### Simplifying Matching Methods for Causal Inference

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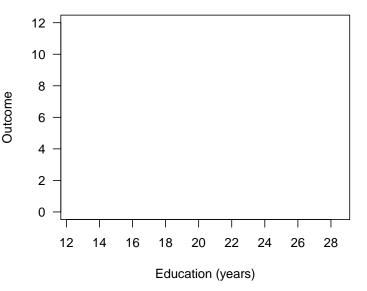
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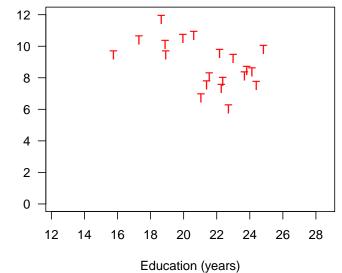
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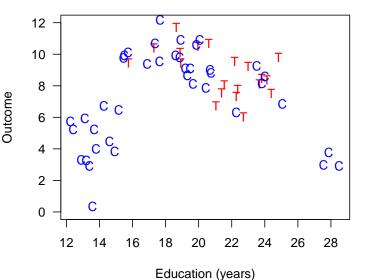


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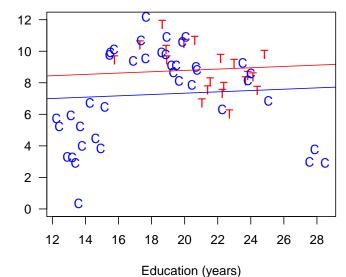


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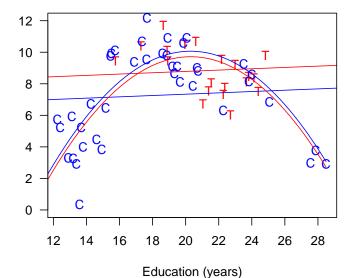


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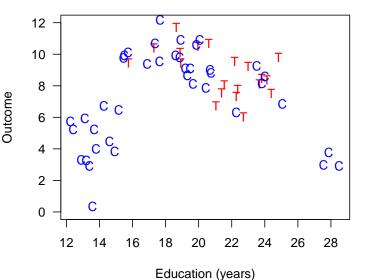


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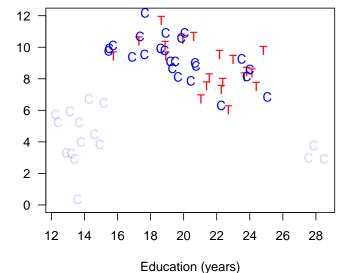
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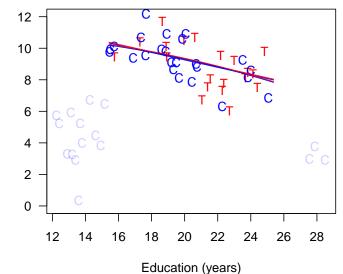


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Outcome

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- "Teaching psychology is mostly a waste of time" (Kahneman 2011)

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# The Problems Matching Solves

Without Matching: Malance ---- Model Dependence ---- Researcher discretion ---- Bias

A central project of statistics: Automating away human discretion

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- Pruning nonmatches makes control vars matter less: reduces imbalance, model dependence, researcher discretion, & bias

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> Complete Randomization

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> Balance Observed Unobserved

Complete Covariates: Randomization Blocked

Fully

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On average

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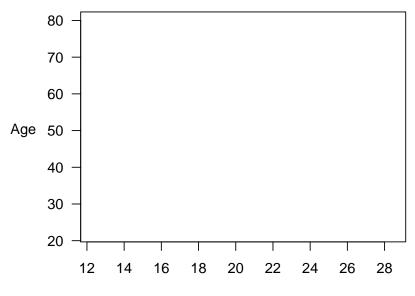
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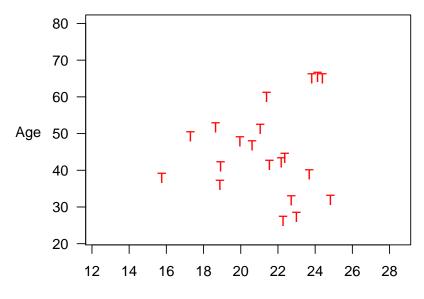
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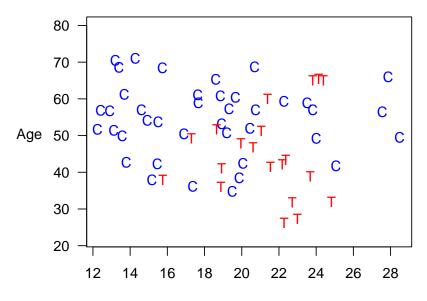
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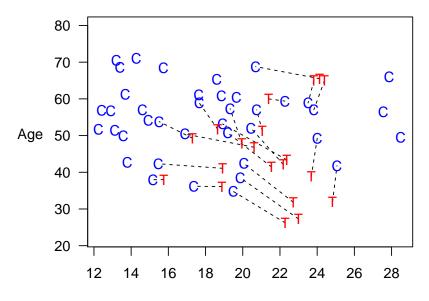
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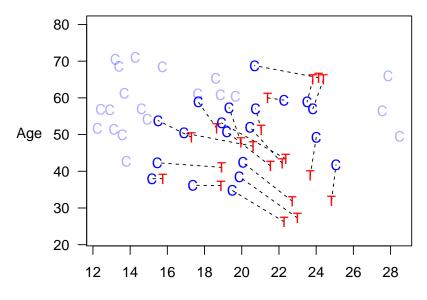
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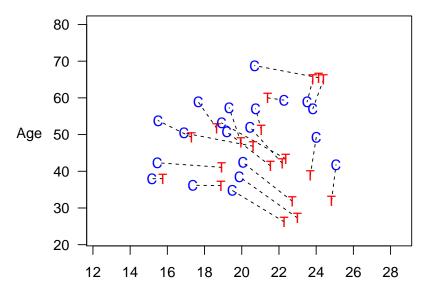


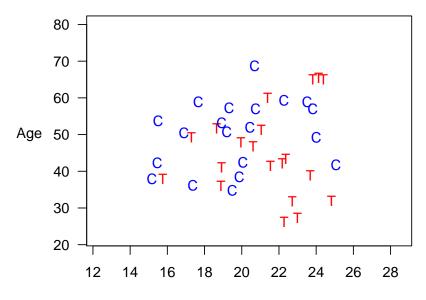






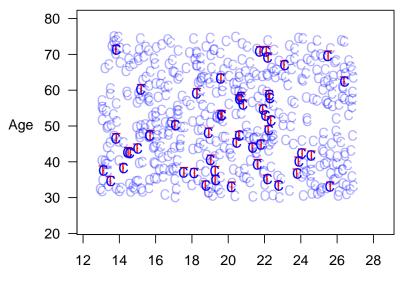




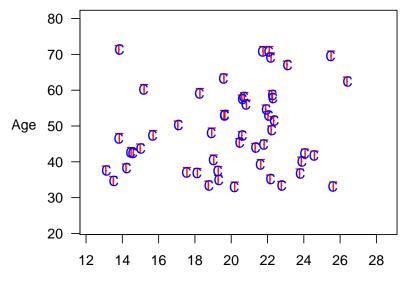


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Method 2: Coarsened Exact Matching (Most powerful easy-to-use approach)

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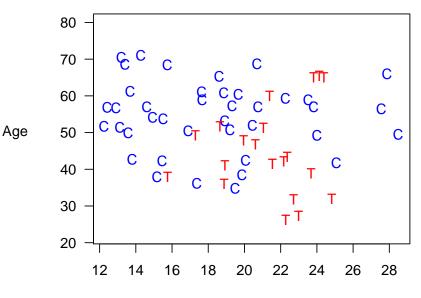
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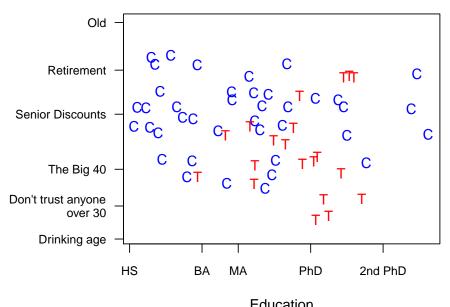
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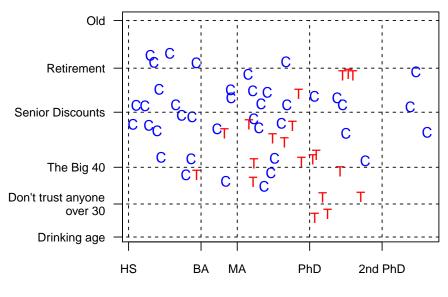
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  - Weight controls in each stratum to equal treateds

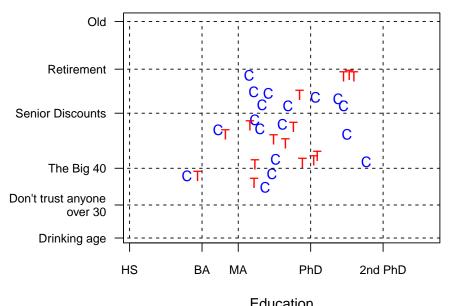


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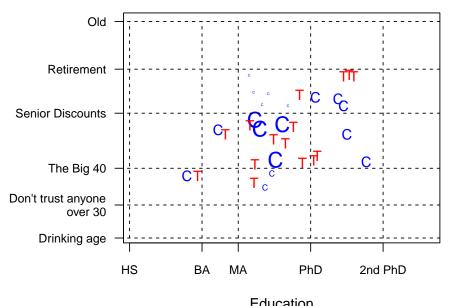


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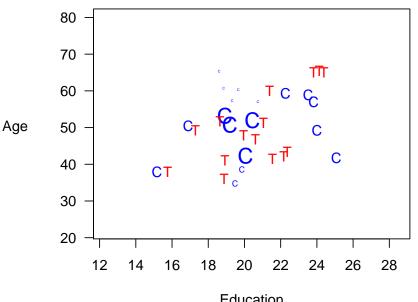
 $11 \, / \, 25$ 

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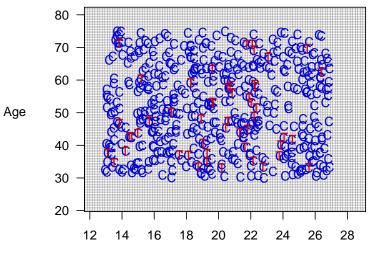


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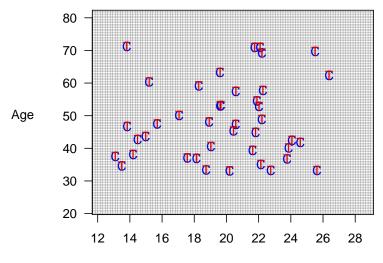
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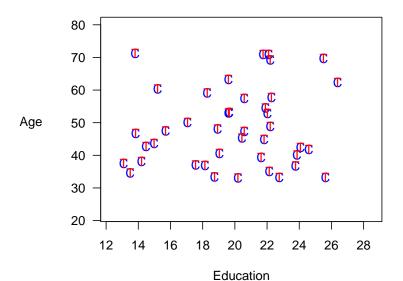
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• Reduce k elements of X to scalar  $\pi_i \equiv \Pr(T_i = 1|X) = \frac{1}{1 + e^{-X_i\beta}}$ 

• Distance
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- Match each treated unit to the nearest control unit
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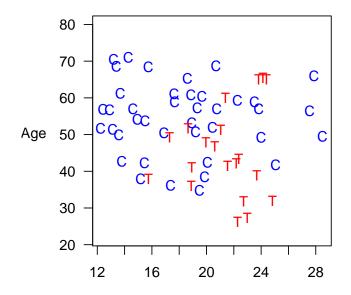
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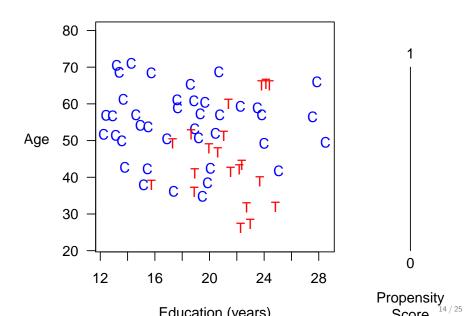
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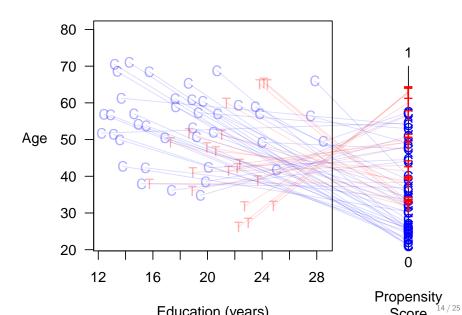
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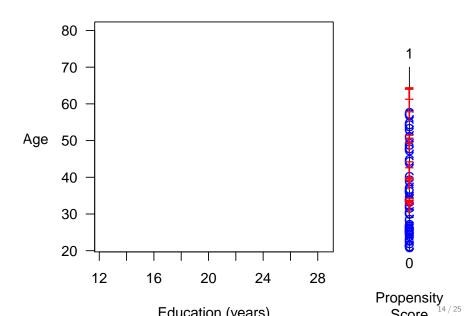
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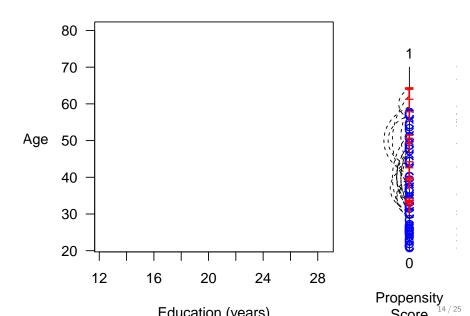


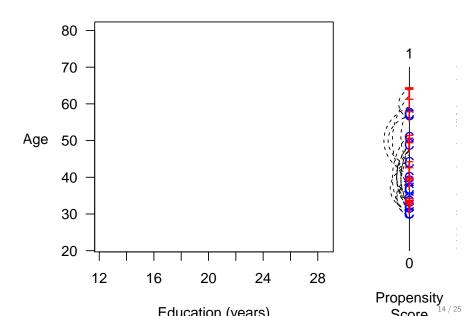
Education (years)

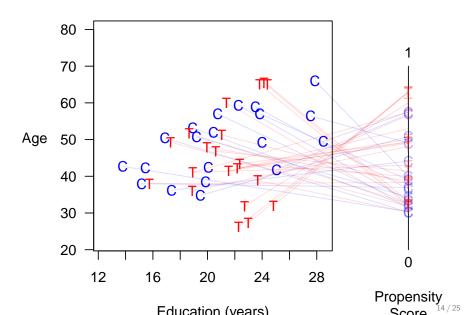


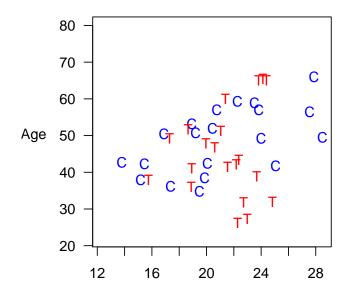




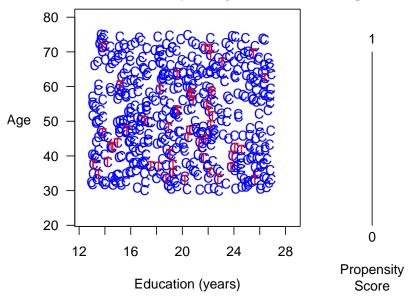


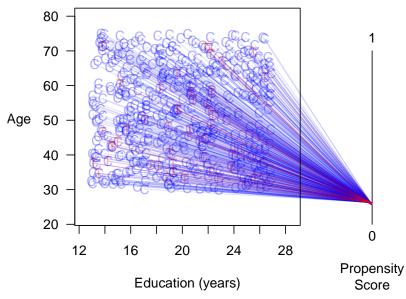


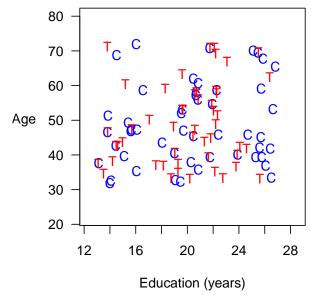




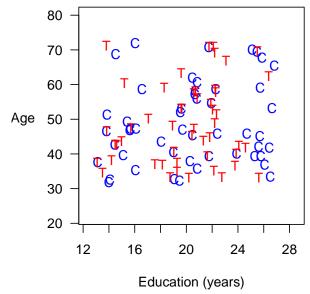
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### Best Case: Propensity Score Matching is Suboptimal



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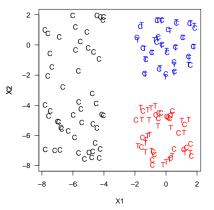
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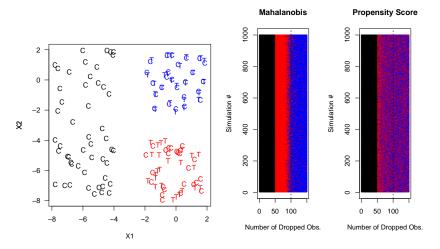
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17 / 25

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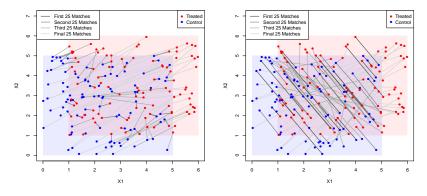


17 / 25

### What Does PSM Match?

#### MDM Matches

#### **PSM Matches**

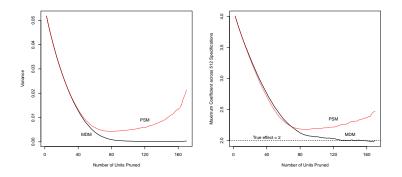


Controls:  $X_1, X_2 \sim \text{Uniform}(0,5)$ Treateds:  $X_1, X_2 \sim \text{Uniform}(1,6)$ 

### PSM Increases Model Dependence & Bias

Model Dependence

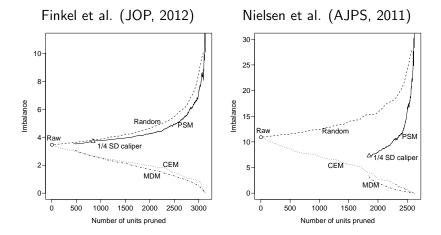
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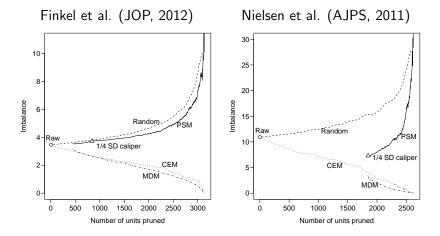
$$Y_i = 2T_i + X_{1i} + X_{2i} + \epsilon_i$$
  
$$\epsilon_i \sim N(0, 1)$$

### The Propensity Score Paradox in Real Data

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Similar pattern for > 20 other real data sets we checked

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- Choose an imbalance metric, then run.

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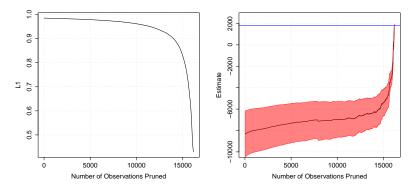
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### Job Training Data: Frontier and Causal Estimates



- 185 Ts; pruning most 16,252 Cs won't increase variance much
- Huge bias-variance trade-off after pruning most Cs
- Estimates converge to experiment after removing bias
- No mysteries: basis of inference clearly revealed

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### For more information, articles, & software

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