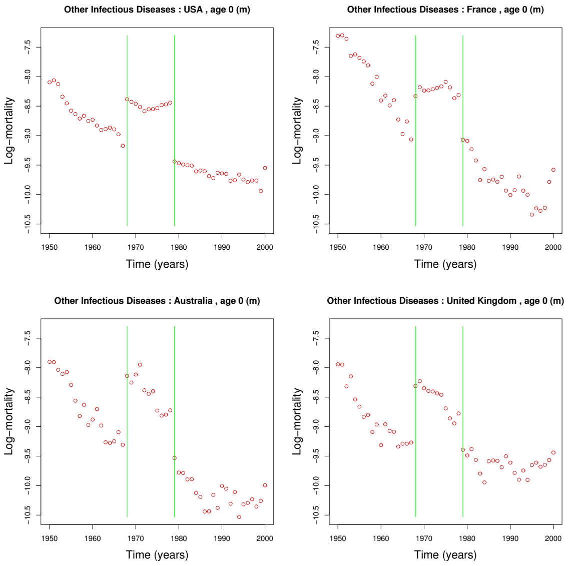
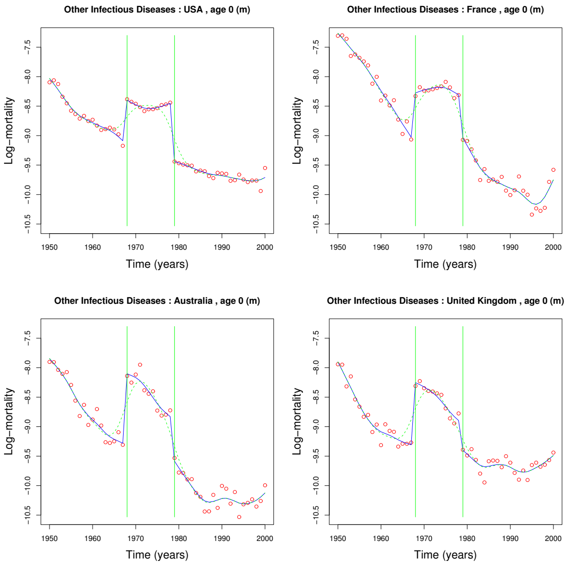




What about ICD Changes?



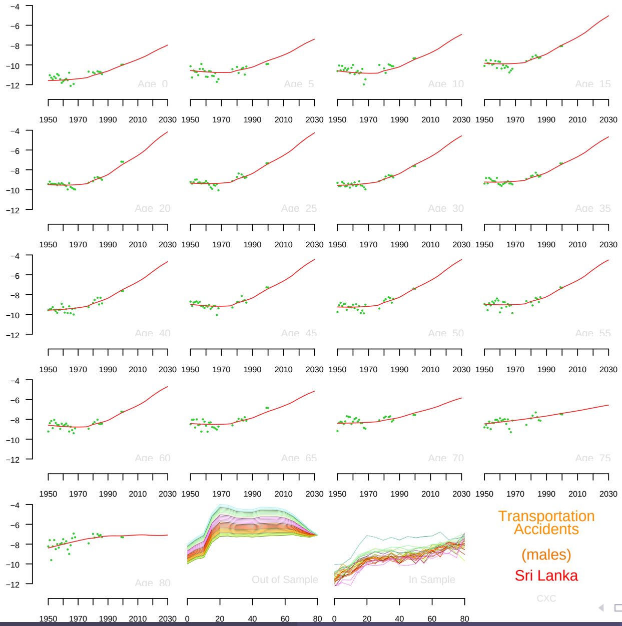
Fixing ICD Changes



A book manuscript, YourCast software, etc.

http://GKing.Harvard.edu

Without Country Smoothing



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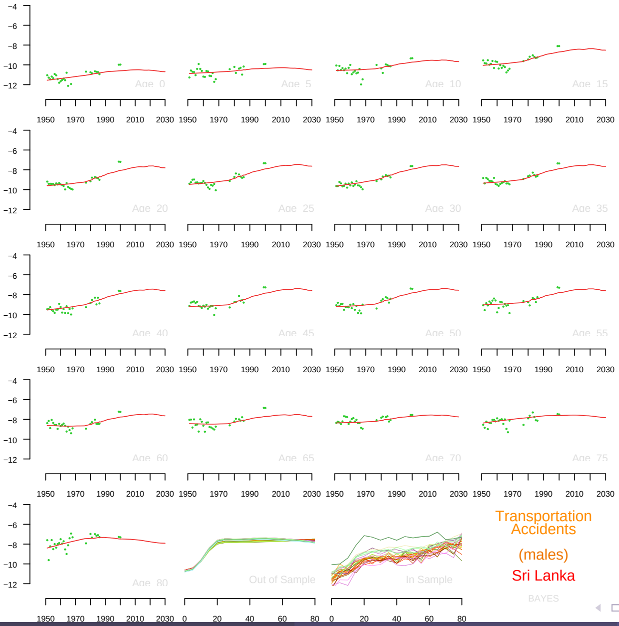
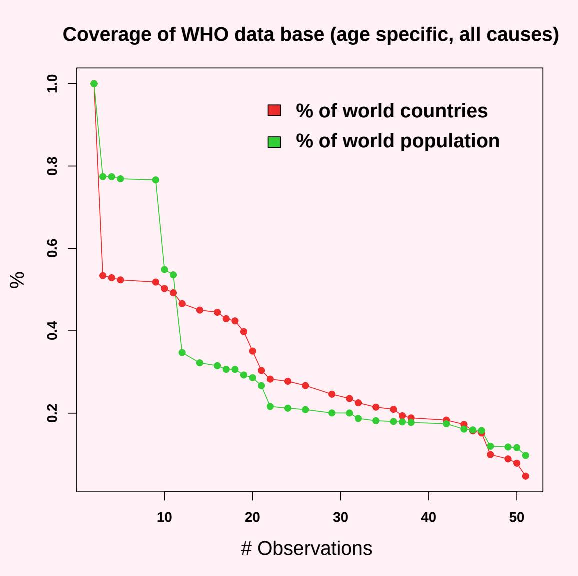


Many Short Time Series



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- where $p(a,t)$ is a polynomial in a (whose degree is the degree of the derivative in the prior)
- Prior information is about relative (not absolute) levels of log-mortality

Preview of Results: Out-of-Sample Evaluation

Preview of Results: Out-of-Sample Evaluation

Mean Absolute Error in Males (over age and country)

Preview of Results: Out-of-Sample Evaluation

Mean Absolute Error in Males (over age and country)

	% Improvement Over Best Previous	% Improvement to Best Conceivable
Cardiovascular	22	49
Lung Cancer	24	47
Transportation	16	31
Respiratory Chronic	13	30
Other Infectious	12	30
Stomach Cancer	8	24
All-Cause	12	22
Suicide	7	17
Respiratory Infectious	3	7

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- Does *considerably* better with more informative covariates

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Mean Absolute Error in Males (over age and country)

	Mean Absolute Error			% Improvement	
	Best Previous	Our Method	Best Conceivable	Over Best Previous	to Best Conceivable
Cardiovascular	0.34	0.27	0.19	22	49
Lung Cancer	0.36	0.27	0.17	24	47
Transportation	0.37	0.31	0.18	16	31
Respiratory Chronic	0.45	0.39	0.26	13	30
Other Infectious	0.55	0.48	0.32	12	30
Stomach Cancer	0.30	0.27	0.20	8	24
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