Demographic Forecasting: Incorporating Qualitative Insight in Quantitative Modeling

> Gary King Harvard University

# Joint work with Federico Girosi (RAND) with contributions from Kevin Quinn and Gregory Wawro

Demographic Forecasting

æ

イロト イヨト イヨト イヨト

- ∢ ∃ ▶

- Mortality forecasts, which are studied in:
  - demography & sociology

- ∢ ∃ →

- demography & sociology
- public health & biostatistics

.∋...>



- demography & sociology
- public health & biostatistics
- economics & social security and retirement planning

- demography & sociology
- public health & biostatistics
- economics & social security and retirement planning
- actuarial science & insurance companies

- demography & sociology
- public health & biostatistics
- economics & social security and retirement planning
- actuarial science & insurance companies
- medical research & pharmaceutical companies

- demography & sociology
- public health & biostatistics
- economics & social security and retirement planning
- actuarial science & insurance companies
- medical research & pharmaceutical companies
- political science & public policy

- demography & sociology
- public health & biostatistics
- economics & social security and retirement planning
- actuarial science & insurance companies
- medical research & pharmaceutical companies
- political science & public policy
- A better forecasting method

- demography & sociology
- public health & biostatistics
- economics & social security and retirement planning
- actuarial science & insurance companies
- medical research & pharmaceutical companies
- political science & public policy
- A better forecasting method
- A better farcasting method

- demography & sociology
- public health & biostatistics
- economics & social security and retirement planning
- actuarial science & insurance companies
- medical research & pharmaceutical companies
- political science & public policy
- A better forecasting method
- A better farcasting method
- Other results we needed to achieve this original goal



- demography & sociology
- public health & biostatistics
- economics & social security and retirement planning
- actuarial science & insurance companies
- medical research & pharmaceutical companies
- political science & public policy
- A better forecasting method
- A better farcasting method
- Other results we needed to achieve this original goal
- Approach: Formalizing qualitative insights in quantitative models

E

B ▶ < B ▶



- Affects almost every field that studies human behavior
  - Medicine: clinical decisions vs. "evidence-based medicine"



- Medicine: clinical decisions vs. "evidence-based medicine"
- Law: jurisprudence vs. "empirical research"

- Medicine: clinical decisions vs. "evidence-based medicine"
- Law: jurisprudence vs. "empirical research"
- Political Science: Area studies vs. comparative politics

- Medicine: clinical decisions vs. "evidence-based medicine"
- Law: jurisprudence vs. "empirical research"
- Political Science: Area studies vs. comparative politics
- Sociology: qualitative vs. quantitative work

- Medicine: clinical decisions vs. "evidence-based medicine"
- Law: jurisprudence vs. "empirical research"
- Political Science: Area studies vs. comparative politics
- Sociology: qualitative vs. quantitative work
- Psychology: clinicians vs. scientists

- Medicine: clinical decisions vs. "evidence-based medicine"
- Law: jurisprudence vs. "empirical research"
- Political Science: Area studies vs. comparative politics
- Sociology: qualitative vs. quantitative work
- Psychology: clinicians vs. scientists
- Geography: place people vs. space people

- Medicine: clinical decisions vs. "evidence-based medicine"
- Law: jurisprudence vs. "empirical research"
- Political Science: Area studies vs. comparative politics
- Sociology: qualitative vs. quantitative work
- Psychology: clinicians vs. scientists
- Geography: place people vs. space people
- Qualitative information:

- Medicine: clinical decisions vs. "evidence-based medicine"
- Law: jurisprudence vs. "empirical research"
- Political Science: Area studies vs. comparative politics
- Sociology: qualitative vs. quantitative work
- Psychology: clinicians vs. scientists
- Geography: place people vs. space people
- Qualitative information:
  - Definition: information not quantified and formalized

- Medicine: clinical decisions vs. "evidence-based medicine"
- Law: jurisprudence vs. "empirical research"
- Political Science: Area studies vs. comparative politics
- Sociology: qualitative vs. quantitative work
- Psychology: clinicians vs. scientists
- Geography: place people vs. space people
- Qualitative information:
  - Definition: information not quantified and formalized
  - Anthropological, ethnographic, archival, participant observation, soaking and poking, contextual...

- Medicine: clinical decisions vs. "evidence-based medicine"
- Law: jurisprudence vs. "empirical research"
- Political Science: Area studies vs. comparative politics
- Sociology: qualitative vs. quantitative work
- Psychology: clinicians vs. scientists
- Geography: place people vs. space people
- Qualitative information:
  - Definition: information not quantified and formalized
  - Anthropological, ethnographic, archival, participant observation, soaking and poking, contextual...
  - All research is qualitative; some is also quantitative.

- Medicine: clinical decisions vs. "evidence-based medicine"
- Law: jurisprudence vs. "empirical research"
- Political Science: Area studies vs. comparative politics
- Sociology: qualitative vs. quantitative work
- Psychology: clinicians vs. scientists
- Geography: place people vs. space people
- Qualitative information:
  - Definition: information not quantified and formalized
  - Anthropological, ethnographic, archival, participant observation, soaking and poking, contextual...
  - All research is qualitative; some is also quantitative.
  - Goal: include as much information as possible from any source

# Other Results (Needed to Develop Improved Forecasts)

ヨト イヨト

Image: Image:

3 K K 3 K

#### • Output: same as linear regression

- Output: same as linear regression
- Estimates a set of linear regressions together (over countries, age groups, years, etc.)

- Output: same as linear regression
- Estimates a set of linear regressions together (over countries, age groups, years, etc.)
- Can include *different covariates* in each regression

- Output: same as linear regression
- Estimates a set of linear regressions together (over countries, age groups, years, etc.)
- Can include *different covariates* in each regression
- New ways of creating Bayesian priors



- Output: same as linear regression
- Estimates a set of linear regressions together (over countries, age groups, years, etc.)
- Can include different covariates in each regression
- New ways of creating Bayesian priors
- Produces forecasts and farcasts using considerably more information

## Resolving Disputes: Comparativists vs. Area Studies

3

過 ト イヨ ト イヨト

## Resolving Disputes: Comparativists vs. Area Studies

• When a variable is not available in all countries, comparativists must choose:

∃ ▶ ∢ ∃ ▶



# Resolving Disputes: Comparativists vs. Area Studies

- When a variable is not available in all countries, comparativists must choose:
  - Run separate regressions in each country

- When a variable is not available in all countries, comparativists must choose:
  - Run separate regressions in each country
    - risking large inefficiencies (huge standard errors)



- When a variable is not available in all countries, comparativists must choose:
  - Run separate regressions in each country
    - risking large inefficiencies (huge standard errors)
  - Omit variables not observed for all countries

- When a variable is not available in all countries, comparativists must choose:
  - Run separate regressions in each country
    - risking large inefficiencies (huge standard errors)
  - Omit variables not observed for all countries
    - risking omitted variable bias

- When a variable is not available in all countries, comparativists must choose:
  - Run separate regressions in each country
    - risking large inefficiencies (huge standard errors)
  - Omit variables not observed for all countries
    - risking omitted variable bias
  - Second text and te

- When a variable is not available in all countries, comparativists must choose:
  - Run separate regressions in each country
    - risking large inefficiencies (huge standard errors)
  - Omit variables not observed for all countries
    - risking omitted variable bias
  - Second transformed and the second transformed and transfo
    - risking selection bias

- When a variable is not available in all countries, comparativists must choose:
  - Run separate regressions in each country
    - risking large inefficiencies (huge standard errors)
  - Omit variables not observed for all countries
    - risking omitted variable bias
  - Second transformed and the second transformed and transfo
    - risking selection bias
- Our methods:

- When a variable is not available in all countries, comparativists must choose:
  - Run separate regressions in each country
    - risking large inefficiencies (huge standard errors)
  - Omit variables not observed for all countries
    - risking omitted variable bias
  - Exclude countries when some variables are not available — risking selection bias
- Our methods:
  - Allows different covariates in each regression

- When a variable is not available in all countries, comparativists must choose:
  - Run separate regressions in each country
    - risking large inefficiencies (huge standard errors)
  - Omit variables not observed for all countries
    - risking omitted variable bias
  - Exclude countries when some variables are not available — risking selection bias
- Our methods:
  - Allows different covariates in each regression
  - All are still estimated together

- When a variable is not available in all countries, comparativists must choose:
  - Run separate regressions in each country
    - risking large inefficiencies (huge standard errors)
  - Omit variables not observed for all countries
    - risking omitted variable bias
  - Exclude countries when some variables are not available — risking selection bias
- Our methods:
  - Allows different covariates in each regression
  - All are still estimated together
  - Can thereby forecast with much more local, contextual information

- When a variable is not available in all countries, comparativists must choose:
  - Run separate regressions in each country
    - risking large inefficiencies (huge standard errors)
  - Omit variables not observed for all countries
    - risking omitted variable bias
  - Second the second text and the second text and text an
    - risking selection bias
- Our methods:
  - Allows different covariates in each regression
  - All are still estimated together
  - Can thereby forecast with much more local, contextual information
  - Resolves analogous issues in predicting mortality by age, sex, and cause

• Multidimensional Data Structures: 24 causes of death, 17 age groups, 2 sexes, 191 countries, 50 annual observations.



- Multidimensional Data Structures: 24 causes of death, 17 age groups, 2 sexes, 191 countries, 50 annual observations.
- One time series analysis for each of 155,856 cross-sections:



- Multidimensional Data Structures: 24 causes of death, 17 age groups, 2 sexes, 191 countries, 50 annual observations.
- One time series analysis for each of 155,856 cross-sections: with 1 minute to analyze each, one run takes 108 days



- Multidimensional Data Structures: 24 causes of death, 17 age groups, 2 sexes, 191 countries, 50 annual observations.
- One time series analysis for each of 155,856 cross-sections: with 1 minute to analyze each, one run takes 108 days
- Every decision must be automated, systematized, and formalized:

- Multidimensional Data Structures: 24 causes of death, 17 age groups, 2 sexes, 191 countries, 50 annual observations.
- One time series analysis for each of 155,856 cross-sections: with 1 minute to analyze each, one run takes 108 days
- Every decision must be automated, systematized, and formalized: the same goal as including qualitative information in the model

- Multidimensional Data Structures: 24 causes of death, 17 age groups, 2 sexes, 191 countries, 50 annual observations.
- One time series analysis for each of 155,856 cross-sections: with 1 minute to analyze each, one run takes 108 days
- Every decision must be automated, systematized, and formalized: the same goal as including qualitative information in the model
- Explanatory variables:

- Multidimensional Data Structures: 24 causes of death, 17 age groups, 2 sexes, 191 countries, 50 annual observations.
- One time series analysis for each of 155,856 cross-sections: with 1 minute to analyze each, one run takes 108 days
- Every decision must be automated, systematized, and formalized: the same goal as including qualitative information in the model
- Explanatory variables:
  - Available in many countries: tobacco consumption, GDP, human capital, trends, fat consumption, total fertility rates, etc.

- Multidimensional Data Structures: 24 causes of death, 17 age groups, 2 sexes, 191 countries, 50 annual observations.
- One time series analysis for each of 155,856 cross-sections: with 1 minute to analyze each, one run takes 108 days
- Every decision must be automated, systematized, and formalized: the same goal as including qualitative information in the model
- Explanatory variables:
  - Available in many countries: tobacco consumption, GDP, human capital, trends, fat consumption, total fertility rates, etc.
  - Numerous variables specific to a cause, age group, sex, and country

- Multidimensional Data Structures: 24 causes of death, 17 age groups, 2 sexes, 191 countries, 50 annual observations.
- One time series analysis for each of 155,856 cross-sections: with 1 minute to analyze each, one run takes 108 days
- Every decision must be automated, systematized, and formalized: the same goal as including qualitative information in the model
- Explanatory variables:
  - Available in many countries: tobacco consumption, GDP, human capital, trends, fat consumption, total fertility rates, etc.
  - Numerous variables specific to a cause, age group, sex, and country
- Most time series are very short.

- Multidimensional Data Structures: 24 causes of death, 17 age groups, 2 sexes, 191 countries, 50 annual observations.
- One time series analysis for each of 155,856 cross-sections: with 1 minute to analyze each, one run takes 108 days
- Every decision must be automated, systematized, and formalized: the same goal as including qualitative information in the model
- Explanatory variables:
  - Available in many countries: tobacco consumption, GDP, human capital, trends, fat consumption, total fertility rates, etc.
  - Numerous variables specific to a cause, age group, sex, and country
- Most time series are very short. A majority of countries have only a few isolated annual observations;

- Multidimensional Data Structures: 24 causes of death, 17 age groups, 2 sexes, 191 countries, 50 annual observations.
- One time series analysis for each of 155,856 cross-sections: with 1 minute to analyze each, one run takes 108 days
- Every decision must be automated, systematized, and formalized: the same goal as including qualitative information in the model
- Explanatory variables:
  - Available in many countries: tobacco consumption, GDP, human capital, trends, fat consumption, total fertility rates, etc.
  - Numerous variables specific to a cause, age group, sex, and country
- Most time series are very short. A majority of countries have only a few isolated annual observations; only 54 countries have at least 20 observations;

- Multidimensional Data Structures: 24 causes of death, 17 age groups, 2 sexes, 191 countries, 50 annual observations.
- One time series analysis for each of 155,856 cross-sections: with 1 minute to analyze each, one run takes 108 days
- Every decision must be automated, systematized, and formalized: the same goal as including qualitative information in the model
- Explanatory variables:
  - Available in many countries: tobacco consumption, GDP, human capital, trends, fat consumption, total fertility rates, etc.
  - Numerous variables specific to a cause, age group, sex, and country
- Most time series are very short. A majority of countries have only a few isolated annual observations; only 54 countries have at least 20 observations; Africa, AIDS, & Malaria are real problems

E

(日) (周) (三) (三)

#### Procedures:

Demographic Forecasting

B ▶ < B ▶



#### Procedures:

• Develop private forecasts qualitatively (i.e., informally)



- Develop private forecasts qualitatively (i.e., informally)
- Adopt a 'toy' statistical model

- Develop private forecasts qualitatively (i.e., informally)
- Adopt a 'toy' statistical model
- Get data; produce tentative forecasts with the model

- Develop private forecasts qualitatively (i.e., informally)
- Adopt a 'toy' statistical model
- Get data; produce tentative forecasts with the model
- Adjust model until forecasts fit private views

- Develop private forecasts qualitatively (i.e., informally)
- Adopt a 'toy' statistical model
- Get data; produce tentative forecasts with the model
- Adjust model until forecasts fit private views
- Present forecasts, with statistical model as your "method"

#### Procedures:

- Develop private forecasts qualitatively (i.e., informally)
- Adopt a 'toy' statistical model
- Get data; produce tentative forecasts with the model
- Adjust model until forecasts fit private views
- Present forecasts, with statistical model as your "method"

#### Meaning of procedures

- 一司

#### Procedures:

- Develop private forecasts qualitatively (i.e., informally)
- Adopt a 'toy' statistical model
- Get data; produce tentative forecasts with the model
- Adjust model until forecasts fit private views
- Present forecasts, with statistical model as your "method"

#### Meaning of procedures

Forecasts use qualitative information (good!)

#### Procedures:

- Develop private forecasts qualitatively (i.e., informally)
- Adopt a 'toy' statistical model
- Get data; produce tentative forecasts with the model
- Adjust model until forecasts fit private views
- Present forecasts, with statistical model as your "method"

#### Meaning of procedures

- Forecasts use qualitative information (good!)
- Statistical models add little (bad!)

#### Procedures:

- Develop private forecasts qualitatively (i.e., informally)
- Adopt a 'toy' statistical model
- Get data; produce tentative forecasts with the model
- Adjust model until forecasts fit private views
- Present forecasts, with statistical model as your "method"

#### Meaning of procedures

- Forecasts use qualitative information (good!)
- Statistical models add little (bad!)
- Method is invulnerable to being proven wrong

#### Procedures:

- Develop private forecasts qualitatively (i.e., informally)
- Adopt a 'toy' statistical model
- Get data; produce tentative forecasts with the model
- Adjust model until forecasts fit private views
- Present forecasts, with statistical model as your "method"

#### Meaning of procedures

- Forecasts use qualitative information (good!)
- Statistical models add little (bad!)
- Method is invulnerable to being proven wrong
- Subtitle of my talk should be reversed:
  "Incorporating Quantitative Modeling into Qualitative Forecasts"

#### Preview of Results: Out-of-Sample Evaluation

æ

Mean Absolute Error in Males (over age and country)

Image: Image:

∃ ► < ∃ ►</p>

Mean Absolute Error in Males (over age and country)

|                        | % Improvement |             |
|------------------------|---------------|-------------|
|                        | Over Best     | to Best     |
|                        | Previous      | Conceivable |
| Cardiovascular         | 22            | 49          |
| Lung Cancer            | 24            | 47          |
| Transportation         | 16            | 31          |
| Respiratory Chronic    | 13            | 30          |
| Other Infectious       | 12            | 30          |
| Stomach Cancer         | 8             | 24          |
| All-Cause              | 12            | 22          |
| Suicide                | 7             | 17          |
| Respiratory Infectious | 3             | 7           |

イロト イポト イヨト イヨト



Mean Absolute Error in Males (over age and country)

|                        | % Improvement |             |
|------------------------|---------------|-------------|
|                        | Over Best     | to Best     |
|                        | Previous      | Conceivable |
| Cardiovascular         | 22            | 49          |
| Lung Cancer            | 24            | 47          |
| Transportation         | 16            | 31          |
| Respiratory Chronic    | 13            | 30          |
| Other Infectious       | 12            | 30          |
| Stomach Cancer         | 8             | 24          |
| All-Cause              | 12            | 22          |
| Suicide                | 7             | 17          |
| Respiratory Infectious | 3             | 7           |

• Each row averages 6,800 forecast errors (17 age groups, 40 countries, and 10 out-of-sample years).

イロト イポト イヨト イヨト



Mean Absolute Error in Males (over age and country)

|                        | % Improvement |             |
|------------------------|---------------|-------------|
|                        | Over Best     | to Best     |
|                        | Previous      | Conceivable |
| Cardiovascular         | 22            | 49          |
| Lung Cancer            | 24            | 47          |
| Transportation         | 16            | 31          |
| Respiratory Chronic    | 13            | 30          |
| Other Infectious       | 12            | 30          |
| Stomach Cancer         | 8             | 24          |
| All-Cause              | 12            | 22          |
| Suicide                | 7             | 17          |
| Respiratory Infectious | 3             | 7           |

- Each row averages 6,800 forecast errors (17 age groups, 40 countries, and 10 out-of-sample years).
- % to best conceivable = % of the way our method takes us from the best existing to the best conceivable forecast.

< ロ > < 同 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ >

Mean Absolute Error in Males (over age and country)

|                        | % Improvement |             |
|------------------------|---------------|-------------|
|                        | Over Best     | to Best     |
|                        | Previous      | Conceivable |
| Cardiovascular         | 22            | 49          |
| Lung Cancer            | 24            | 47          |
| Transportation         | 16            | 31          |
| Respiratory Chronic    | 13            | 30          |
| Other Infectious       | 12            | 30          |
| Stomach Cancer         | 8             | 24          |
| All-Cause              | 12            | 22          |
| Suicide                | 7             | 17          |
| Respiratory Infectious | 3             | 7           |

- Each row averages 6,800 forecast errors (17 age groups, 40 countries, and 10 out-of-sample years).
- % to best conceivable = % of the way our method takes us from the best existing to the best conceivable forecast.
- The new method out-performs with the same covariates, for most countries, causes, sexes, and age groups.

< ロ > < 同 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ >

Mean Absolute Error in Males (over age and country)

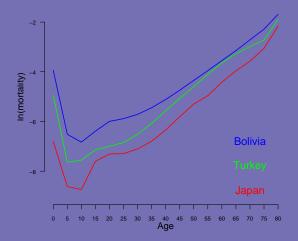
|                        | % Improvement |             |
|------------------------|---------------|-------------|
|                        | Over Best     | to Best     |
|                        | Previous      | Conceivable |
| Cardiovascular         | 22            | 49          |
| Lung Cancer            | 24            | 47          |
| Transportation         | 16            | 31          |
| Respiratory Chronic    | 13            | 30          |
| Other Infectious       | 12            | 30          |
| Stomach Cancer         | 8             | 24          |
| All-Cause              | 12            | 22          |
| Suicide                | 7             | 17          |
| Respiratory Infectious | 3             | 7           |

- Each row averages 6,800 forecast errors (17 age groups, 40 countries, and 10 out-of-sample years).
- % to best conceivable = % of the way our method takes us from the best existing to the best conceivable forecast.
- The new method out-performs with the same covariates, for most countries, causes, sexes, and age groups.
- Does *considerably* better with more informative covariates

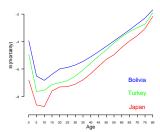
< ロ > < 同 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ >

#### All-Cause Mortality Age Profile Patterns

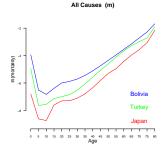
All Causes (m)

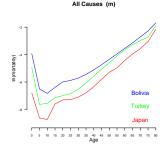


()



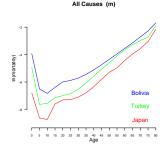
All Causes (m)



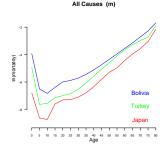


• Gompertz (1825): log-mortality is linear in age after age 20

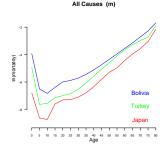
reduces 17 age-specific mortality rates to 2 parameters (intercept and slope)



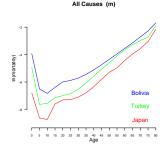
- reduces 17 age-specific mortality rates to 2 parameters (intercept and slope)
- Then forecast only these 2 parameters



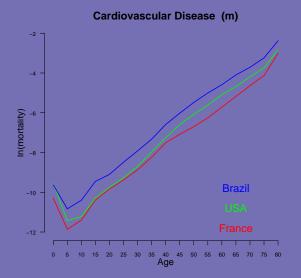
- reduces 17 age-specific mortality rates to 2 parameters (intercept and slope)
- Then forecast only these 2 parameters
- Reduces variance, constrains forecasts



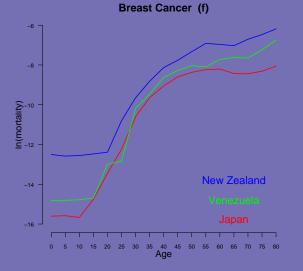
- reduces 17 age-specific mortality rates to 2 parameters (intercept and slope)
- Then forecast only these 2 parameters
- Reduces variance, constrains forecasts
- Dozens of more general functional forms proposed



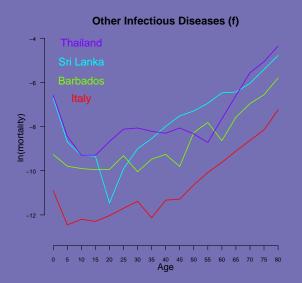
- reduces 17 age-specific mortality rates to 2 parameters (intercept and slope)
- Then forecast only these 2 parameters
- Reduces variance, constrains forecasts
- Dozens of more general functional forms proposed
- But does it fit anything else?



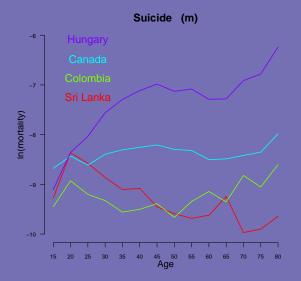
()



Demographic Forecasting



()



#### Parameterizing Age Profiles Does Not Work

Demographic Forecasting

3

(4 個)ト イヨト イヨト

## Parameterizing Age Profiles Does Not Work

• No mathematical form fits all or even most age profiles

ヨト イヨト

- No mathematical form fits all or even most age profiles
- Out-of-sample age profiles often unrealistic

- No mathematical form fits all or even most age profiles
- Out-of-sample age profiles often unrealistic
- The key empirical patterns are qualitative:

- No mathematical form fits all or even most age profiles
- Out-of-sample age profiles often unrealistic
- The key empirical patterns are qualitative:
  - Adjacent age groups have similar mortality rates

- No mathematical form fits all or even most age profiles
- Out-of-sample age profiles often unrealistic
- The key empirical patterns are qualitative:
  - Adjacent age groups have similar mortality rates
  - Age profiles are more variable for younger ages

- No mathematical form fits all or even most age profiles
- Out-of-sample age profiles often unrealistic
- The key empirical patterns are qualitative:
  - Adjacent age groups have similar mortality rates
  - Age profiles are more variable for younger ages
  - We don't know much about levels or exact shapes

- No mathematical form fits all or even most age profiles
- Out-of-sample age profiles often unrealistic
- The key empirical patterns are qualitative:
  - Adjacent age groups have similar mortality rates
  - Age profiles are more variable for younger ages
  - We don't know much about levels or exact shapes
- Key question: how to include this qualitative information

- No mathematical form fits all or even most age profiles
- Out-of-sample age profiles often unrealistic
- The key empirical patterns are qualitative:
  - Adjacent age groups have similar mortality rates
  - Age profiles are more variable for younger ages
  - We don't know much about levels or exact shapes
- Key question: how to include this qualitative information
- Also: Method ignores covariate information; the leading current method (McNown-Rogers) not replicable

## **Deterministic Projections**

Demographic Forecasting

E

Image: A Image: A

## **Deterministic Projections**

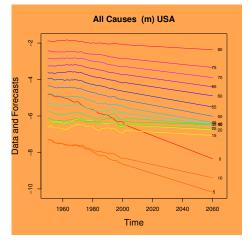


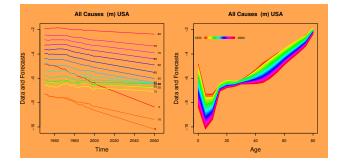
표 문 표

## **Deterministic Projections**

All Causes (m) USA All Causes (m) USA Ņ Ņ 4 4 Data and Forecasts Data and Forecasts φ φ 15 ထု ထု 우 6-10 1960 1980 2020 2040 2060 20 40 60 80 2000 0 Time Age

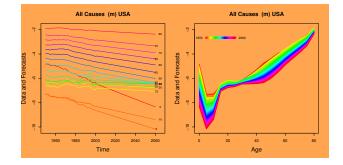
3

()

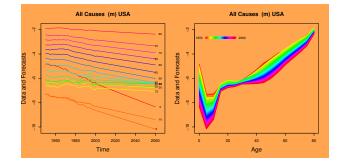


Ξ

- 4 ⊒ →

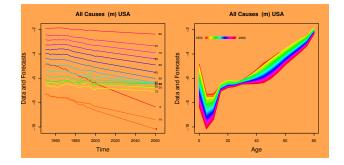


• Random walk with drift; Lee-Carter; least squares on linear trend

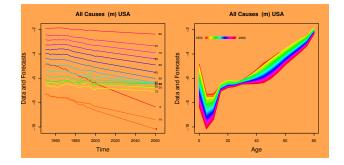


Random walk with drift; Lee-Carter; least squares on linear trend

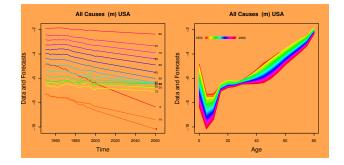
• Pros: simple, fast, works well in appropriate data



- Random walk with drift; Lee-Carter; least squares on linear trend
- Pros: simple, fast, works well in appropriate data
- Cons: omits covariates

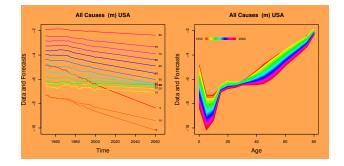


- Random walk with drift; Lee-Carter; least squares on linear trend
- Pros: simple, fast, works well in appropriate data
- Cons: omits covariates; forecasts fan out



- Random walk with drift; Lee-Carter; least squares on linear trend
- Pros: simple, fast, works well in appropriate data
- Cons: omits covariates; forecasts fan out; age profile becomes less smooth

### Existing Method 2: Deterministic Projections



- Random walk with drift; Lee-Carter; least squares on linear trend
- Pros: simple, fast, works well in appropriate data
- Cons: omits covariates; forecasts fan out; age profile becomes less smooth
- Does it fit elsewhere?

### The same pattern?

Demographic Forecasting

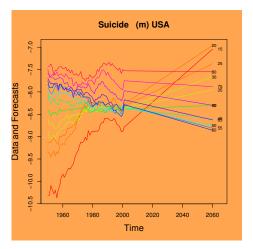
E

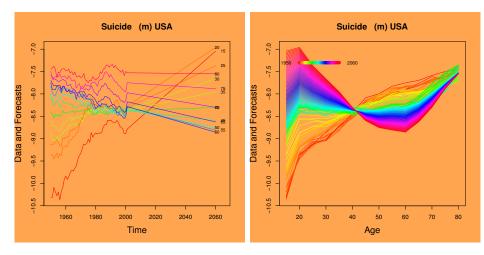
<ロト <回ト < 回ト < 回ト

# The same pattern? Random Walk with Drift $\approx$ Lee-Carter $\approx$ Least Squares

э

イロト イポト イヨト イヨト





### The same pattern?

Demographic Forecasting

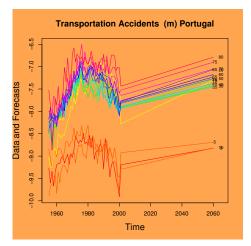
E

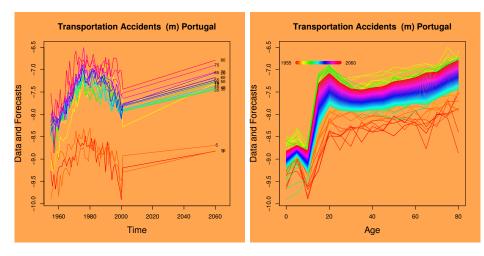
<ロト <回ト < 回ト < 回ト

# The same pattern? Random Walk with Drift $\approx$ Lee-Carter $\approx$ Least Squares

3

イロト イポト イヨト イヨト





19 / 76

### Deterministic Projections Do Not Work

3

∃ ► < ∃ ►</p>

### Deterministic Projections Do Not Work

• Linearity does not fit most time series data



- Linearity does not fit most time series data
- Out-of-sample age profiles become unrealistic over time

Demographic Forecasting

Э

- 4 伺 ト 4 ヨ ト 4 ヨ ト

Our Goal: Use all available information

3

イロト イポト イヨト イヨト

Our Goal: Use all available information

• Quantitative data

э

• • = • • = •

Our Goal: Use all available information

- Quantitative data
  - Use all mortality data

ヨト イヨト

Our Goal: Use all available information

#### • Quantitative data

- Use all mortality data
- Allows covariates (smoking causes lung cancer!)

ヨト イヨト

Our Goal: Use all available information

#### • Quantitative data

- Use all mortality data
- Allows covariates (smoking causes lung cancer!)
- Allow different covariates in each regression (smoking doesn't help forecast infant mortality!)

Our Goal: Use all available information

#### • Quantitative data

- Use all mortality data
- Allows covariates (smoking causes lung cancer!)
- Allow different covariates in each regression (smoking doesn't help forecast infant mortality!)
- Qualitative information

Our Goal: Use all available information

#### • Quantitative data

- Use all mortality data
- Allows covariates (smoking causes lung cancer!)
- Allow different covariates in each regression (smoking doesn't help forecast infant mortality!)
- Qualitative information
  - Mortality age profiles are smooth

Our Goal: Use all available information

#### • Quantitative data

- Use all mortality data
- Allows covariates (smoking causes lung cancer!)
- Allow different covariates in each regression (smoking doesn't help forecast infant mortality!)

- Mortality age profiles are smooth
- Younger age groups are more variable

Our Goal: Use all available information

#### • Quantitative data

- Use all mortality data
- Allows covariates (smoking causes lung cancer!)
- Allow different covariates in each regression (smoking doesn't help forecast infant mortality!)

- Mortality age profiles are smooth
- Younger age groups are more variable
- Mortality trends smoothly over time

Our Goal: Use all available information

#### • Quantitative data

- Use all mortality data
- Allows covariates (smoking causes lung cancer!)
- Allow different covariates in each regression (smoking doesn't help forecast infant mortality!)

- Mortality age profiles are smooth
- Younger age groups are more variable
- Mortality trends smoothly over time
- Neighboring age groups have similar mortality trends

Our Goal: Use all available information

#### • Quantitative data

- Use all mortality data
- Allows covariates (smoking causes lung cancer!)
- Allow different covariates in each regression (smoking doesn't help forecast infant mortality!)

- Mortality age profiles are smooth
- Younger age groups are more variable
- Mortality trends smoothly over time
- Neighboring age groups have similar mortality trends
- Neighboring countries have similar trends in mortality

Our Goal: Use all available information

#### • Quantitative data

- Use all mortality data
- Allows covariates (smoking causes lung cancer!)
- Allow different covariates in each regression (smoking doesn't help forecast infant mortality!)

- Mortality age profiles are smooth
- Younger age groups are more variable
- Mortality trends smoothly over time
- Neighboring age groups have similar mortality trends
- Neighboring countries have similar trends in mortality
- Statistical Modeling

Our Goal: Use all available information

#### • Quantitative data

- Use all mortality data
- Allows covariates (smoking causes lung cancer!)
- Allow different covariates in each regression (smoking doesn't help forecast infant mortality!)

- Mortality age profiles are smooth
- Younger age groups are more variable
- Mortality trends smoothly over time
- Neighboring age groups have similar mortality trends
- Neighboring countries have similar trends in mortality
- Statistical Modeling
  - Priors on expected mortality rather than coefficients

Our Goal: Use all available information

#### • Quantitative data

- Use all mortality data
- Allows covariates (smoking causes lung cancer!)
- Allow different covariates in each regression (smoking doesn't help forecast infant mortality!)

- Mortality age profiles are smooth
- Younger age groups are more variable
- Mortality trends smoothly over time
- Neighboring age groups have similar mortality trends
- Neighboring countries have similar trends in mortality
- Statistical Modeling
  - Priors on expected mortality rather than coefficients
  - Only choose parameter values we know something about

Our Goal: Use all available information

#### • Quantitative data

- Use all mortality data
- Allows covariates (smoking causes lung cancer!)
- Allow different covariates in each regression (smoking doesn't help forecast infant mortality!)

### Qualitative information

- Mortality age profiles are smooth
- Younger age groups are more variable
- Mortality trends smoothly over time
- Neighboring age groups have similar mortality trends
- Neighboring countries have similar trends in mortality

### • Statistical Modeling

- Priors on expected mortality rather than coefficients
- Only choose parameter values we know something about
- Allow ignorance about specific patterns

Our Goal: Use all available information

### • Quantitative data

- Use all mortality data
- Allows covariates (smoking causes lung cancer!)
- Allow different covariates in each regression (smoking doesn't help forecast infant mortality!)

### Qualitative information

- Mortality age profiles are smooth
- Younger age groups are more variable
- Mortality trends smoothly over time
- Neighboring age groups have similar mortality trends
- Neighboring countries have similar trends in mortality

### • Statistical Modeling

- Priors on expected mortality rather than coefficients
- Only choose parameter values we know something about
- Allow ignorance about specific patterns
- Allow variables to change meaning in different countries (such as GDP) or time periods (ICD changes)

### How to Forecast Two Short Time Series?

Demographic Forecasting

Ξ

(4) E (4) (4) E (4)

э

∃ ► < ∃ ►</p>

| U.S.:   | $y_t = X_{t-1}\beta + \epsilon_t$ | $(t = 1950, \dots, 2005)$ |
|---------|-----------------------------------|---------------------------|
| Mexico: | $y_t = X_{t-1}\beta + \epsilon_t$ | $(t = 1950, \dots, 2005)$ |

э

∃ ► < ∃ ►</p>

| U.S.:   | $y_t = X_{t-1}\beta + \epsilon_t$ | $(t = 1950, \dots, 2005)$ |
|---------|-----------------------------------|---------------------------|
| Mexico: | $y_t = X_{t-1}\beta + \epsilon_t$ | $(t = 1950, \dots, 2005)$ |

Options:

3

イロト イポト イヨト イヨト

U.S.: 
$$y_t = X_{t-1}\beta + \epsilon_t$$
  $(t = 1950, ..., 2005)$   
Mexico:  $y_t = X_{t-1}\beta + \epsilon_t$   $(t = 1950, ..., 2005)$ 

### Options:

• Estimate regressions separately:

U.S.: 
$$y_t = X_{t-1}\beta + \epsilon_t$$
  $(t = 1950, ..., 2005)$   
Mexico:  $y_t = X_{t-1}\beta + \epsilon_t$   $(t = 1950, ..., 2005)$ 

### Options:

- Estimate regressions separately:
  - too few observations

U.S.: 
$$y_t = X_{t-1}\beta + \epsilon_t$$
  $(t = 1950, ..., 2005)$   
Mexico:  $y_t = X_{t-1}\beta + \epsilon_t$   $(t = 1950, ..., 2005)$ 

### Options:

- Estimate regressions separately:
  - too few observations
  - confidence intervals too wide

U.S.: 
$$y_t = X_{t-1}\beta + \epsilon_t$$
  $(t = 1950, ..., 2005)$   
Mexico:  $y_t = X_{t-1}\beta + \epsilon_t$   $(t = 1950, ..., 2005)$ 

- Estimate regressions separately:
  - too few observations
  - confidence intervals too wide
- Pooling (Murray and Lopez, 1996):

U.S.: 
$$y_t = X_{t-1}\beta + \epsilon_t$$
  $(t = 1950, ..., 2005)$   
Mexico:  $y_t = X_{t-1}\beta + \epsilon_t$   $(t = 1950, ..., 2005)$ 

- Estimate regressions separately:
  - too few observations
  - confidence intervals too wide
- Pooling (Murray and Lopez, 1996):
  - Pool over countries (political scientists mortified)

U.S.: 
$$y_t = X_{t-1}\beta + \epsilon_t$$
  $(t = 1950, ..., 2005)$   
Mexico:  $y_t = X_{t-1}\beta + \epsilon_t$   $(t = 1950, ..., 2005)$ 

- Estimate regressions separately:
  - too few observations
  - confidence intervals too wide
- Pooling (Murray and Lopez, 1996):
  - Pool over countries (political scientists mortified)
  - Pool over age groups (public health scholars mortified)

U.S.: 
$$y_t = X_{t-1}\beta + \epsilon_t$$
  $(t = 1950, ..., 2005)$   
Mexico:  $y_t = X_{t-1}\beta + \epsilon_t$   $(t = 1950, ..., 2005)$ 

- Estimate regressions separately:
  - too few observations
  - confidence intervals too wide
- Pooling (Murray and Lopez, 1996):
  - Pool over countries (political scientists mortified)
  - Pool over age groups (public health scholars mortified)
  - Enormous biases either way

U.S.: 
$$y_t = X_{t-1}\beta + \epsilon_t$$
  $(t = 1950, ..., 2005)$   
Mexico:  $y_t = X_{t-1}\beta + \epsilon_t$   $(t = 1950, ..., 2005)$ 

- Estimate regressions separately:
  - too few observations
  - confidence intervals too wide
- Pooling (Murray and Lopez, 1996):
  - Pool over countries (political scientists mortified)
  - Pool over age groups (public health scholars mortified)
  - Enormous biases either way
  - Requires covariates with the same meaning in all cross-sections

U.S.: 
$$y_t = X_{t-1}\beta + \epsilon_t$$
  $(t = 1950, ..., 2005)$   
Mexico:  $y_t = X_{t-1}\beta + \epsilon_t$   $(t = 1950, ..., 2005)$ 

- Estimate regressions separately:
  - too few observations
  - confidence intervals too wide
- Pooling (Murray and Lopez, 1996):
  - Pool over countries (political scientists mortified)
  - Pool over age groups (public health scholars mortified)
  - Enormous biases either way
  - Requires covariates with the same meaning in all cross-sections
- Qualitative knowledge: patterns are similar, not identical.

Demographic Forecasting

E

・ロト ・四ト ・ヨト ・ヨト

Just three easy steps:

E

イロト イヨト イヨト

#### How to do it?

#### Just three easy steps:

$$\begin{split} \mathsf{P}(y_{i}|\eta_{i}) &= \left\{ \prod_{s=1}^{S} \prod_{k=1}^{K_{s}} \left[ F(\tau_{is}^{k}|\mu_{i},1) - F(\tau_{is}^{k-1}|\mu_{i},1) \right]^{\mathsf{I}(y_{is}=k)} \right\} \frac{\sqrt{\mathfrak{B}}P_{10}P_{11}}{\sqrt{\mathfrak{B}}P_{10} + P_{11}}, \\ L_{s}(\beta,\omega^{2},\gamma|y) &\propto \prod_{i=1}^{n} \int_{-\infty}^{\infty} \prod_{s=1}^{S} \prod_{k=1}^{K_{s}} \left[ F(\tau_{is}^{k}|X_{i}\beta + \eta_{i},1) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi \right] \\ \mathsf{RD}_{\gamma} &= \sqrt{\mathfrak{B}}/(1 + \sqrt{\mathfrak{B}}) - 1/(1 + \sqrt{\mathfrak{B}}) - F(\tau_{is}^{k-1}|X_{i}\beta + \eta_{i},1) \right]^{\mathsf{I}(y_{is}=k)} N(\eta_{i}|0, \\ \Theta_{ab} &= \mathsf{Pr}(X_{a}|Y = b), \,\mathfrak{B} = (\Theta_{11}\Theta_{00})/(\Theta_{01}\Theta_{10}). \,\phi = (\mathfrak{B}\zeta 01^{2}/\zeta 11^{2})^{1/2} \\ &= \sqrt{\mathfrak{B}}\zeta 01/\zeta 11, \,\mathrm{and} \,\,\gamma = \sqrt{\mathfrak{B}}/(\sqrt{\mathfrak{B}} + \eta_{11}/\eta_{10}). \,\, \mathsf{Then}, \,\, \mathsf{RD}_{\gamma} \\ \eta_{11}\gamma &= \frac{\sqrt{\mathfrak{B}}\eta_{10}\Lambda_{11}}{\sqrt{\mathfrak{B}}\Lambda_{10} + \Lambda_{11}}, \qquad \Lambda_{01}\gamma &= \frac{\sqrt{\mathfrak{B}}\Lambda_{01}\Gamma_{10}}{\sqrt{\mathfrak{B}}\Gamma_{10} + \Gamma_{11}}, \,\zeta\Gamma GK \boxtimes \Phi\phi \\ \Gamma_{10}(1 - \gamma) &= \frac{\Gamma_{10}\Gamma_{11}}{\sqrt{\mathfrak{B}}\Gamma_{10} + P_{11}}, \qquad P_{00}(1 - \gamma) &= \frac{P_{11}P_{00}}{\sqrt{\mathfrak{B}}P_{10} + P_{11}}. \\ \mathsf{rd} \in [\mathsf{min}[\mathsf{rd}(\underline{\tau}_{j}), \mathsf{rd}(\overline{\tau}_{j})], \,\, \mathsf{max}[\mathsf{rd}(\underline{\tau}_{j}), \mathsf{rd}(\overline{\tau}_{j})]] \xrightarrow{\mathbb{R}} \to \mathbb{R} \to$$

Demographic Forecasting

E

イロト イヨト イヨト イヨト

• Standard Bayesian technology smooths coefficients, requires considerable prior information

э

ヨト イヨト

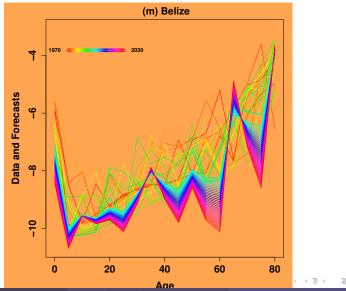
- Standard Bayesian technology smooths coefficients, requires considerable prior information
- We translate assumptions about mortality into assumptions about coefficients (E(y) = Xβ) so standard Bayesian machinery can be used

- Standard Bayesian technology smooths coefficients, requires considerable prior information
- We translate assumptions about mortality into assumptions about coefficients  $(E(y) = X\beta)$  so standard Bayesian machinery can be used
- No extraneous assumptions; few adjustable parameters

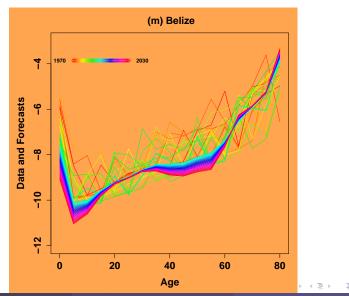
- Standard Bayesian technology smooths coefficients, requires considerable prior information
- We translate assumptions about mortality into assumptions about coefficients (E(y) = Xβ) so standard Bayesian machinery can be used
- No extraneous assumptions; few adjustable parameters
- Remaining parameters chosen based on real qualitative information

- Standard Bayesian technology smooths coefficients, requires considerable prior information
- We translate assumptions about mortality into assumptions about coefficients  $(E(y) = X\beta)$  so standard Bayesian machinery can be used
- No extraneous assumptions; few adjustable parameters
- Remaining parameters chosen based on real qualitative information
- Added a wide array of ways to combine cross-sections

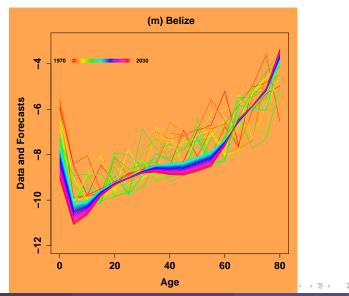
# Mortality from Respiratory Infections, Males



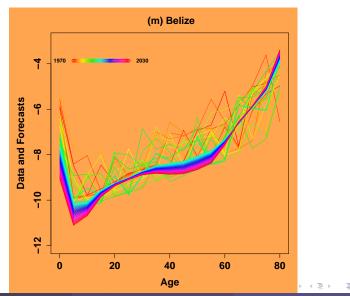
Demographic Forecasting



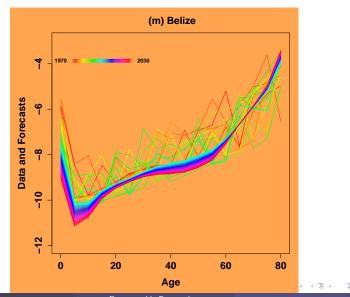
Demographic Forecasting



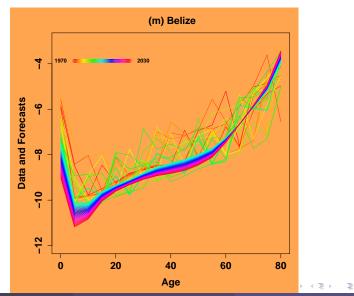
Demographic Forecasting



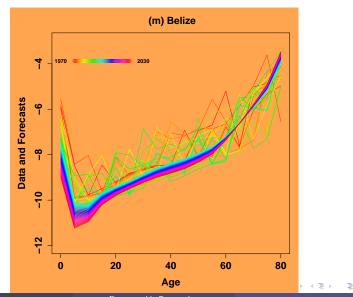
Demographic Forecasting



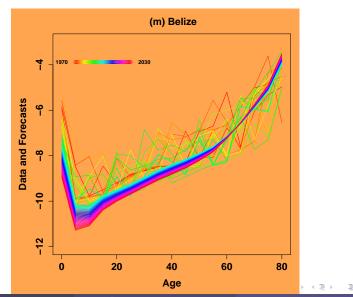
Demographic Forecasting



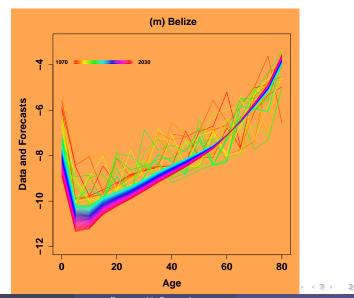
Demographic Forecasting



Demographic Forecasting

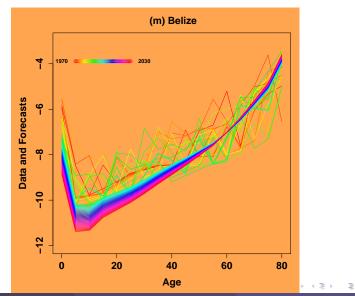


Demographic Forecasting

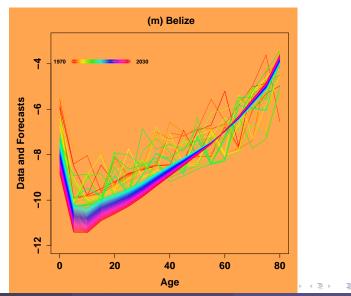


Demographic Forecasting

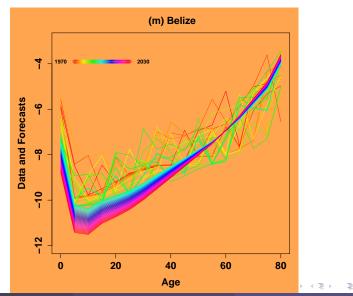
#### Mortality from Respiratory Infections, males, $\sigma = 0.21$ Smoothing over Age Groups



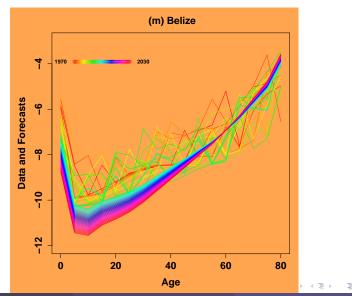
Demographic Forecasting



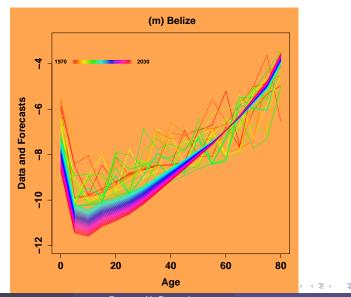
Demographic Forecasting



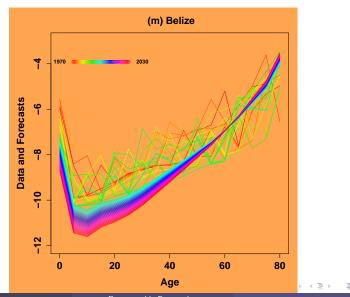
Demographic Forecasting



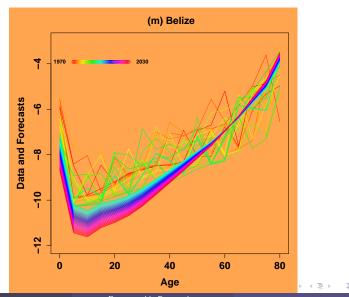
Demographic Forecasting



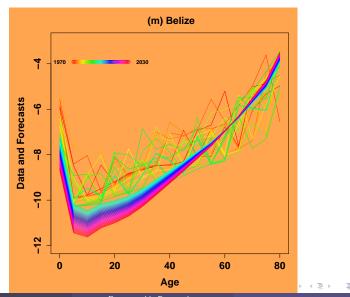
Demographic Forecasting



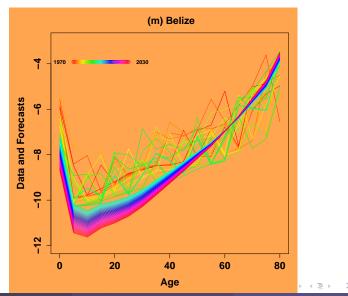
Demographic Forecasting



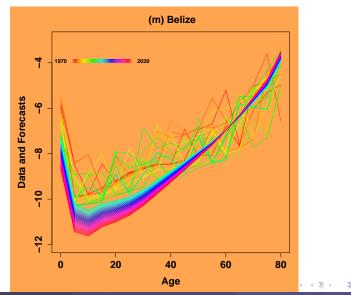
Demographic Forecasting



Demographic Forecasting

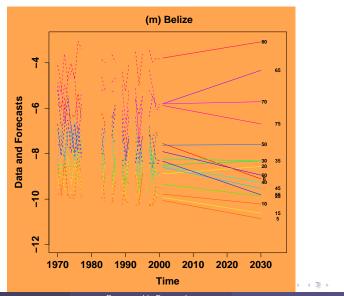


Demographic Forecasting



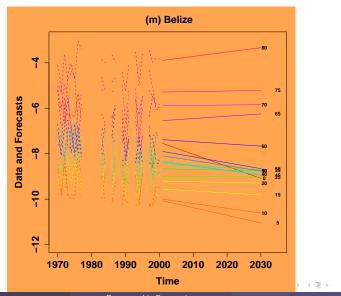
Demographic Forecasting

#### Mortality from Respiratory Infections, males Least Squares



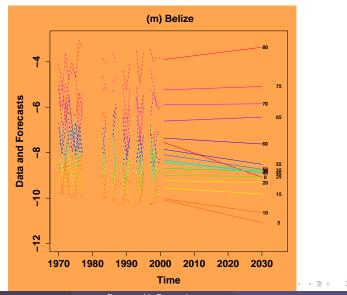
Demographic Forecasting

#### Mortality from Respiratory Infections, males, $\sigma = 2.00$ Smoothing over Age Groups



Demographic Forecasting

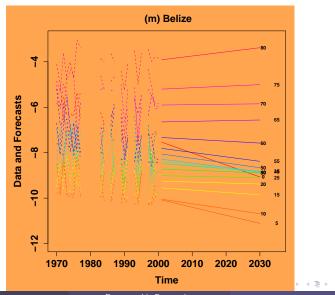
# Mortality from Respiratory Infections, males, $\sigma = 1.51$



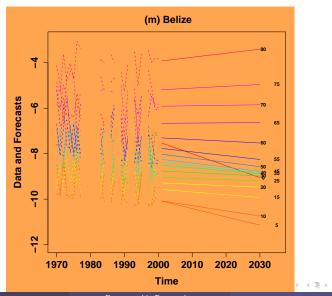
Demographic Forecasting

()

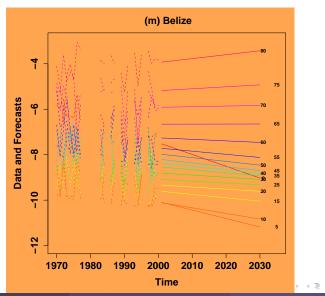
# Mortality from Respiratory Infections, males, $\sigma=1.15$



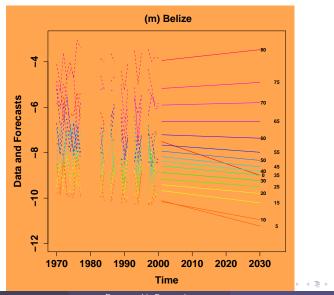
#### Mortality from Respiratory Infections, males, $\sigma = 0.87$ Smoothing over Age Groups



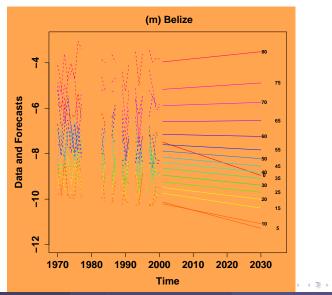
# Mortality from Respiratory Infections, males, $\sigma = 0.66$



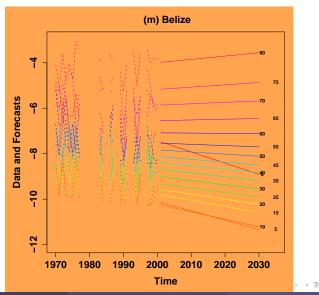
#### Mortality from Respiratory Infections, males, $\sigma = 0.50$ Smoothing over Age Groups



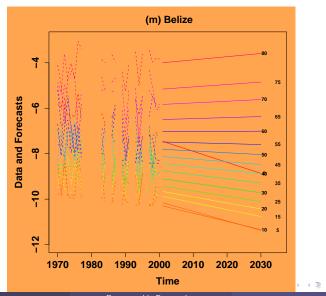
# Mortality from Respiratory Infections, males, $\sigma=0.38$



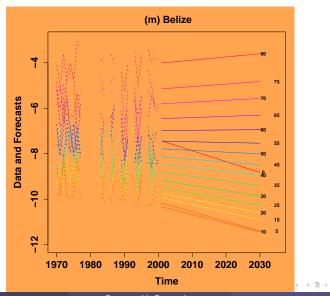
# Mortality from Respiratory Infections, males, $\sigma = 0.28$



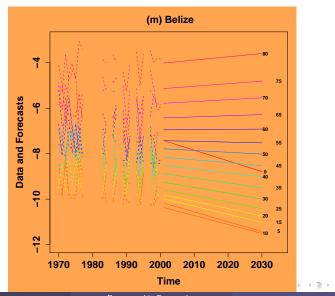
#### Mortality from Respiratory Infections, males, $\sigma = 0.21$ Smoothing over Age Groups



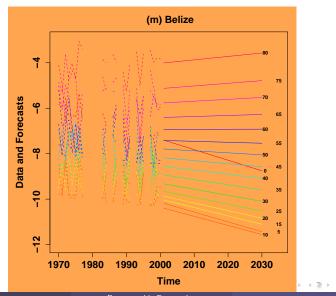
# Mortality from Respiratory Infections, males, $\sigma=0.16$



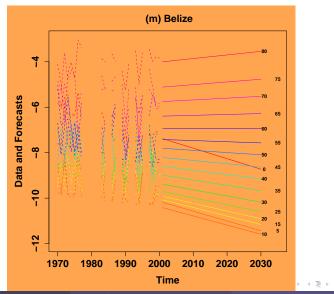
# Mortality from Respiratory Infections, males, $\sigma=0.12$



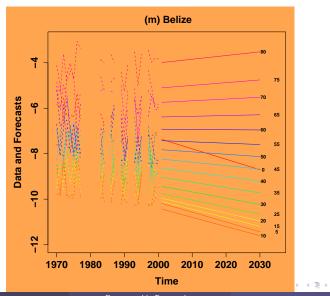
# Mortality from Respiratory Infections, males, $\sigma = 0.09$



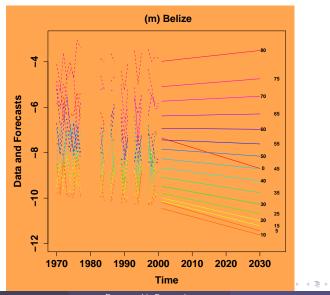
#### Mortality from Respiratory Infections, males, $\sigma = 0.07$ Smoothing over Age Groups



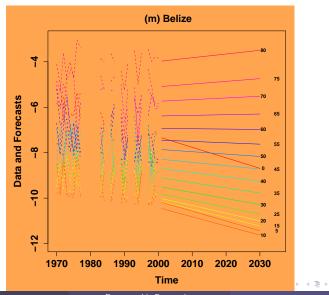
# Mortality from Respiratory Infections, males, $\sigma = 0.05$



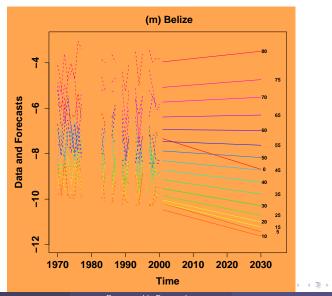
# Mortality from Respiratory Infections, males, $\sigma = 0.04$



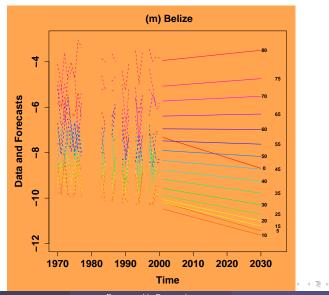
#### Mortality from Respiratory Infections, males, $\sigma = 0.03$ Smoothing over Age Groups



# Mortality from Respiratory Infections, males, $\sigma=0.02$



#### Mortality from Respiratory Infections, males, $\sigma = 0.01$ Smoothing over Age Groups



Demographic Forecasting

э

Log-mortality in Belize males from respiratory infections

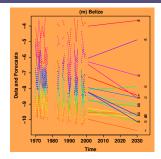
э

Log-mortality in Belize males from respiratory infections

Least Squares



Log-mortality in Belize males from respiratory infections



#### Least Squares

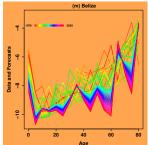
Log-mortality in Belize males from respiratory infections

4 ŝ Data and Forecasts 8

> 1970 1980 1990 2000 2010 2020 2030

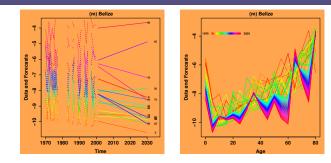
(m) Belize

Time



Least Squares

Log-mortality in Belize males from respiratory infections



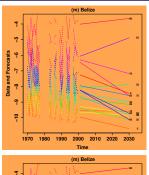
Least Squares

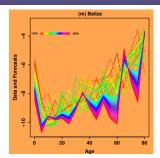
Smoothing Age Groups

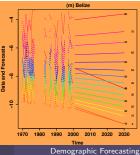
Log-mortality in Belize males from respiratory infections

Least Squares

Smoothing Age Groups





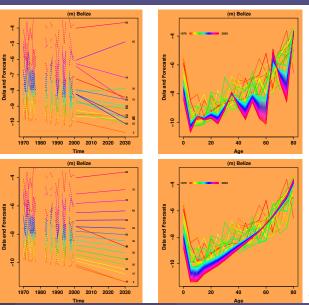


()

Log-mortality in Belize males from respiratory infections

Least Squares

Smoothing Age Groups



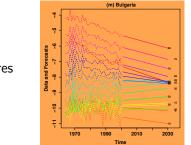
Log-Mortality in Bulgarian males from respiratory infections

Log-Mortality in Bulgarian males from respiratory infections

Least Squares

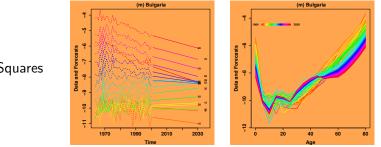
Demographic Forecasting

Log-Mortality in Bulgarian males from respiratory infections



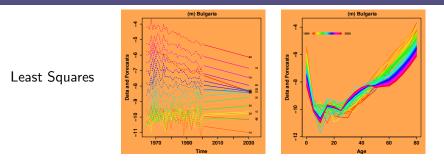
Least Squares

#### Smoothing Trends over Age Groups and Time Log-Mortality in Bulgarian males from respiratory infections



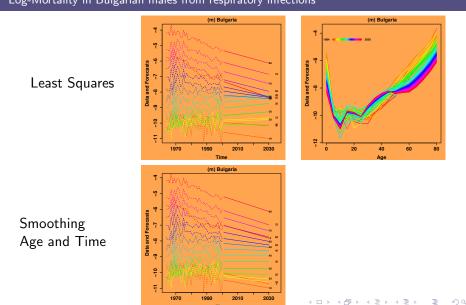
Least Squares

# Smoothing Trends over Age Groups and Time Log-Mortality in Bulgarian males from respiratory infections

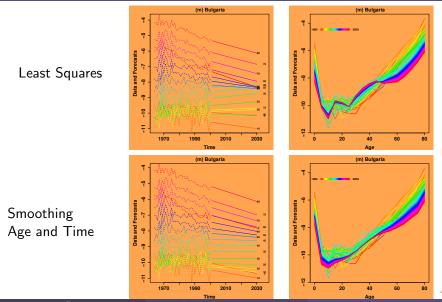


Smoothing Age and Time

# Smoothing Trends over Age Groups and Time Log-Mortality in Bulgarian males from respiratory infections



# Smoothing Trends over Age Groups and Time Log-Mortality in Bulgarian males from respiratory infections



### Using Covariates (GDP, tobacco, trend, log trend)

Demographic Forecasting

(4 回 ト 4 ヨ ト 4 ヨ ト

#### Using Covariates (GDP, tobacco, trend, log trend) Lung cancer in Korean Males

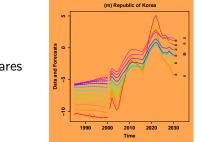
3

#### Using Covariates (GDP, tobacco, trend, log trend) Lung cancer in Korean Males

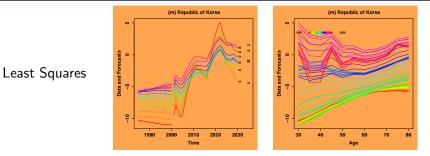
Least Squares

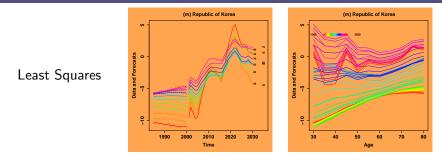


∃ ► < ∃ ►</p>

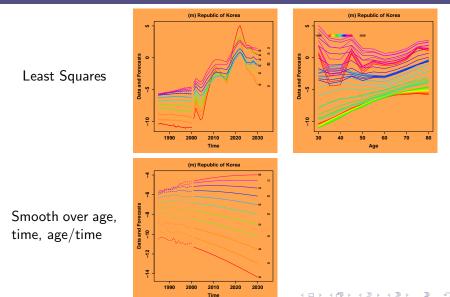


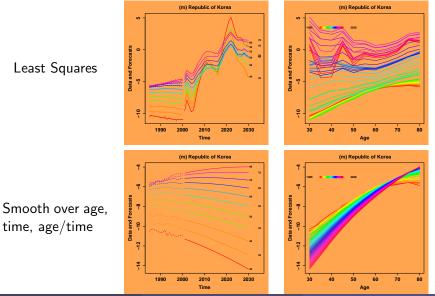
Least Squares





Smooth over age, time, age/time





65 / 76

### Using Covariates (GDP, tobacco, trend, log trend)

Demographic Forecasting

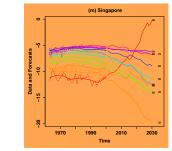
(4 回 ト 4 ヨ ト 4 ヨ ト

イロト イポト イヨト イヨト

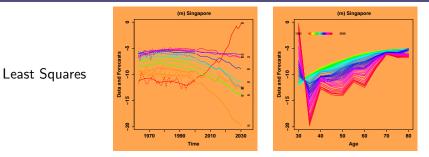
Least Squares



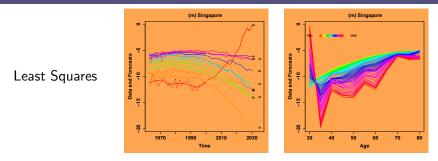
E + 4 E +



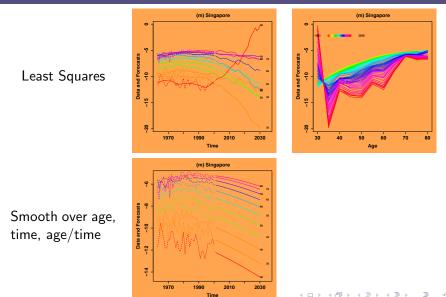
Least Squares

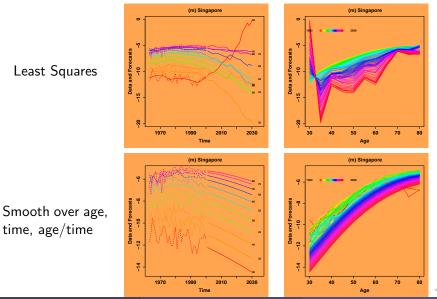


Demographic Forecasting



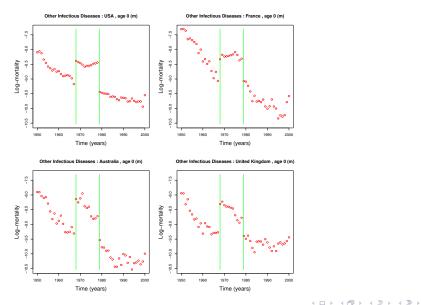
Smooth over age, time, age/time





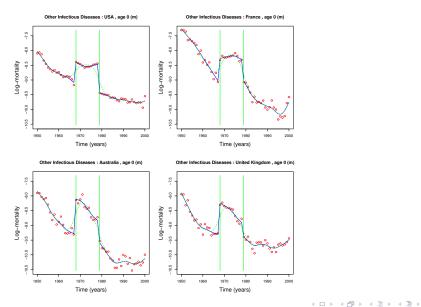
66 / 76

### What about ICD Changes?



Ξ

# Fixing ICD Changes



E

### Formalizing (Prior) Indifference

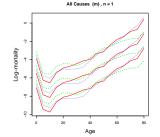
equal color = equal probability

3

イロト イポト イヨト イヨト

# Formalizing (Prior) Indifference

equal color = equal probability



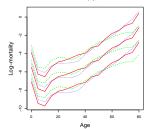
Level indifference

Ξ

E + 4 E +

# Formalizing (Prior) Indifference

equal color = equal probability

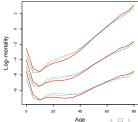


All Causes (m), n = 1

All Causes (m), n = 2

Level and slope indifference

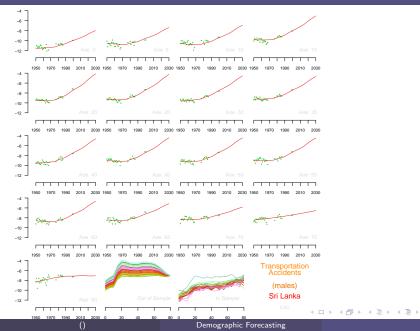
Level indifference



### A book manuscript, YourCast software, etc.

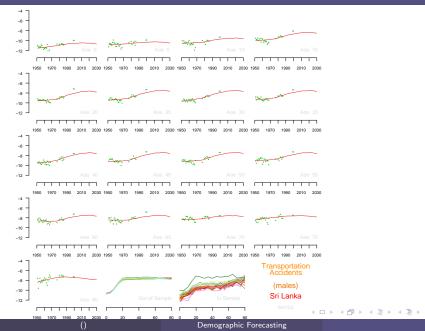
# http://GKing.Harvard.edu

### Without Country Smoothing



E

## With Country Smoothing



E

Demographic Forecasting

E

<ロト <回ト < 回ト < 回ト

Standard Bayesian Approach

3

-

. < □ > < □ > <</p>

#### Standard Bayesian Approach

• Assume coefficients are similar

- Assume coefficients are similar
  - But we know little about the coefficients

- Assume coefficients are similar
  - But we know little about the coefficients
- Requires the same covariates in each cross-section

- Assume coefficients are similar
  - But we know little about the coefficients
- Requires the same covariates in each cross-section
  - Why measure water quality in the U.S.?

- Assume coefficients are similar
  - But we know little about the coefficients
- Requires the same covariates in each cross-section
  - Why measure water quality in the U.S.?
- Requires covariates with the same meaning in each cross-section

- Assume coefficients are similar
  - But we know little about the coefficients
- Requires the same covariates in each cross-section
  - Why measure water quality in the U.S.?
- Requires covariates with the same meaning in each cross-section
  - Does GDP mean the same thing in Botswana and the U.S.?

- Assume coefficients are similar
  - But we know little about the coefficients
- Requires the same covariates in each cross-section
  - Why measure water quality in the U.S.?
- Requires covariates with the same meaning in each cross-section
   Does GDP mean the same thing in Botswana and the U.S.?
- Imposes no assumptions on covariates or mortality

#### Standard Bayesian Approach

- Assume coefficients are similar
  - But we know little about the coefficients
- Requires the same covariates in each cross-section
  - Why measure water quality in the U.S.?
- Requires covariates with the same meaning in each cross-section
   Does GDP mean the same thing in Botswana and the U.S.?

Imposes no assumptions on covariates or mortality

— If covariates are dissimilar, then making coefficients similar makes mortality dissimilar [since  $E(y_t) = X_t\beta$  in each cross-section]

#### Standard Bayesian Approach

- Assume coefficients are similar
  - But we know little about the coefficients
- Requires the same covariates in each cross-section
  - Why measure water quality in the U.S.?
- Requires covariates with the same meaning in each cross-section
   Does GDP mean the same thing in Botswana and the U.S.?
- Imposes no assumptions on covariates or mortality

— If covariates are dissimilar, then making coefficients similar makes mortality dissimilar [since  $E(y_t) = X_t\beta$  in each cross-section]

#### Standard Bayesian Approach

- Assume coefficients are similar
  - But we know little about the coefficients
- Requires the same covariates in each cross-section
  - Why measure water quality in the U.S.?
- Requires covariates with the same meaning in each cross-section
   Does GDP mean the same thing in Botswana and the U.S.?
- Imposes no assumptions on covariates or mortality

— If covariates are dissimilar, then making coefficients similar makes mortality dissimilar [since  $E(y_t) = X_t\beta$  in each cross-section]

#### Alternative Approach

• Assume expected mortality is similar

#### Standard Bayesian Approach

- Assume coefficients are similar
  - But we know little about the coefficients
- Requires the same covariates in each cross-section
  - Why measure water quality in the U.S.?
- Requires covariates with the same meaning in each cross-section
   Does GDP mean the same thing in Botswana and the U.S.?
- Imposes no assumptions on covariates or mortality

— If covariates are dissimilar, then making coefficients similar makes mortality dissimilar [since  $E(y_t) = X_t\beta$  in each cross-section]

- Assume expected mortality is similar
- Coefficients are unobserved, mortality patterns are well known

#### Standard Bayesian Approach

- Assume coefficients are similar
  - But we know little about the coefficients
- Requires the same covariates in each cross-section
  - Why measure water quality in the U.S.?
- Requires covariates with the same meaning in each cross-section
   Does GDP mean the same thing in Botswana and the U.S.?
- Imposes no assumptions on covariates or mortality

— If covariates are dissimilar, then making coefficients similar makes mortality dissimilar [since  $E(y_t) = X_t\beta$  in each cross-section]

- Assume expected mortality is similar
- Coefficients are unobserved, mortality patterns are well known
- Different covariates allowed in each cross-section

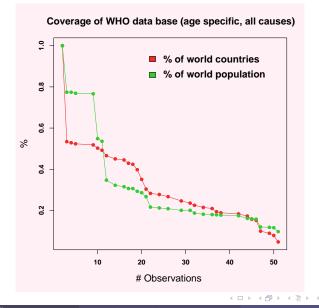
#### Standard Bayesian Approach

- Assume coefficients are similar
  - But we know little about the coefficients
- Requires the same covariates in each cross-section
  - Why measure water quality in the U.S.?
- Requires covariates with the same meaning in each cross-section
   Does GDP mean the same thing in Botswana and the U.S.?
- Imposes no assumptions on covariates or mortality

— If covariates are dissimilar, then making coefficients similar makes mortality dissimilar [since  $E(y_t) = X_t\beta$  in each cross-section]

- Assume expected mortality is similar
- Coefficients are unobserved, mortality patterns are well known
- Different covariates allowed in each cross-section
- Covariates with the same name can have different meanings

### Many Short Time Series



### Preview of Results: Out-of-Sample Evaluation

3

E + 4 E +

# Preview of Results: Out-of-Sample Evaluation

Mean Absolute Error in Males (over age and country)

∃ ► < ∃ ►</p>

|                        | Mean Absolute Error |        |             | % Improvement |             |
|------------------------|---------------------|--------|-------------|---------------|-------------|
|                        | Best                | Our    | Best        | Over Best     | to Best     |
|                        | Previous            | Method | Conceivable | Previous      | Conceivable |
| Cardiovascular         | 0.34                | 0.27   | 0.19        | 22            | 49          |
| Lung Cancer            | 0.36                | 0.27   | 0.17        | 24            | 47          |
| Transportation         | 0.37                | 0.31   | 0.18        | 16            | 31          |
| Respiratory Chronic    | 0.45                | 0.39   | 0.26        | 13            | 30          |
| Other Infectious       | 0.55                | 0.48   | 0.32        | 12            | 30          |
| Stomach Cancer         | 0.30                | 0.27   | 0.20        | 8             | 24          |
| All-Cause              | 0.17                | 0.15   | 0.08        | 12            | 22          |
| Suicide                | 0.31                | 0.29   | 0.18        | 7             | 17          |
| Respiratory Infectious | 0.49                | 0.47   | 0.28        | 3             | 7           |

э

イロト イポト イヨト イヨト

|                        | Mean Absolute Error |        |             | % Improvement |             |
|------------------------|---------------------|--------|-------------|---------------|-------------|
|                        | Best                | Our    | Best        | Over Best     | to Best     |
|                        | Previous            | Method | Conceivable | Previous      | Conceivable |
| Cardiovascular         | 0.34                | 0.27   | 0.19        | 22            | 49          |
| Lung Cancer            | 0.36                | 0.27   | 0.17        | 24            | 47          |
| Transportation         | 0.37                | 0.31   | 0.18        | 16            | 31          |
| Respiratory Chronic    | 0.45                | 0.39   | 0.26        | 13            | 30          |
| Other Infectious       | 0.55                | 0.48   | 0.32        | 12            | 30          |
| Stomach Cancer         | 0.30                | 0.27   | 0.20        | 8             | 24          |
| All-Cause              | 0.17                | 0.15   | 0.08        | 12            | 22          |
| Suicide                | 0.31                | 0.29   | 0.18        | 7             | 17          |
| Respiratory Infectious | 0.49                | 0.47   | 0.28        | 3             | 7           |

• Each row averages 6,800 forecast errors (17 age groups, 40 countries, and 10 out-of-sample years).

|                        | Mean Absolute Error |        |             | % Improvement |             |
|------------------------|---------------------|--------|-------------|---------------|-------------|
|                        | Best                | Our    | Best        | Over Best     | to Best     |
|                        | Previous            | Method | Conceivable | Previous      | Conceivable |
| Cardiovascular         | 0.34                | 0.27   | 0.19        | 22            | 49          |
| Lung Cancer            | 0.36                | 0.27   | 0.17        | 24            | 47          |
| Transportation         | 0.37                | 0.31   | 0.18        | 16            | 31          |
| Respiratory Chronic    | 0.45                | 0.39   | 0.26        | 13            | 30          |
| Other Infectious       | 0.55                | 0.48   | 0.32        | 12            | 30          |
| Stomach Cancer         | 0.30                | 0.27   | 0.20        | 8             | 24          |
| All-Cause              | 0.17                | 0.15   | 0.08        | 12            | 22          |
| Suicide                | 0.31                | 0.29   | 0.18        | 7             | 17          |
| Respiratory Infectious | 0.49                | 0.47   | 0.28        | 3             | 7           |

- Each row averages 6,800 forecast errors (17 age groups, 40 countries, and 10 out-of-sample years).
- % to best conceivable = % of the way our method takes us from the best existing to the best conceivable forecast.

|                        | Mean Absolute Error |        |             | % Improvement |             |
|------------------------|---------------------|--------|-------------|---------------|-------------|
|                        | Best                | Our    | Best        | Over Best     | to Best     |
|                        | Previous            | Method | Conceivable | Previous      | Conceivable |
| Cardiovascular         | 0.34                | 0.27   | 0.19        | 22            | 49          |
| Lung Cancer            | 0.36                | 0.27   | 0.17        | 24            | 47          |
| Transportation         | 0.37                | 0.31   | 0.18        | 16            | 31          |
| Respiratory Chronic    | 0.45                | 0.39   | 0.26        | 13            | 30          |
| Other Infectious       | 0.55                | 0.48   | 0.32        | 12            | 30          |
| Stomach Cancer         | 0.30                | 0.27   | 0.20        | 8             | 24          |
| All-Cause              | 0.17                | 0.15   | 0.08        | 12            | 22          |
| Suicide                | 0.31                | 0.29   | 0.18        | 7             | 17          |
| Respiratory Infectious | 0.49                | 0.47   | 0.28        | 3             | 7           |

- Each row averages 6,800 forecast errors (17 age groups, 40 countries, and 10 out-of-sample years).
- % to best conceivable = % of the way our method takes us from the best existing to the best conceivable forecast.
- The new method out-performs with the same covariates, for most countries, causes, sexes, and age groups.

|                        | Mean Absolute Error |        |             | % Improvement |             |
|------------------------|---------------------|--------|-------------|---------------|-------------|
|                        | Best                | Our    | Best        | Over Best     | to Best     |
|                        | Previous            | Method | Conceivable | Previous      | Conceivable |
| Cardiovascular         | 0.34                | 0.27   | 0.19        | 22            | 49          |
| Lung Cancer            | 0.36                | 0.27   | 0.17        | 24            | 47          |
| Transportation         | 0.37                | 0.31   | 0.18        | 16            | 31          |
| Respiratory Chronic    | 0.45                | 0.39   | 0.26        | 13            | 30          |
| Other Infectious       | 0.55                | 0.48   | 0.32        | 12            | 30          |
| Stomach Cancer         | 0.30                | 0.27   | 0.20        | 8             | 24          |
| All-Cause              | 0.17                | 0.15   | 0.08        | 12            | 22          |
| Suicide                | 0.31                | 0.29   | 0.18        | 7             | 17          |
| Respiratory Infectious | 0.49                | 0.47   | 0.28        | 3             | 7           |

- Each row averages 6,800 forecast errors (17 age groups, 40 countries, and 10 out-of-sample years).
- % to best conceivable = % of the way our method takes us from the best existing to the best conceivable forecast.
- The new method out-performs with the same covariates, for most countries, causes, sexes, and age groups.
- Does much better with better covariates

#### Demographic Forecasting

シックシード エル・ボット 中国・エロ・