

# Simplifying Matching Methods for Causal Inference

Gary King<sup>1</sup>

Institute for Quantitative Social Science  
Harvard University

Abt Associates, Cambridge MA, 6/1/2017

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<sup>1</sup>GaryKing.org

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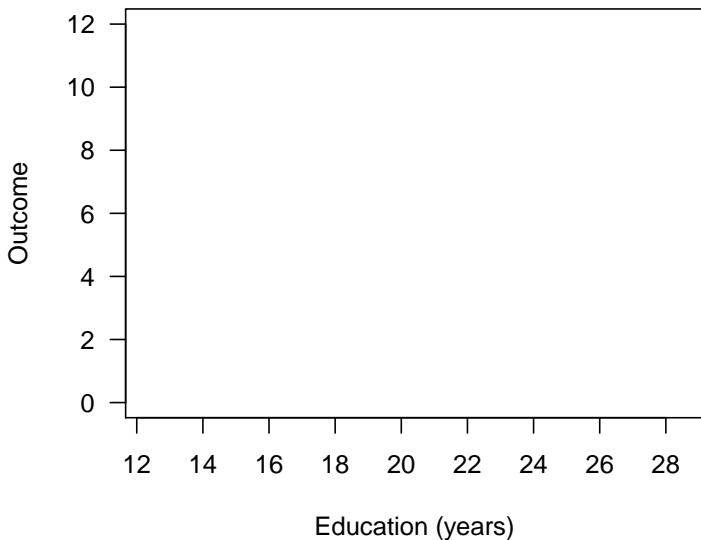
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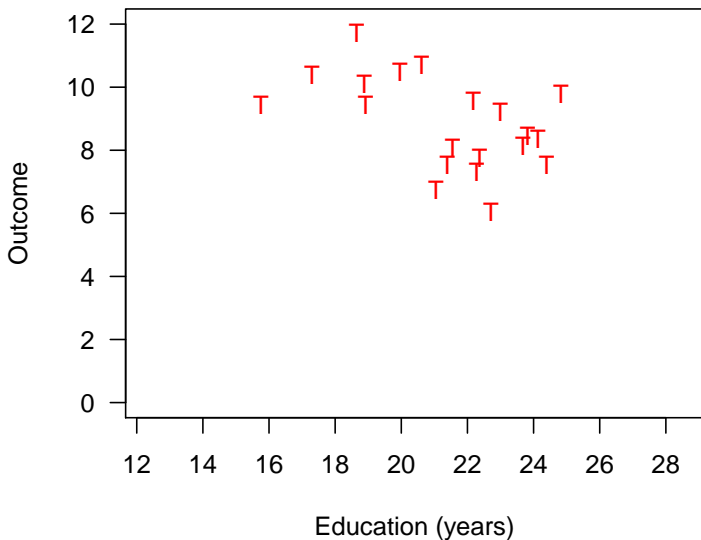
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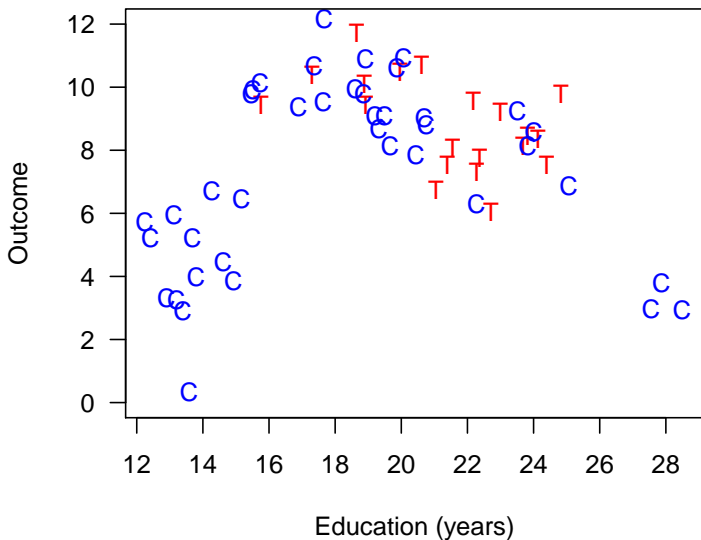
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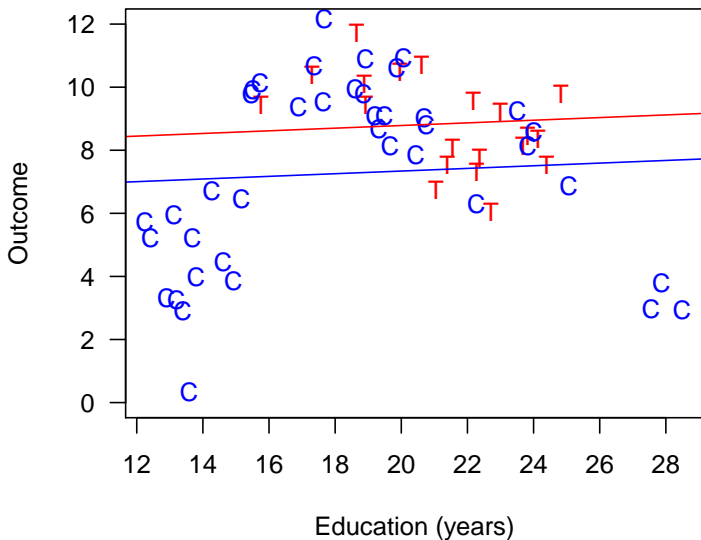
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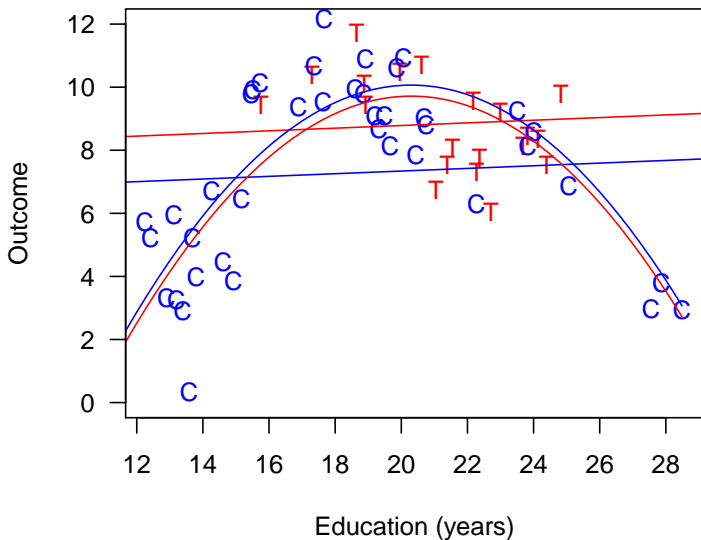
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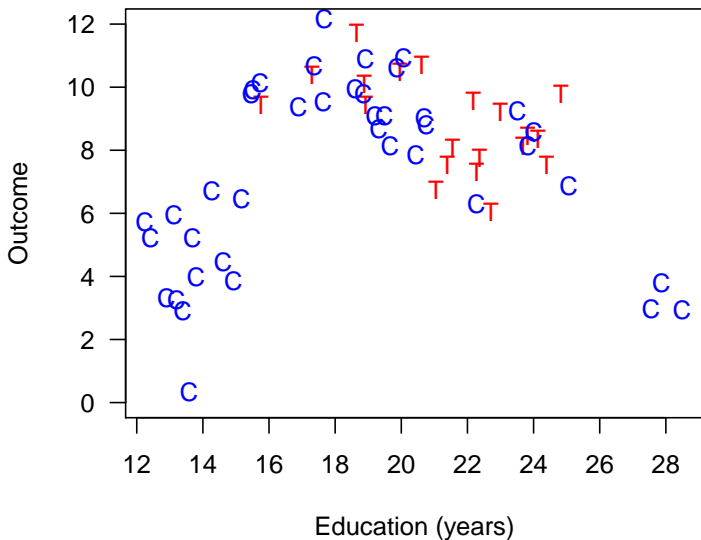
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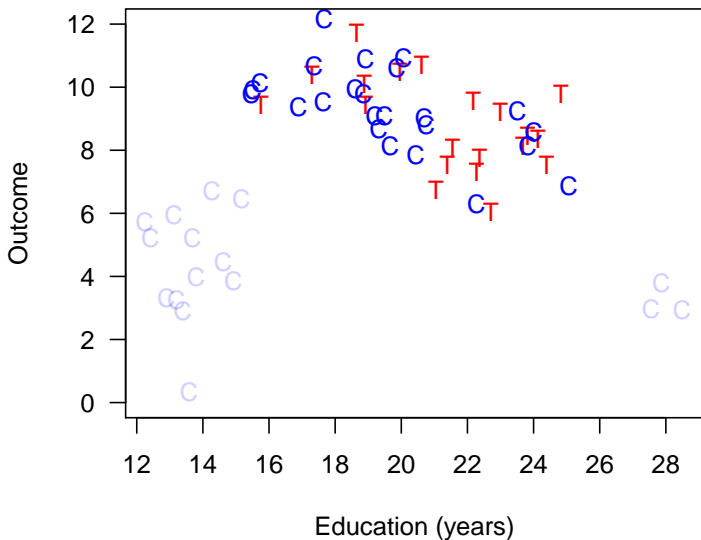
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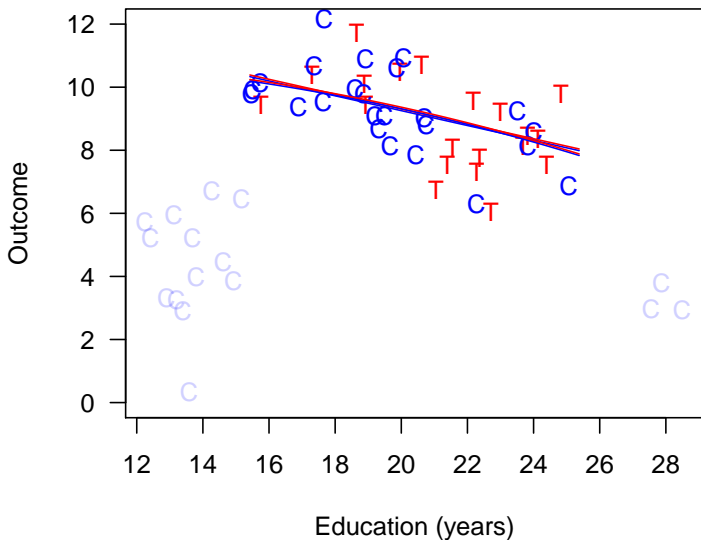
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- “Teaching psychology is mostly a waste of time” (Kahneman 2011)

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A central project of statistics: Automating away human discretion

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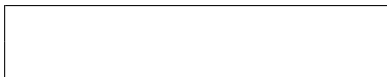
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  - **Pruning nonmatches makes control vars matter less:** reduces imbalance, model dependence, researcher discretion, & bias

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## 2. Estimation Difference in means or a model

# Method 1: Mahalanobis Distance Matching

(Approximates Fully Blocked Experiment)

## 1. Preprocess (Matching)

- $\text{Distance}(X_c, X_t) = \sqrt{(X_c - X_t)'S^{-1}(X_c - X_t)}$
- (Mahalanobis is for methodologists; in applications, use Euclidean!)
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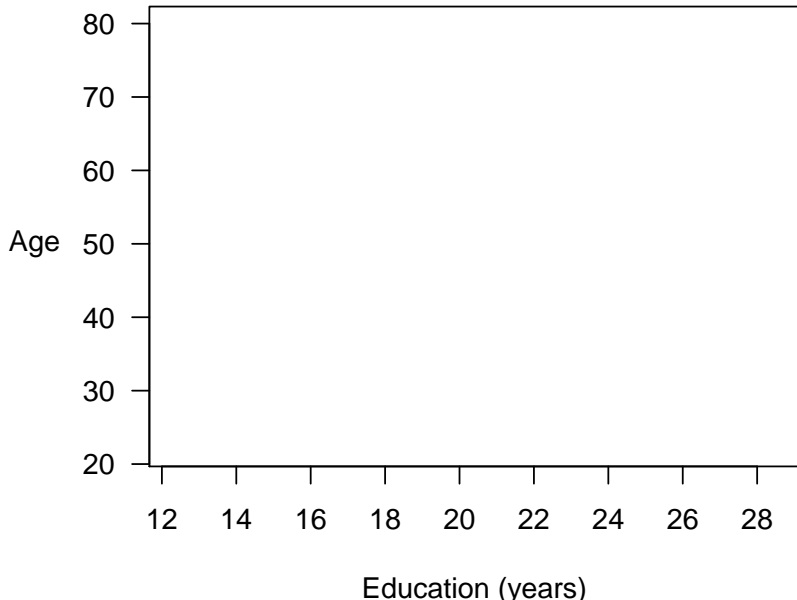
(Approximates Fully Blocked Experiment)

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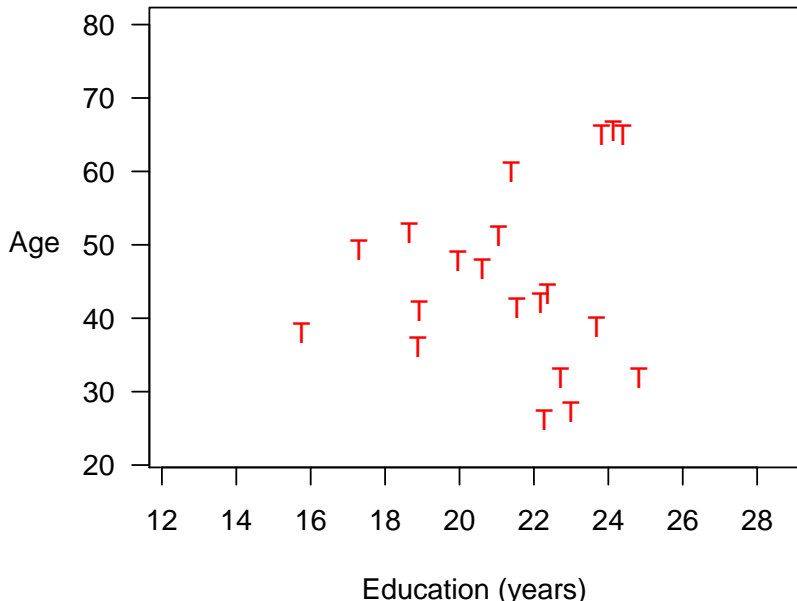
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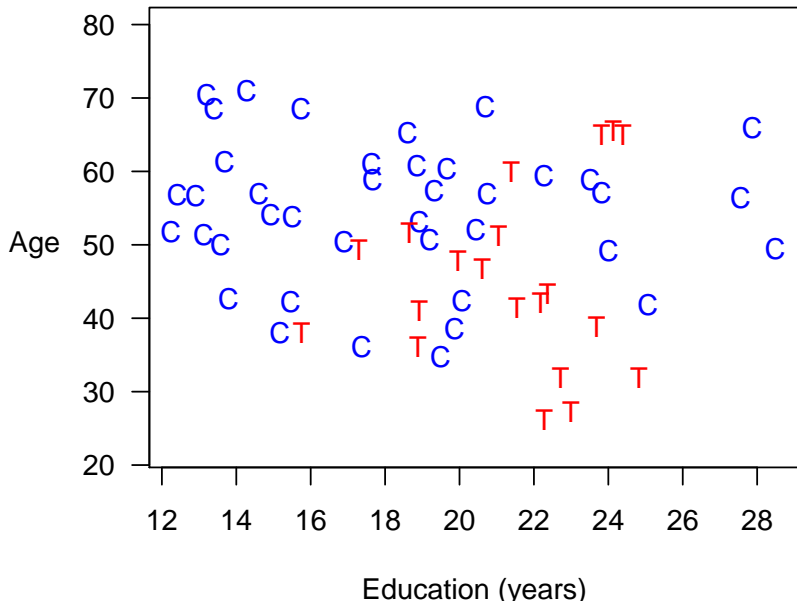
## Mahalanobis Distance Matching



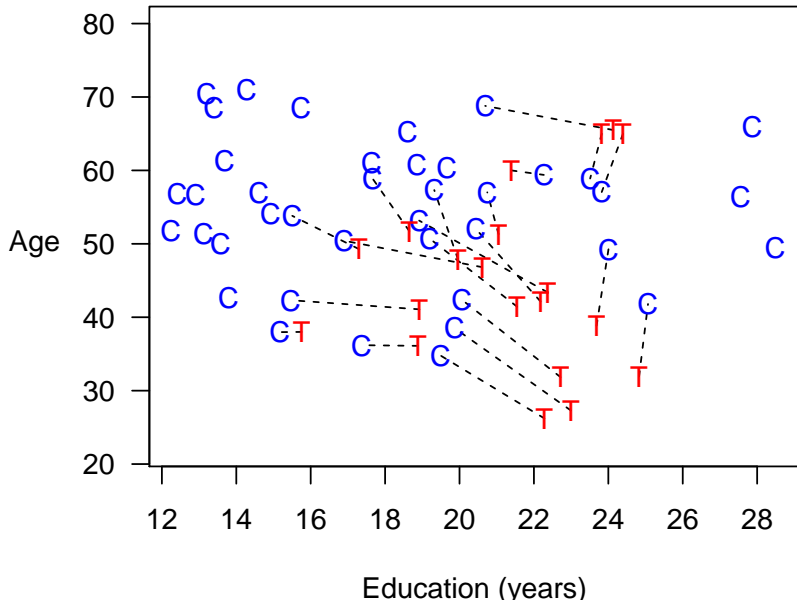
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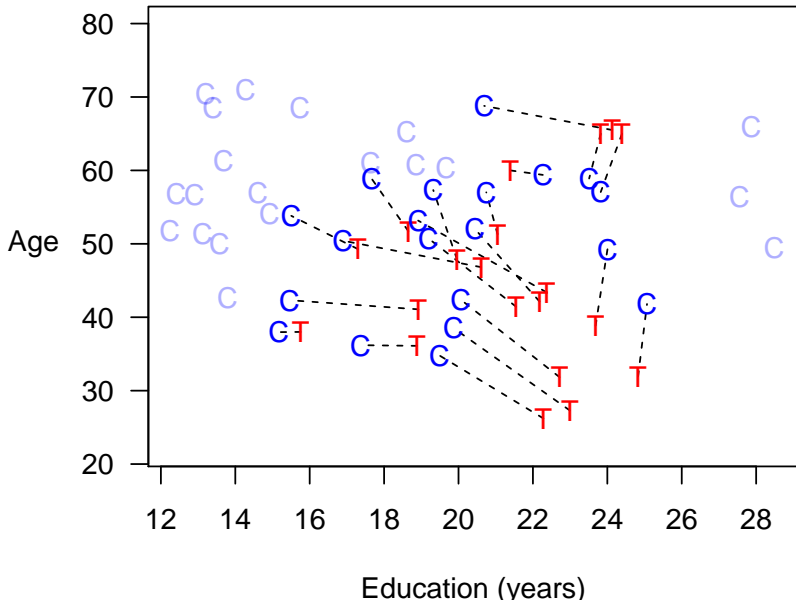
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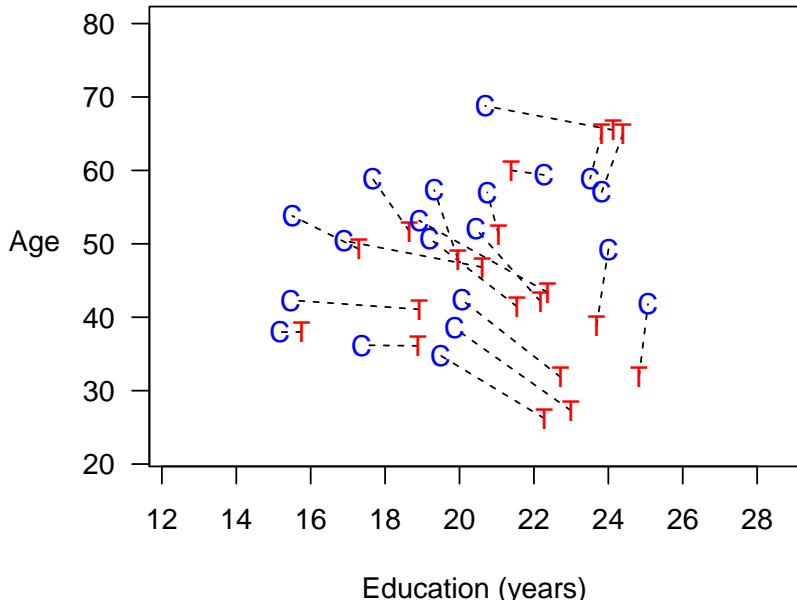


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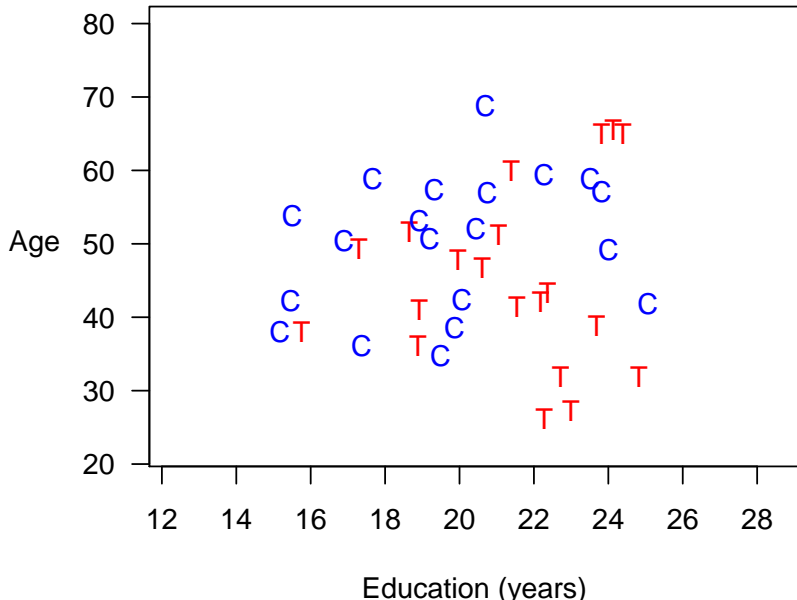




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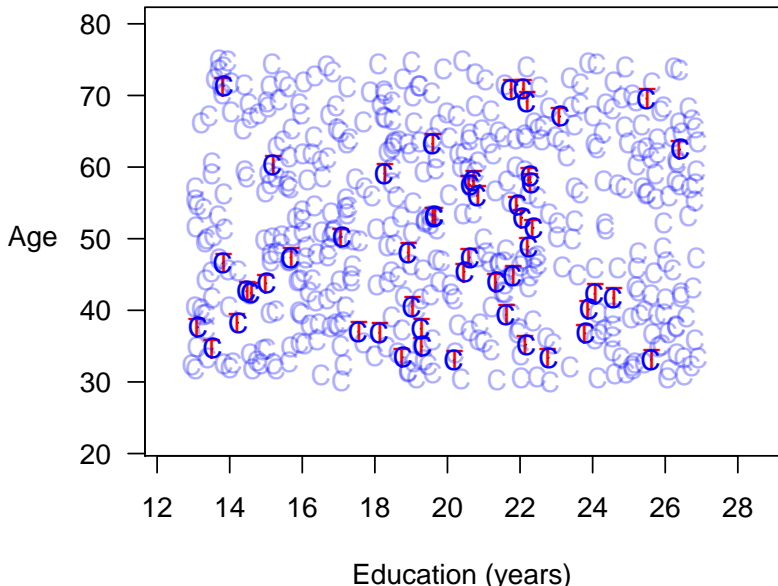


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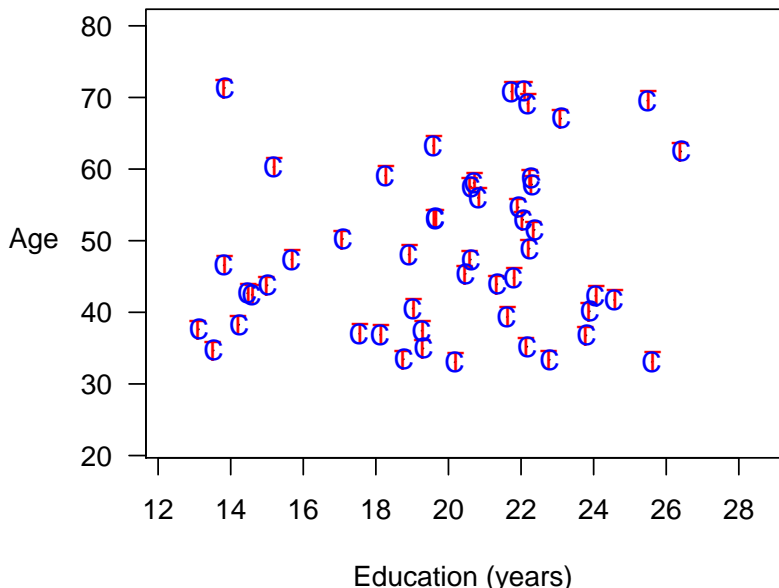


## Best Case: Mahalanobis Distance Matching

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## Method 2: Coarsened Exact Matching

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(Approximates Fully Blocked Experiment)

### 1. Preprocess (Matching)

- Temporarily coarsen  $X$  as much as you're willing

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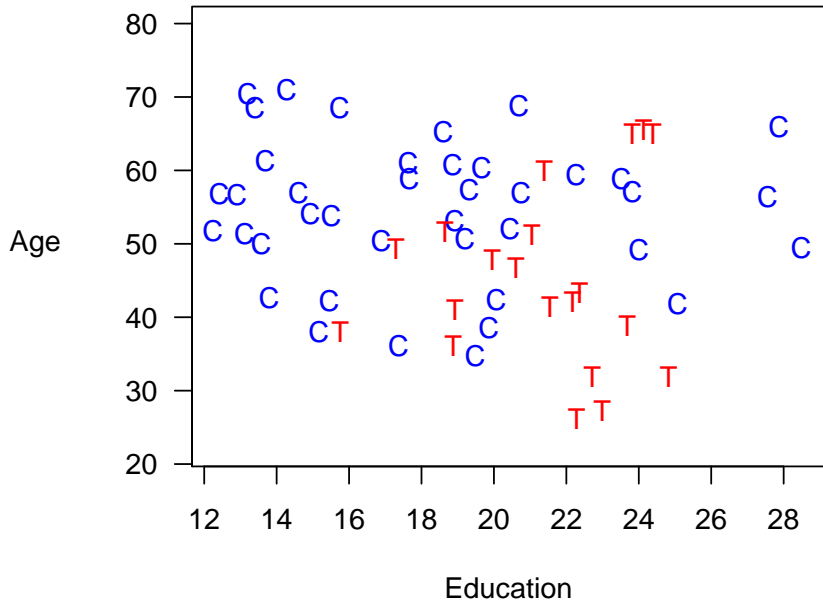
### 2. Estimation Difference in means or a model

- Weight controls in each stratum to equal treated

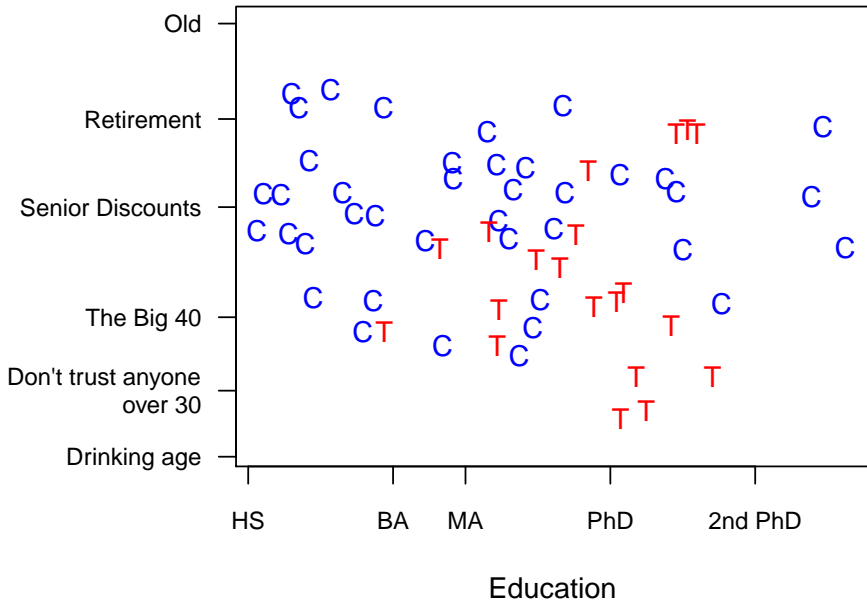
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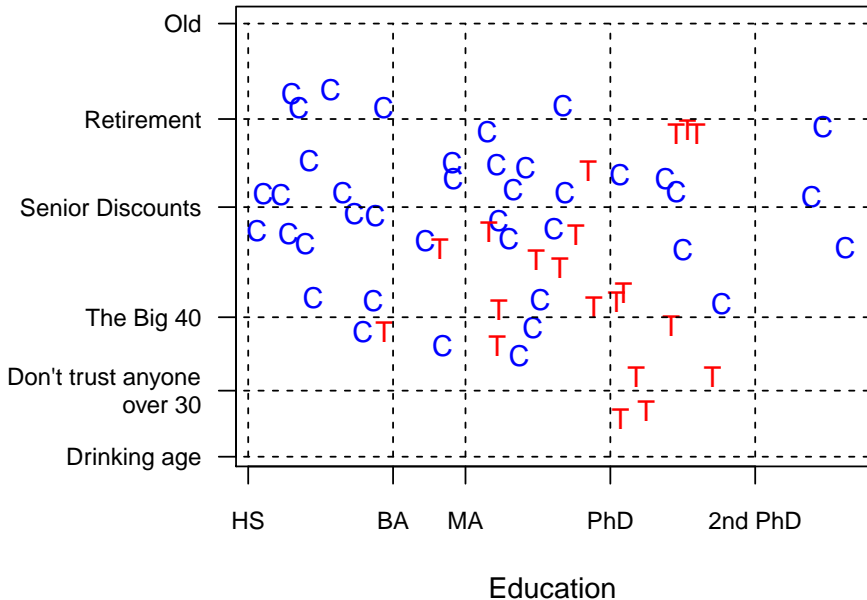
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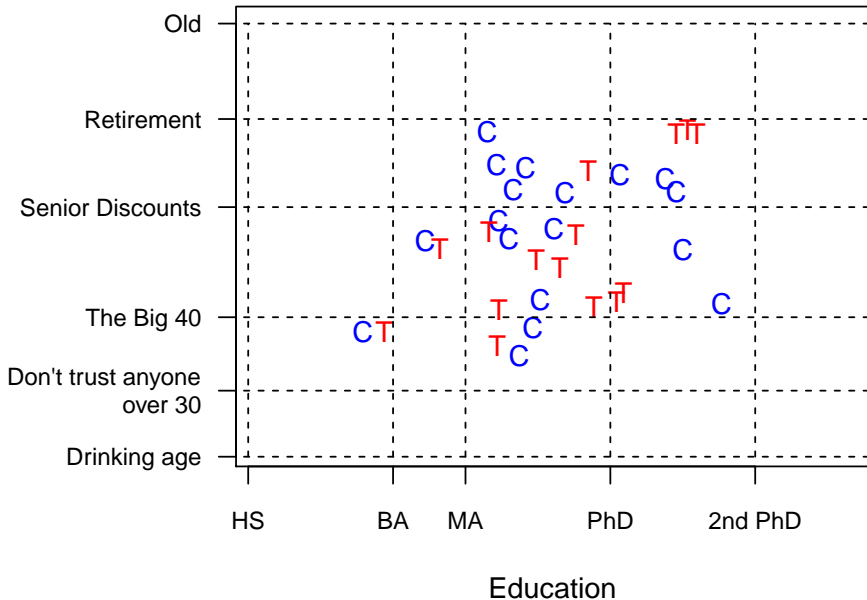
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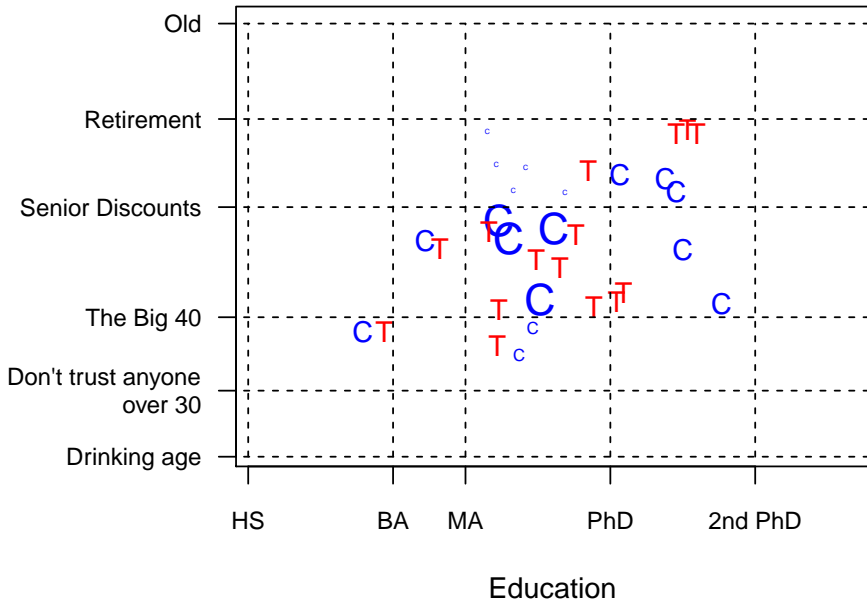
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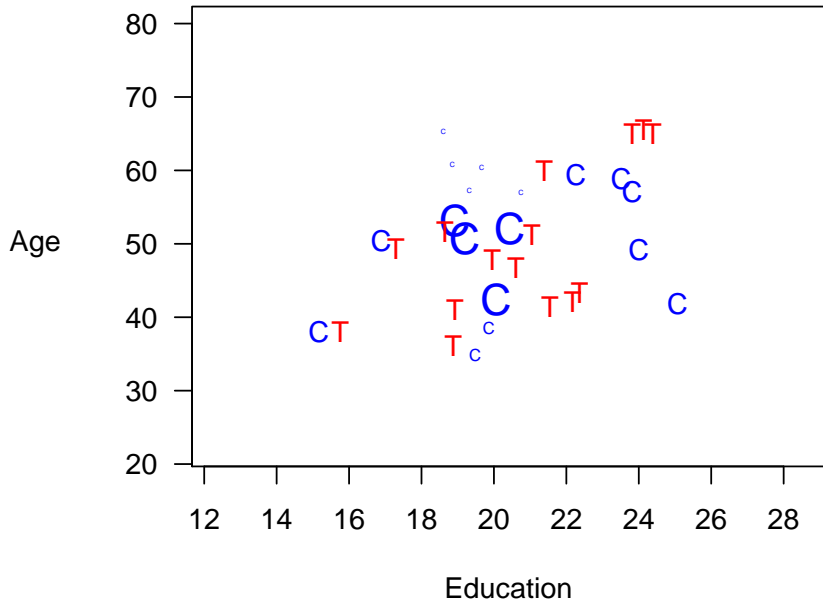
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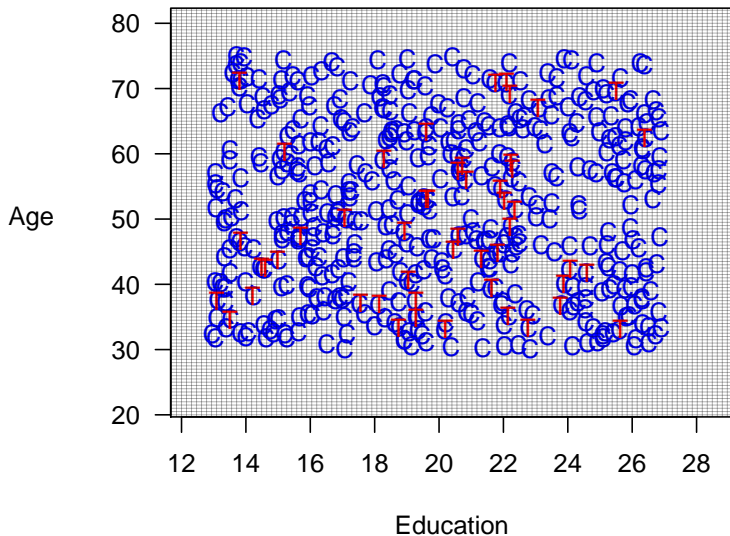


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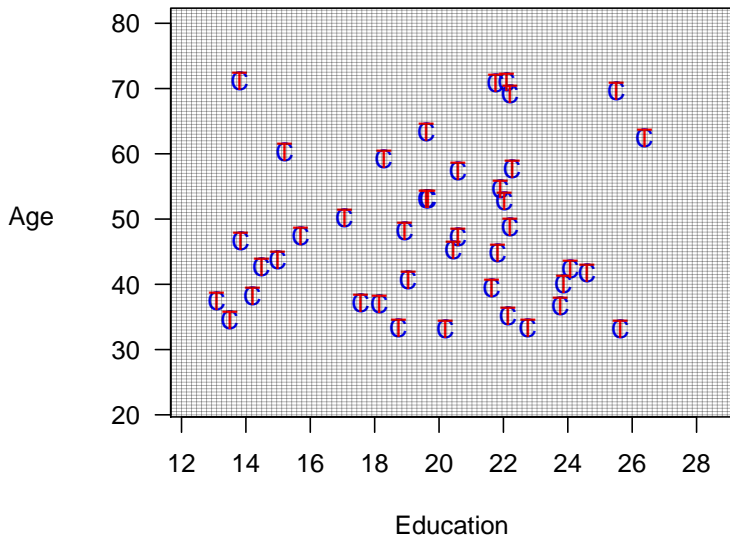
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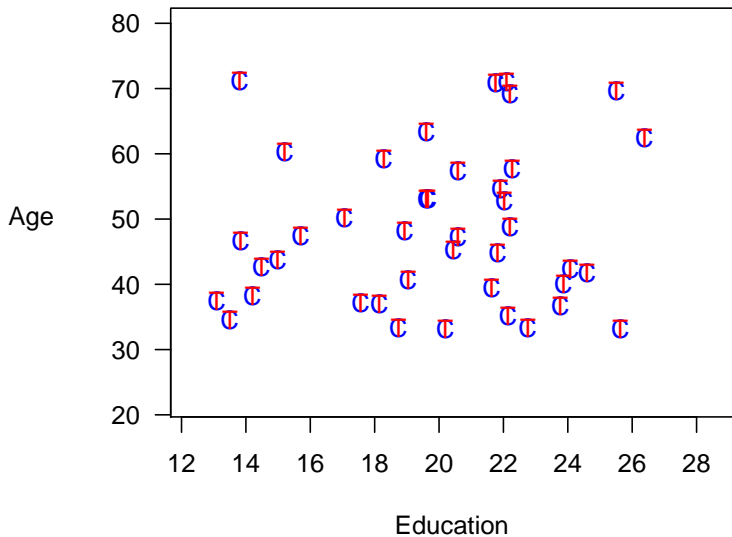




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## Method 3: Propensity Score Matching

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## 1. Preprocess (Matching)

- Reduce  $k$  elements of  $X$  to scalar

$$\pi_i \equiv \Pr(T_i = 1|X) = \frac{1}{1+e^{-X_i\beta}}$$

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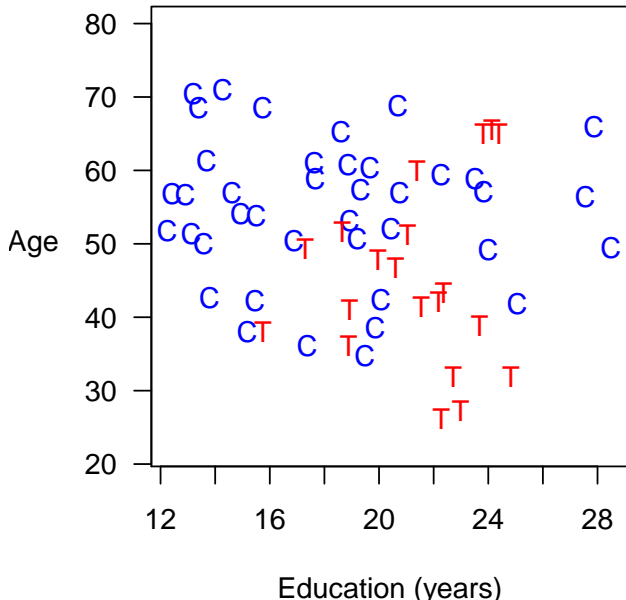
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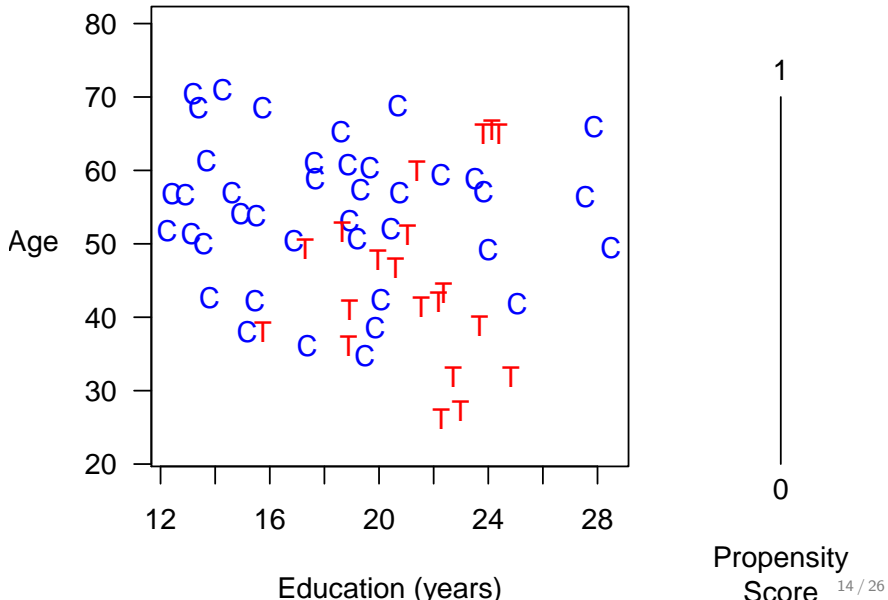
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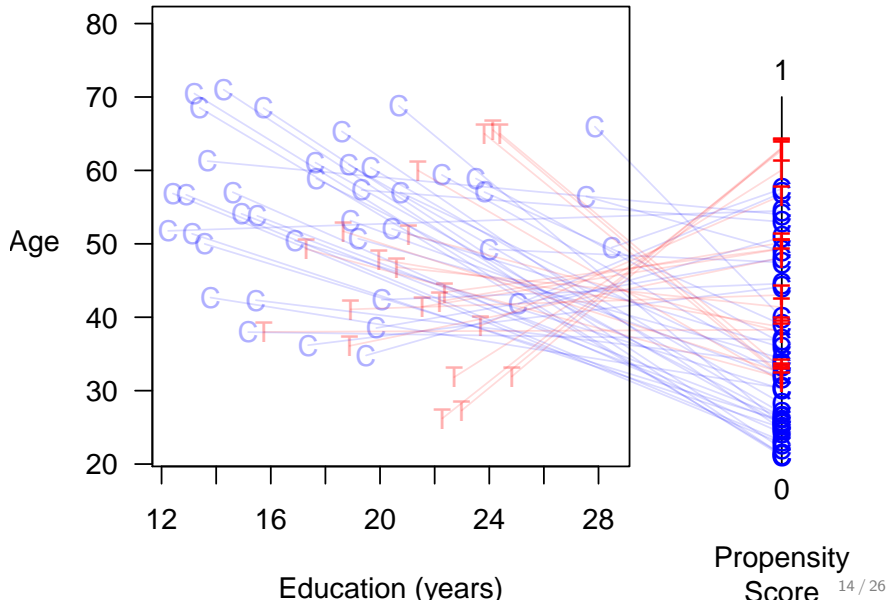
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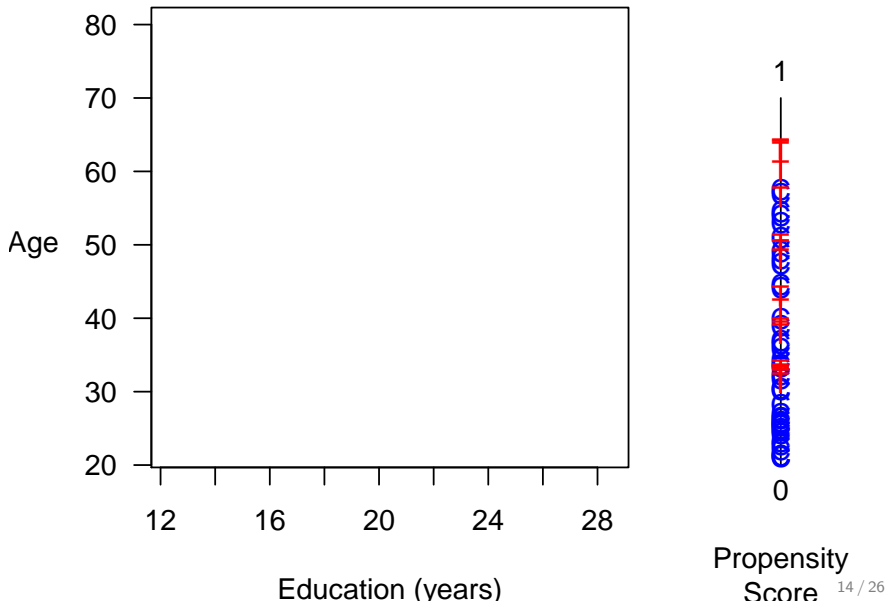
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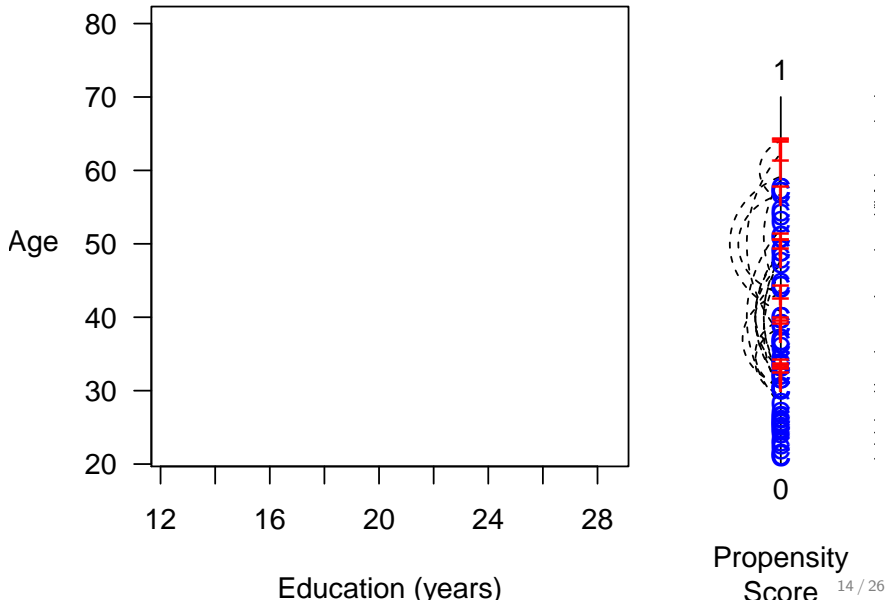
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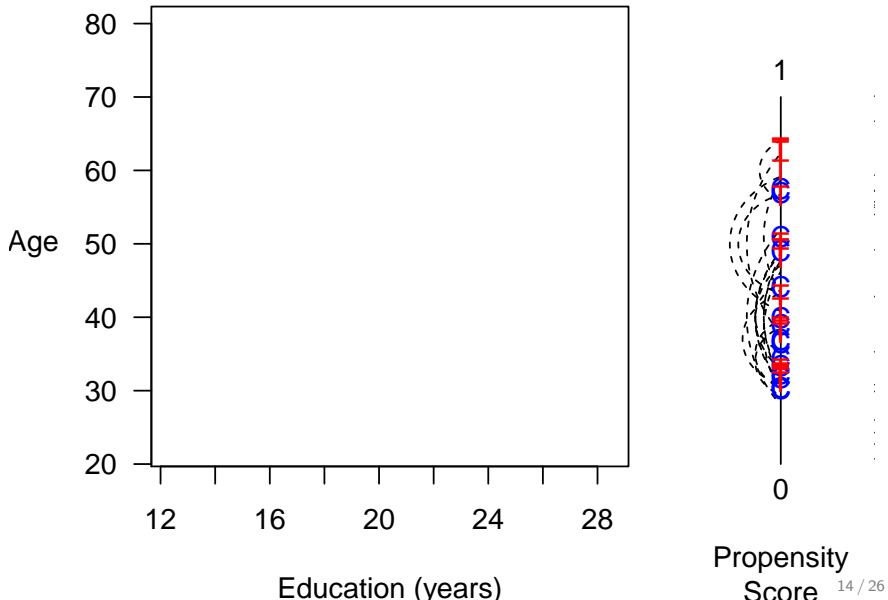


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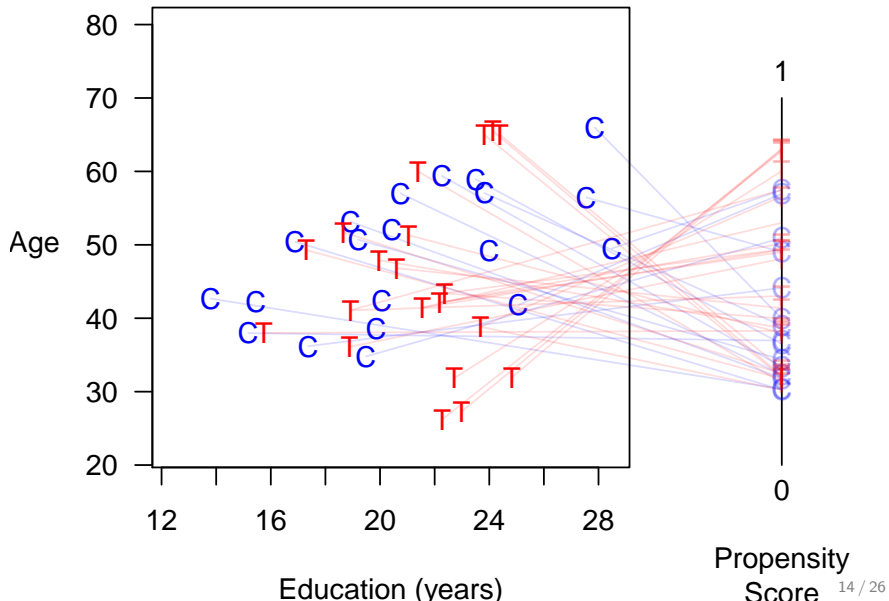




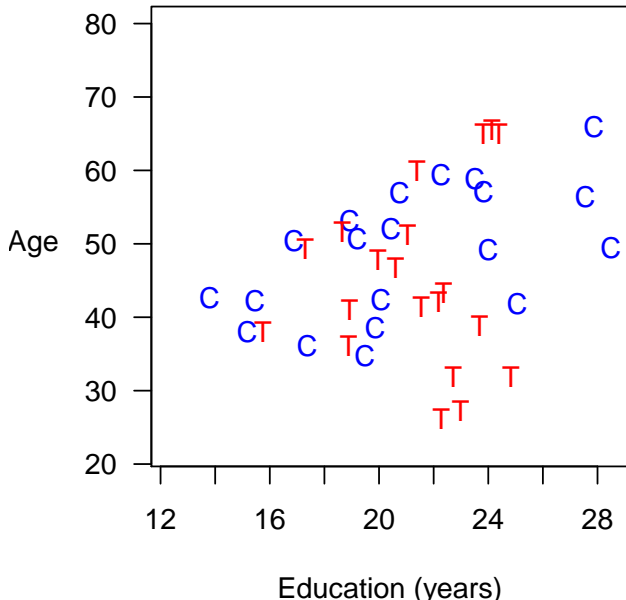
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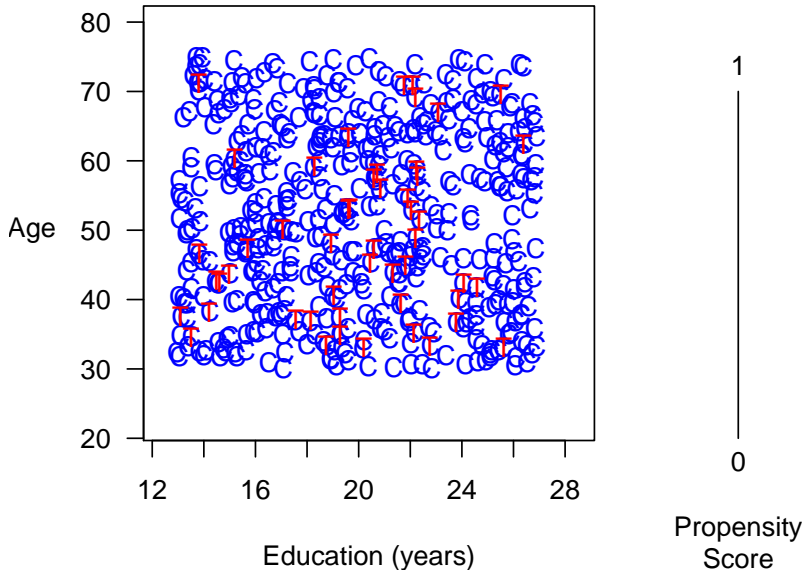


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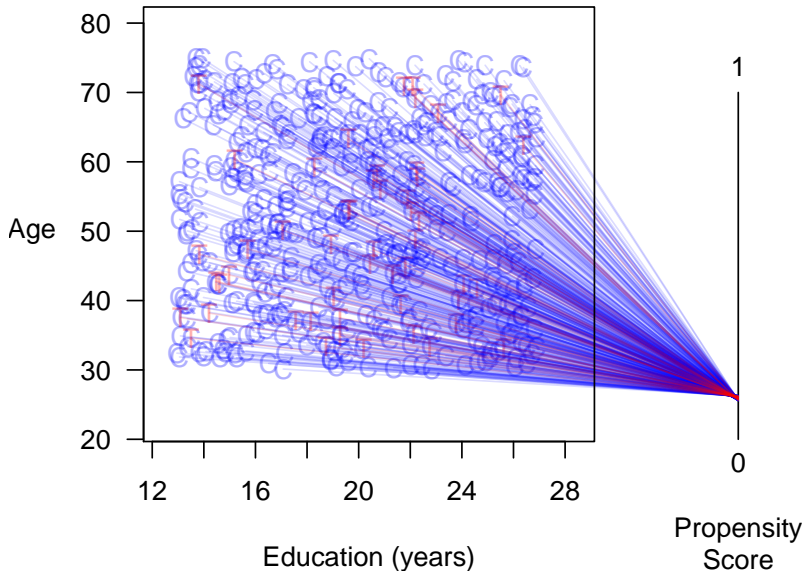


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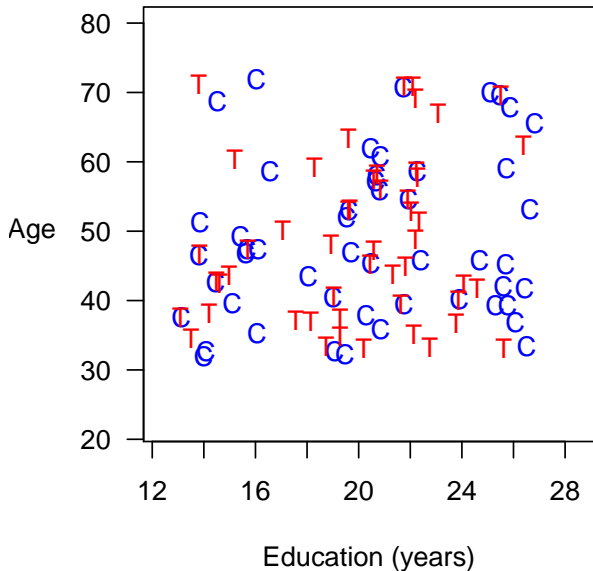
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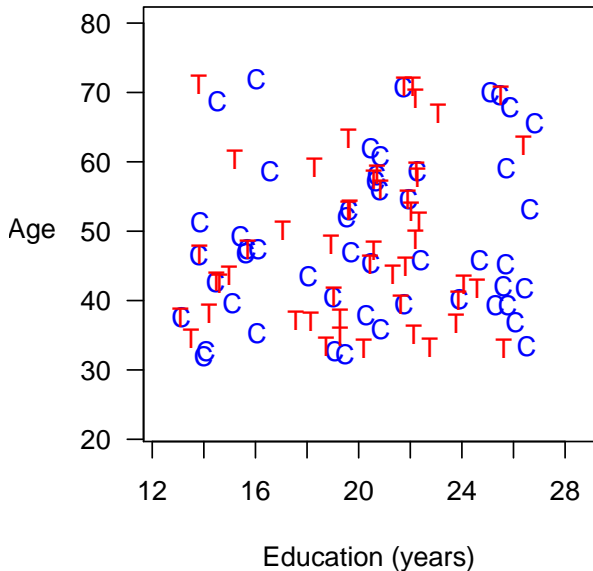
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## Best Case: Propensity Score Matching is Suboptimal





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- Result is completely general (see math in the paper)

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## 2. The PSM Paradox: When you do “better,” you do worse

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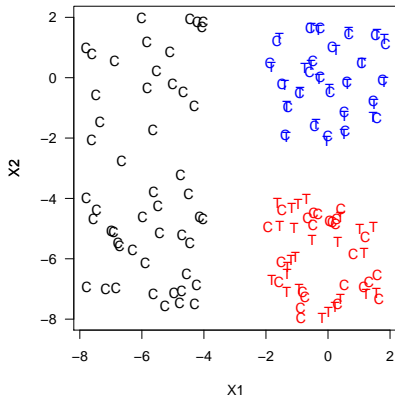
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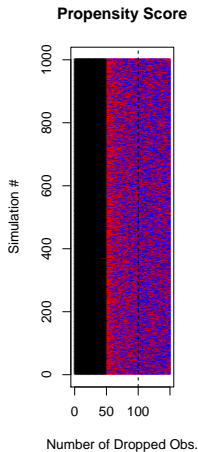
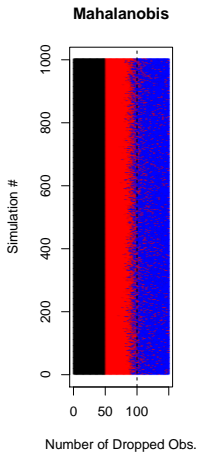
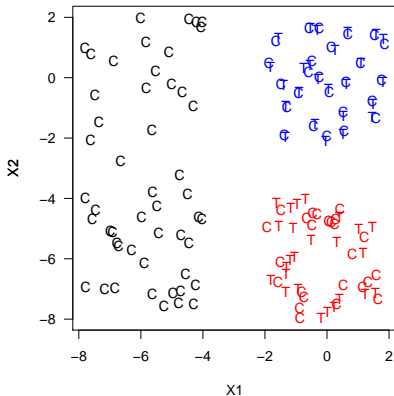
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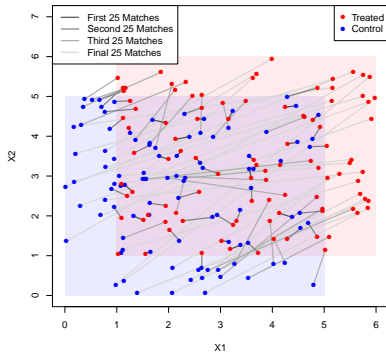


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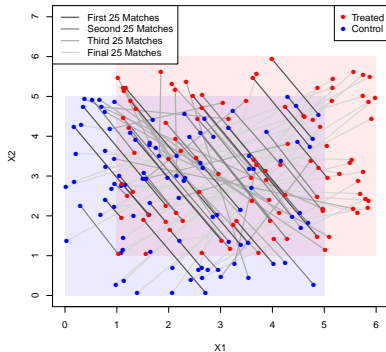


# What Does PSM Match?

## MDM Matches



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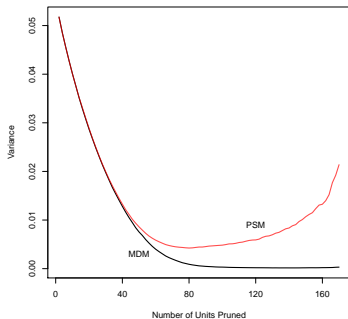


Controls:  $X_1, X_2 \sim \text{Uniform}(0,5)$

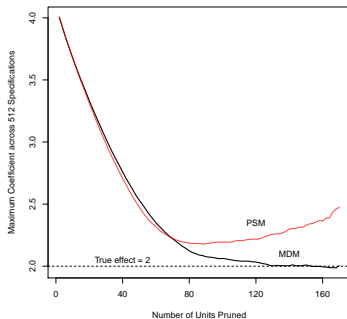
Treateds:  $X_1, X_2 \sim \text{Uniform}(1,6)$

# PSM Increases Model Dependence & Bias

## Model Dependence



## Bias



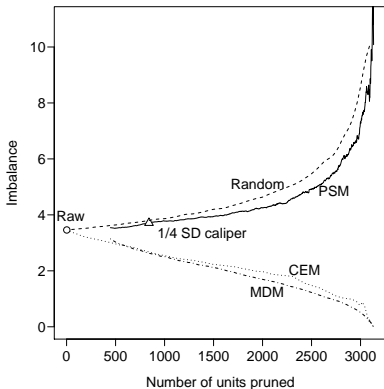
$$Y_i = 2T_i + X_{1i} + X_{2i} + \epsilon_i$$
$$\epsilon_i \sim N(0, 1)$$

# The Propensity Score Paradox in Real Data

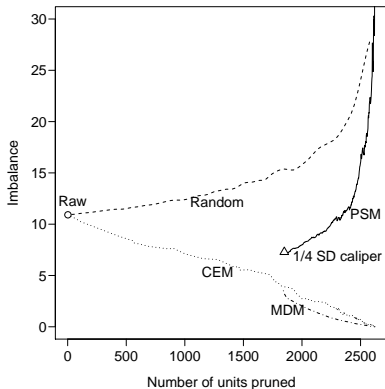


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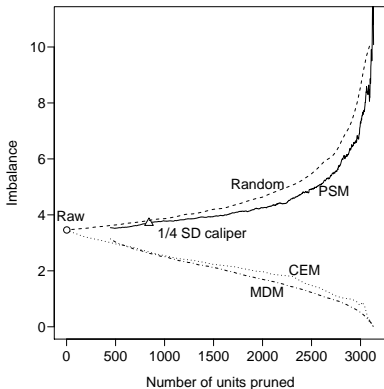


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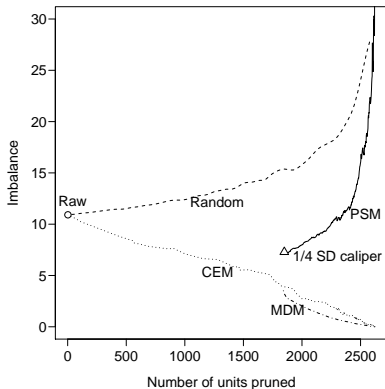


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Similar pattern for > 20 other real data sets we checked

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- Choose an imbalance metric, then run.

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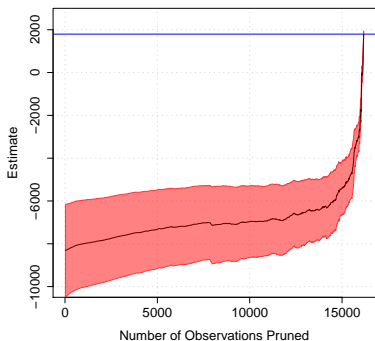
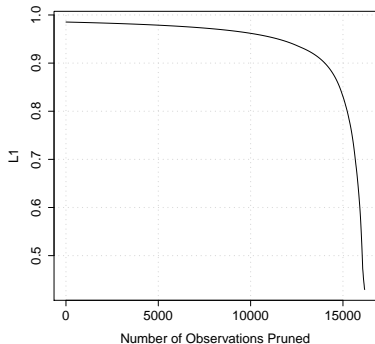
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## Job Training Data: Frontier and Causal Estimates



- 185 Ts; pruning most 16,252 Cs won't increase variance much
- Huge bias-variance trade-off after pruning most Cs
- Estimates converge to experiment after removing bias
- No mysteries: basis of inference clearly revealed

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For more information, articles, & software

GaryKing.org