Simplifying Matching Methods for Causal Inference

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(Talk at MIT, Political Methodology Series, 3/16/2015)

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3 Problems, 3 Solutions

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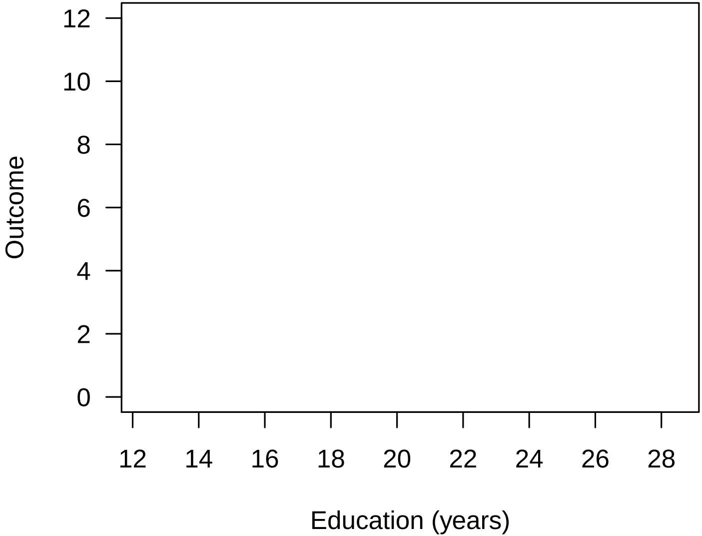

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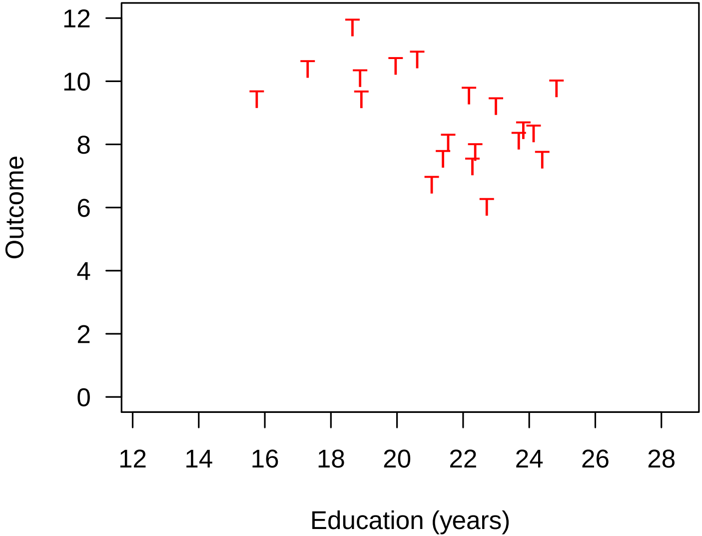

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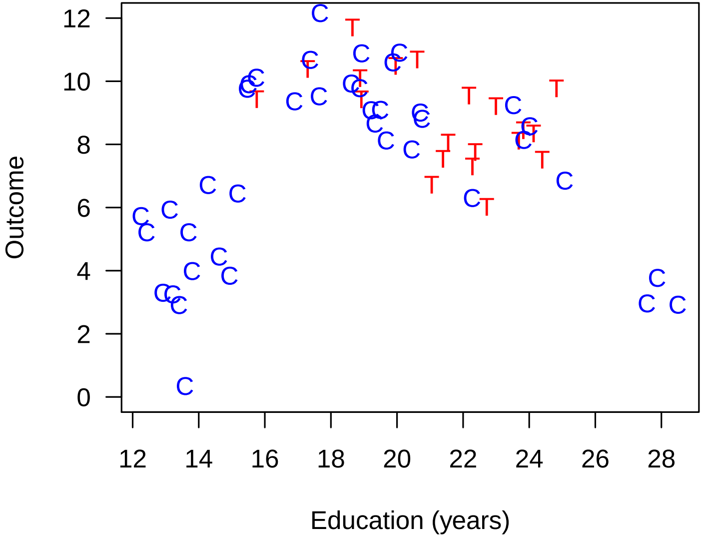

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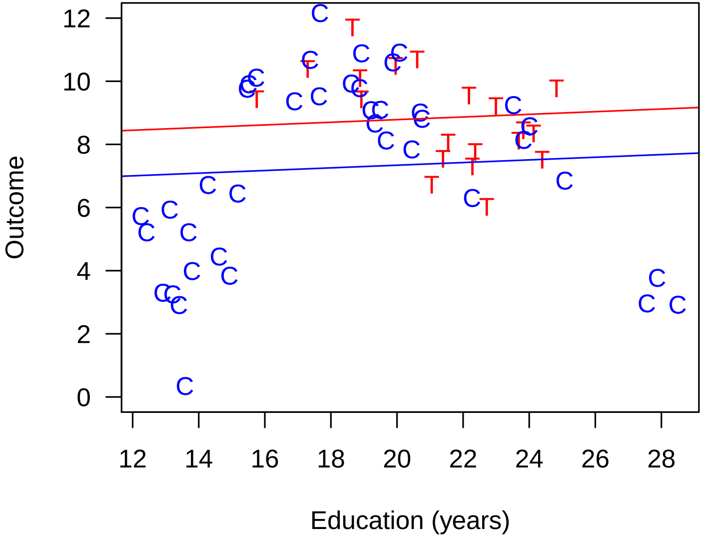

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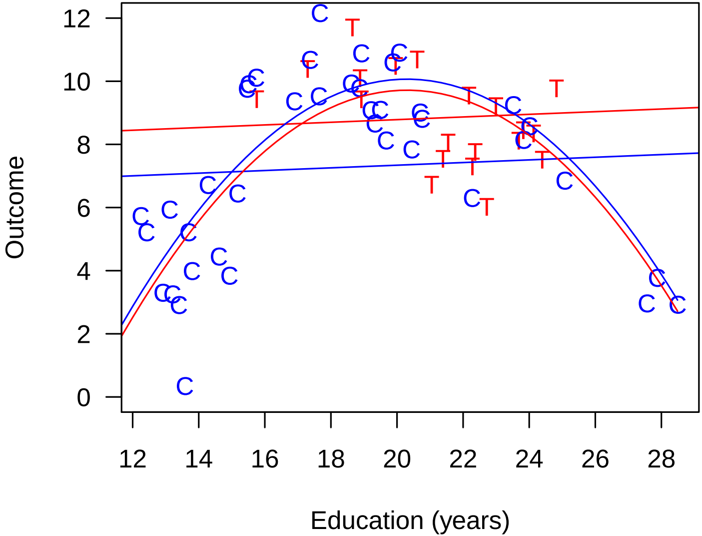

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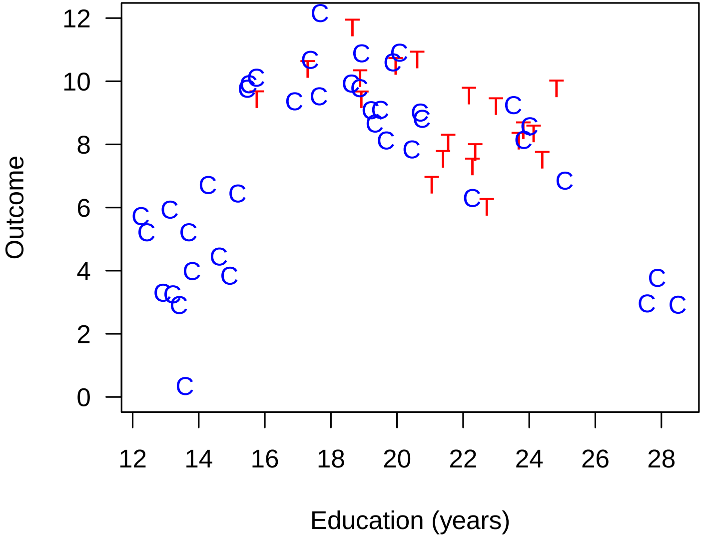

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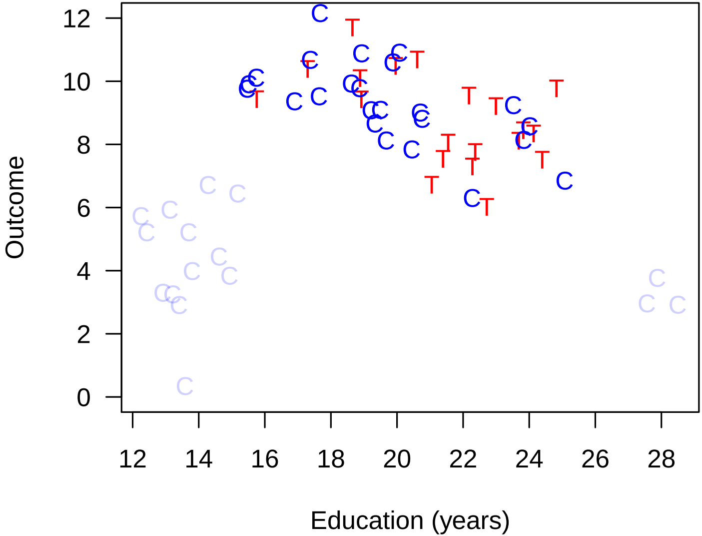

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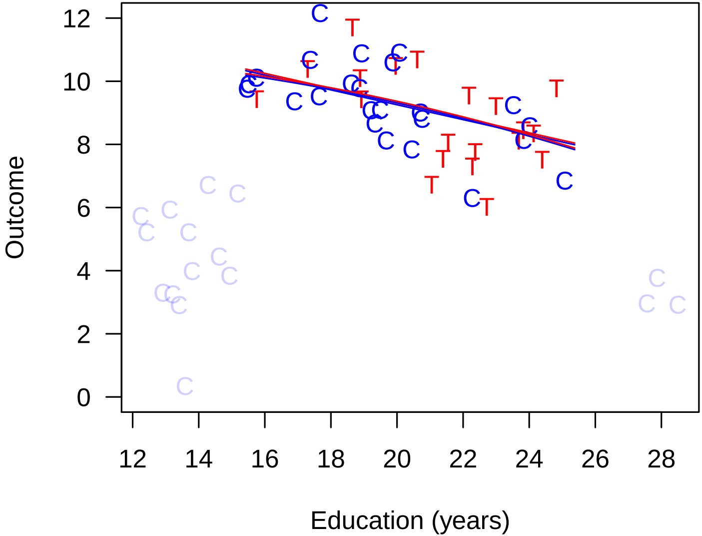

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- Big convenience: Follow preprocessing with whatever statistical method you'd have used without matching

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- Easy extensions for: multi-level, continuous, & mismeasured treatments; A too wide, n too small

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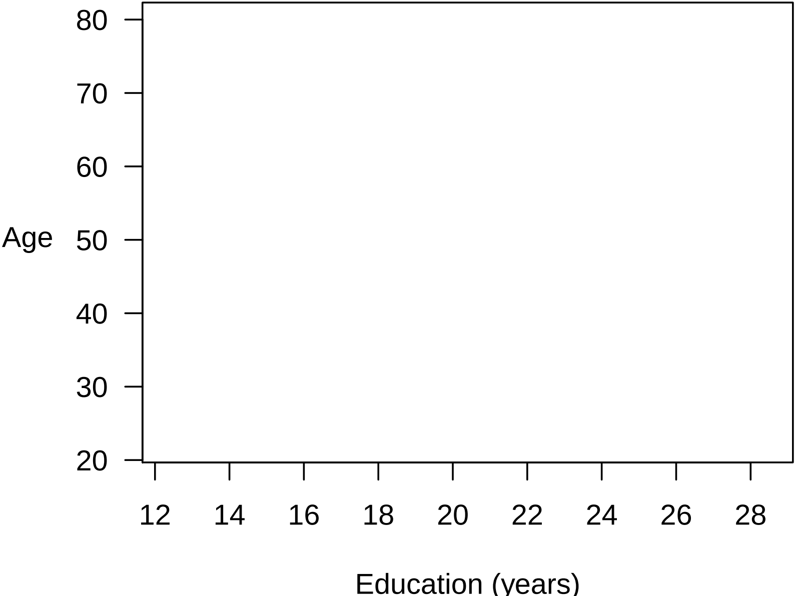

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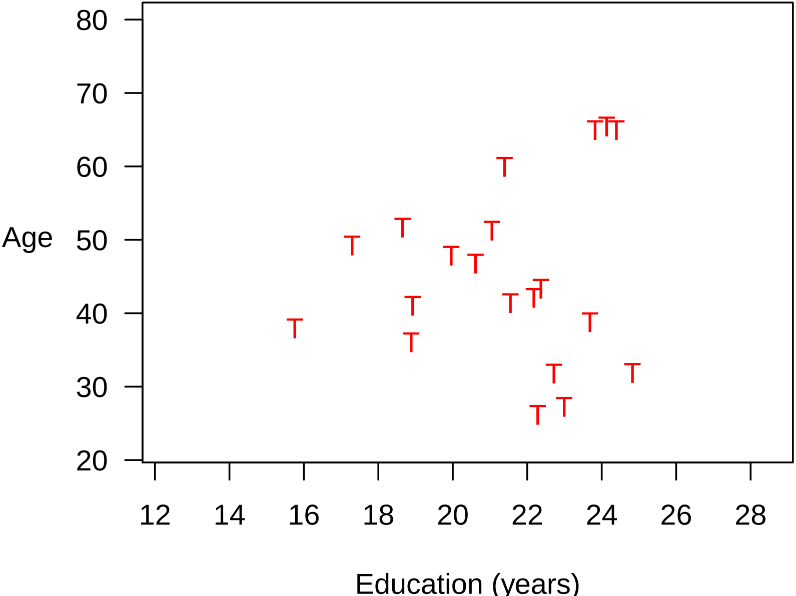

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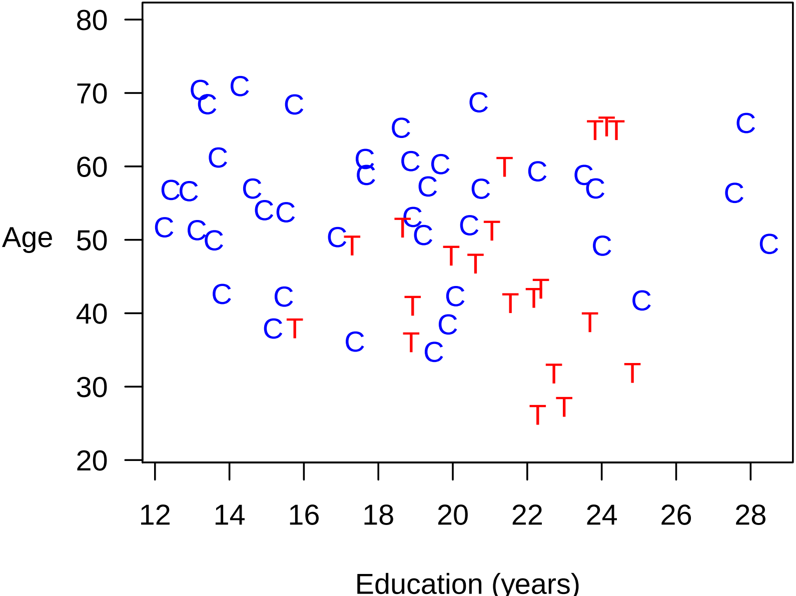

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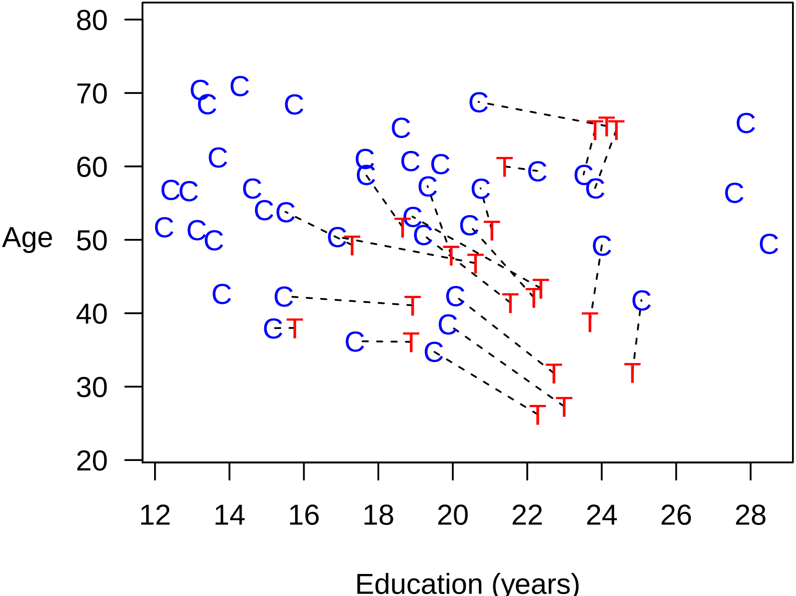

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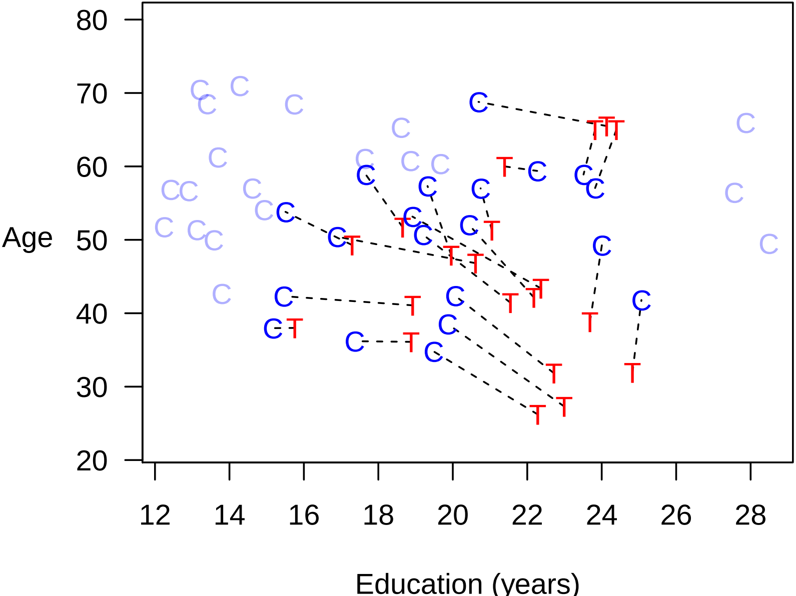

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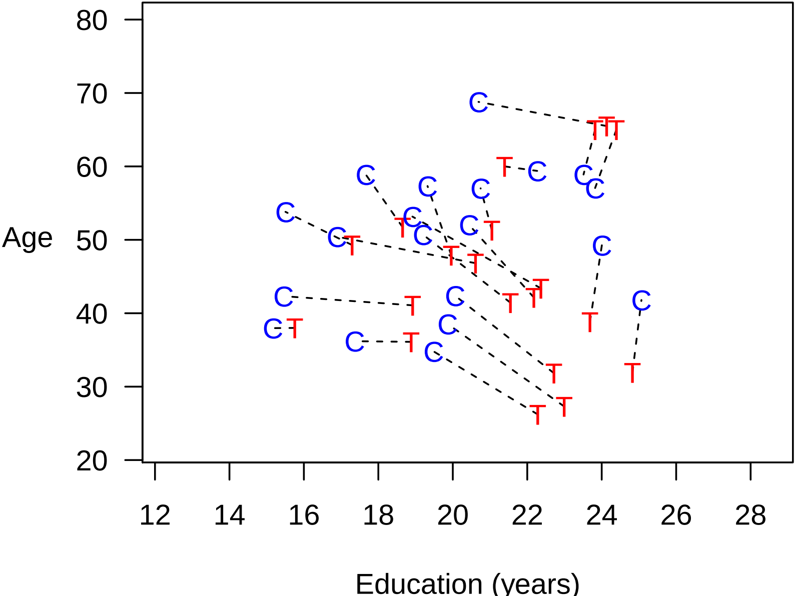

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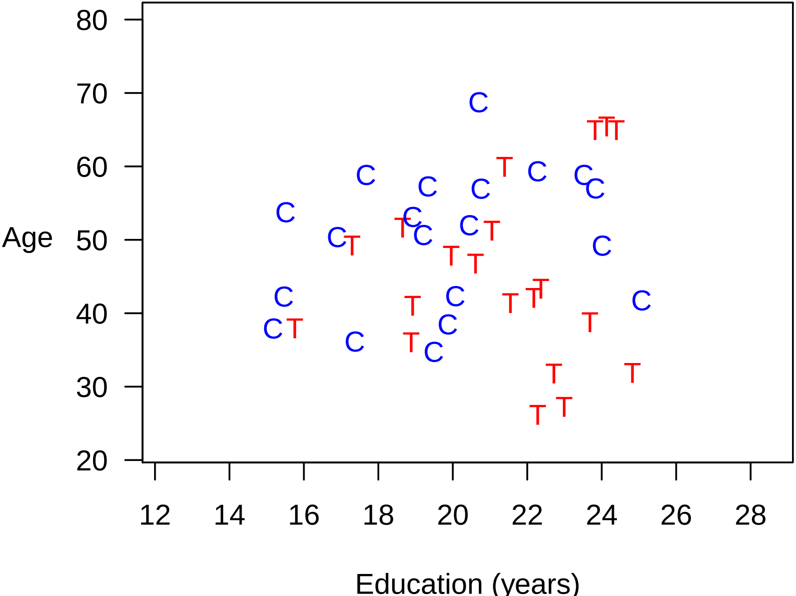

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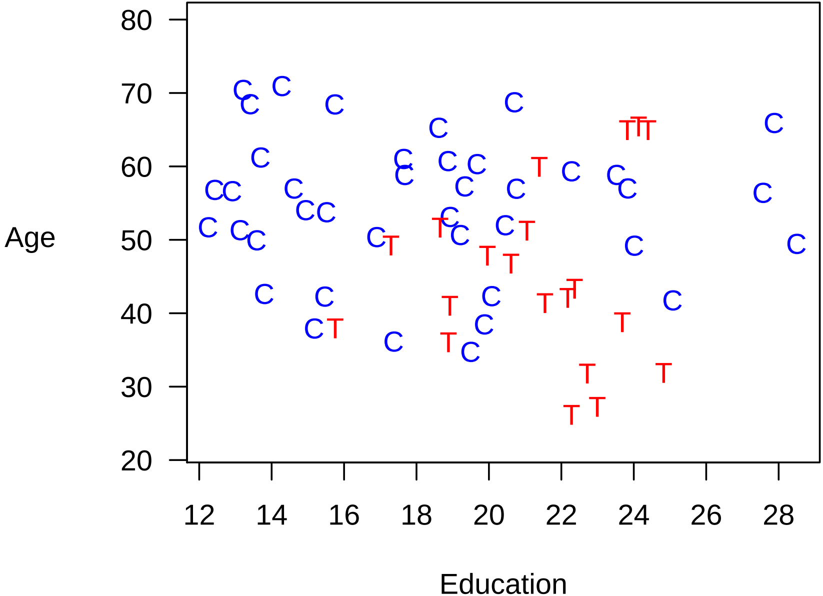

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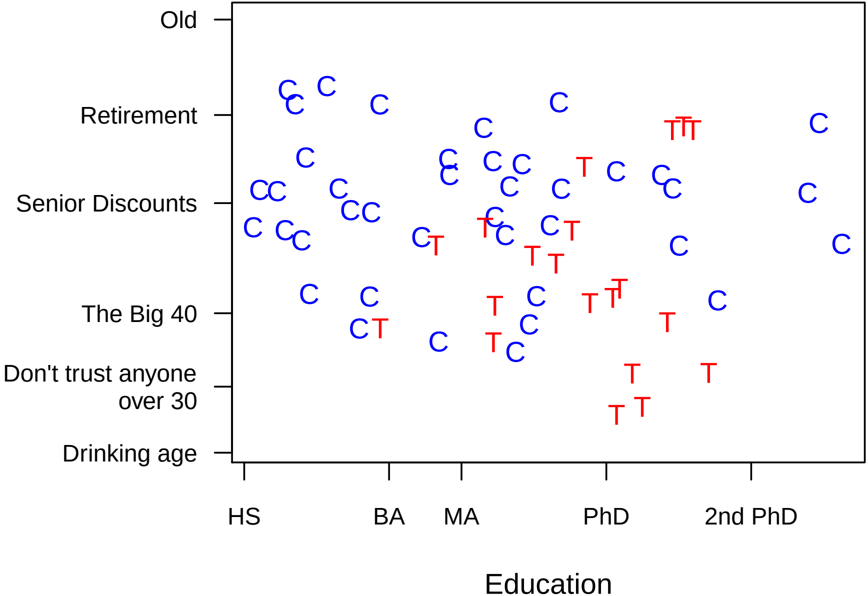

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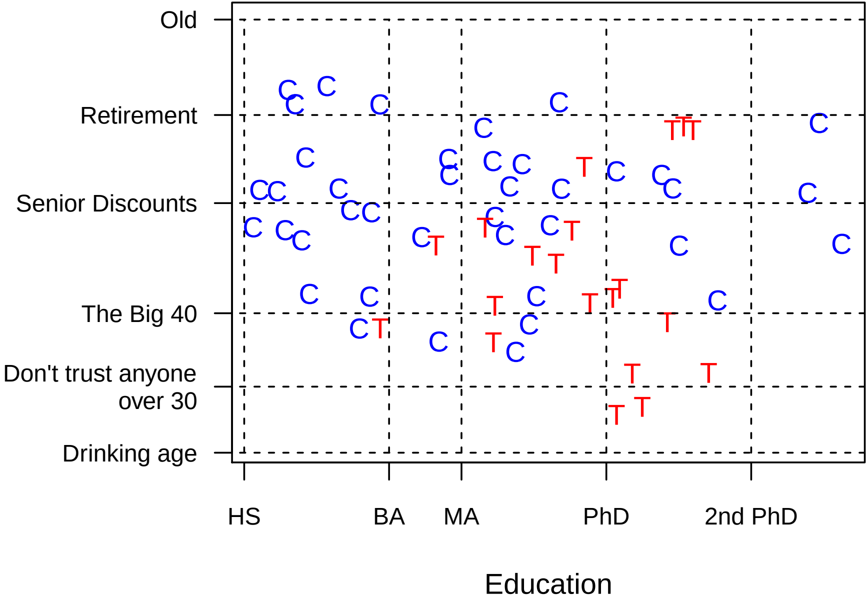

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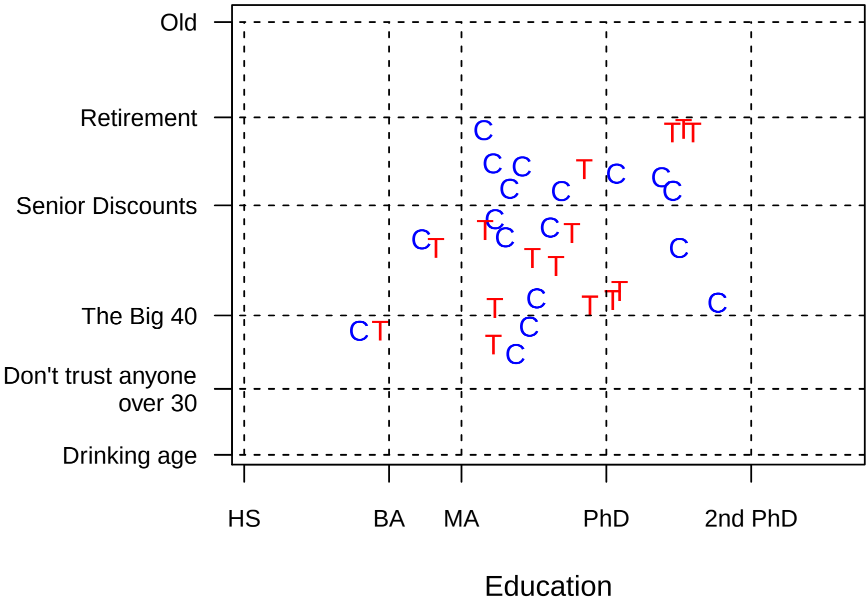

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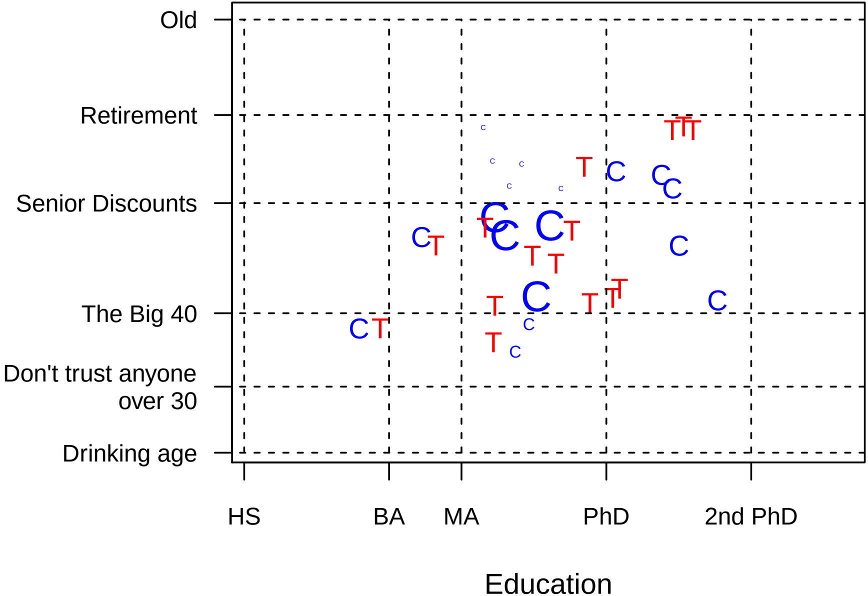

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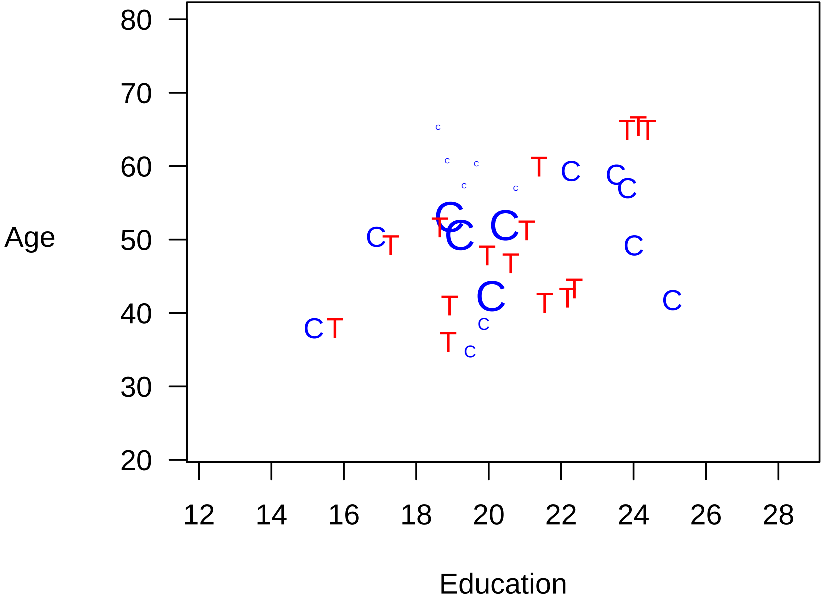

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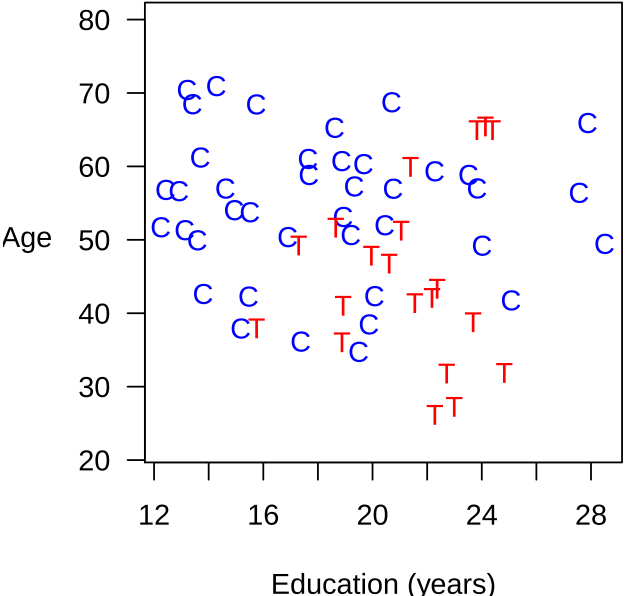

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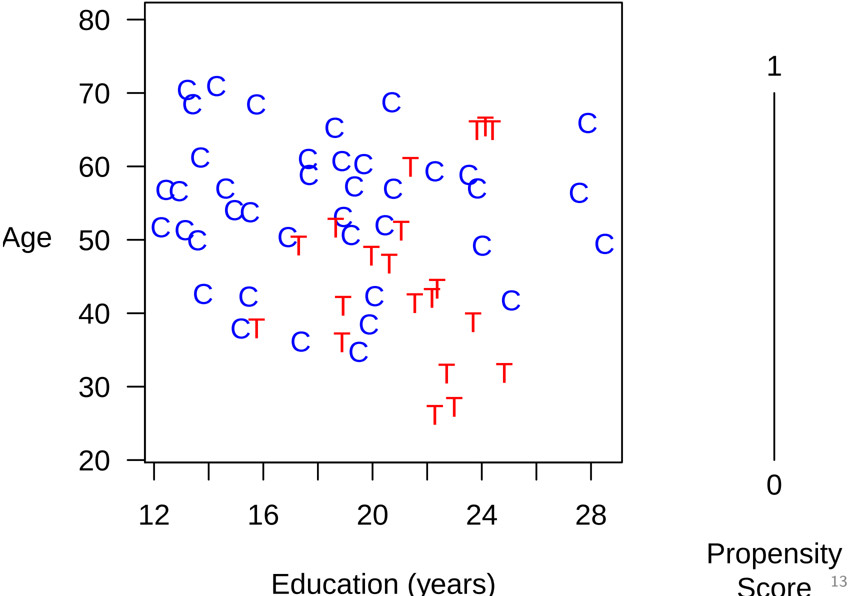

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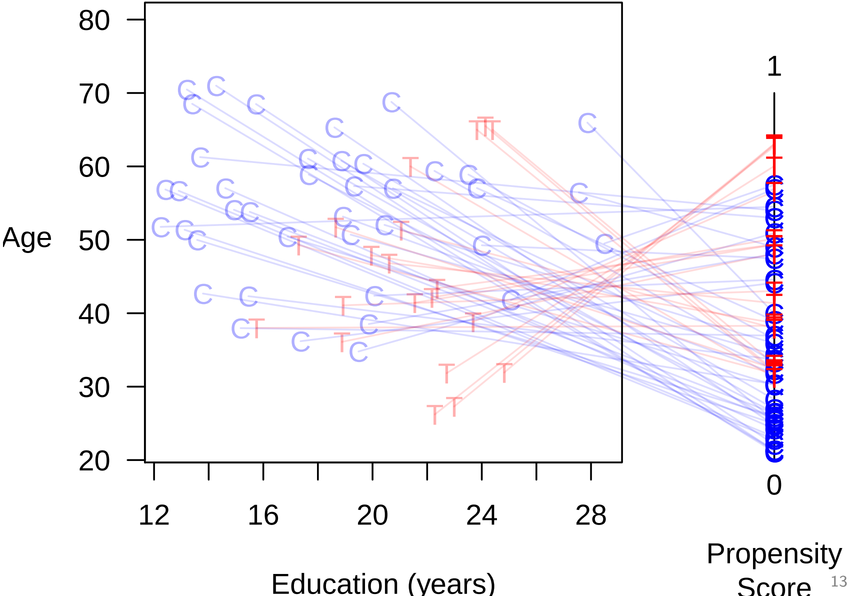

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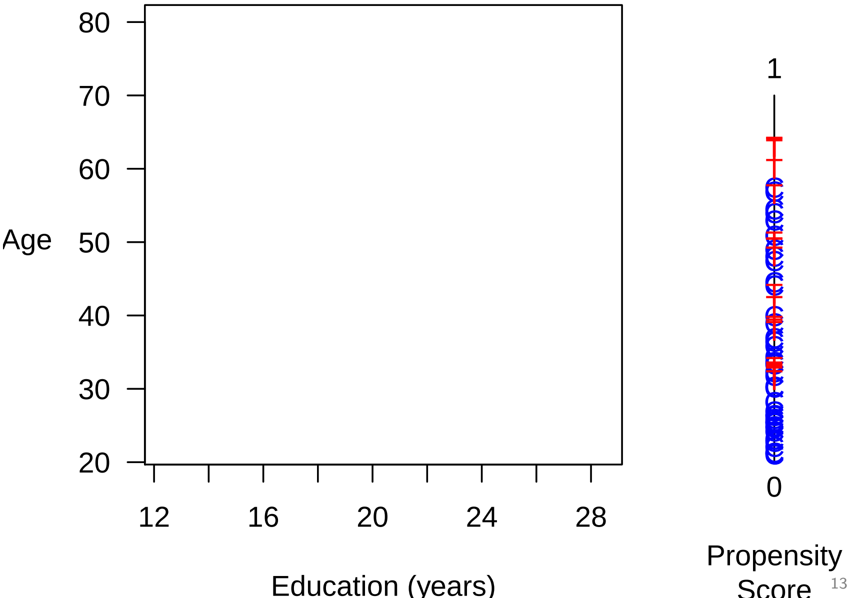

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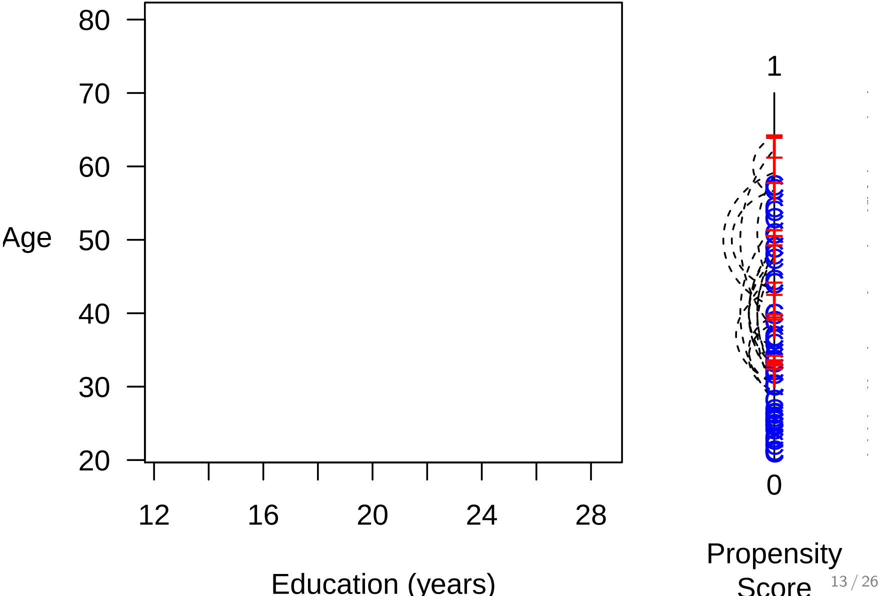

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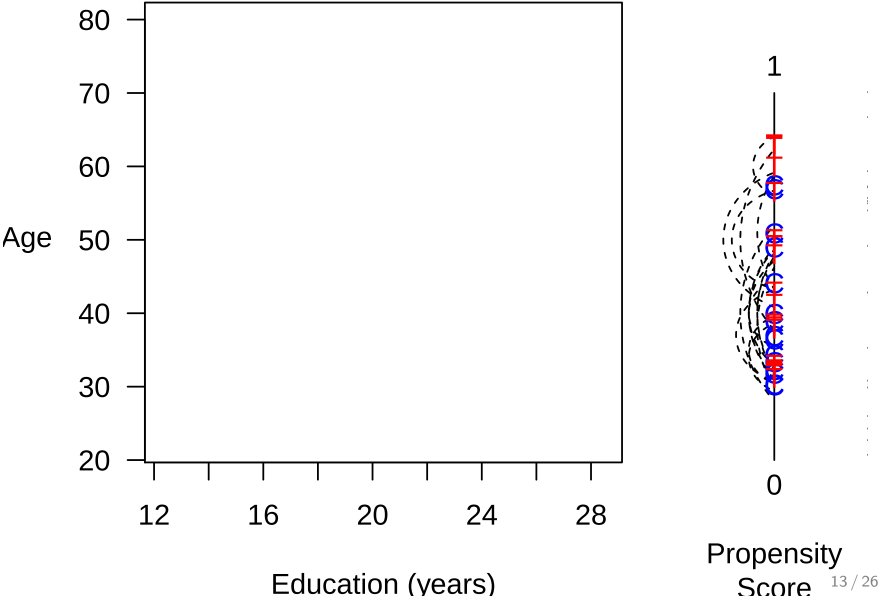

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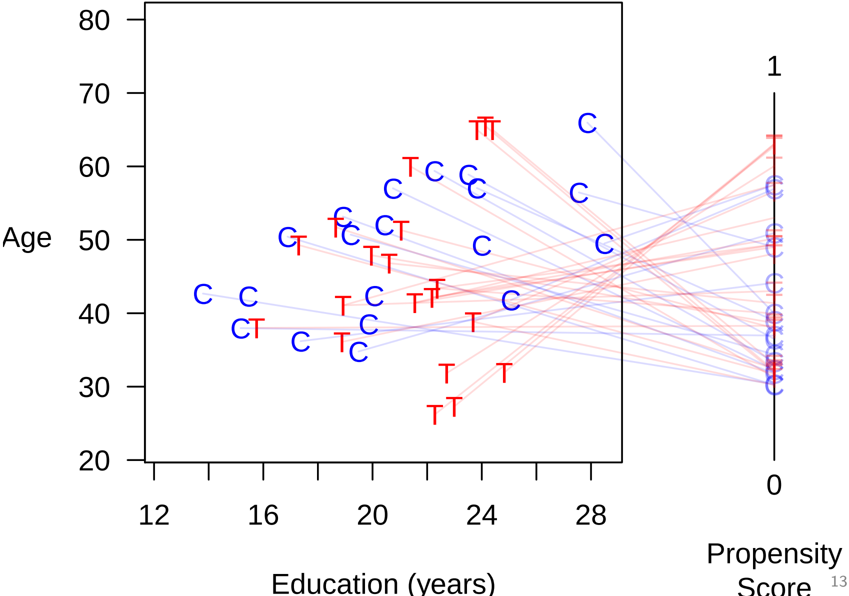

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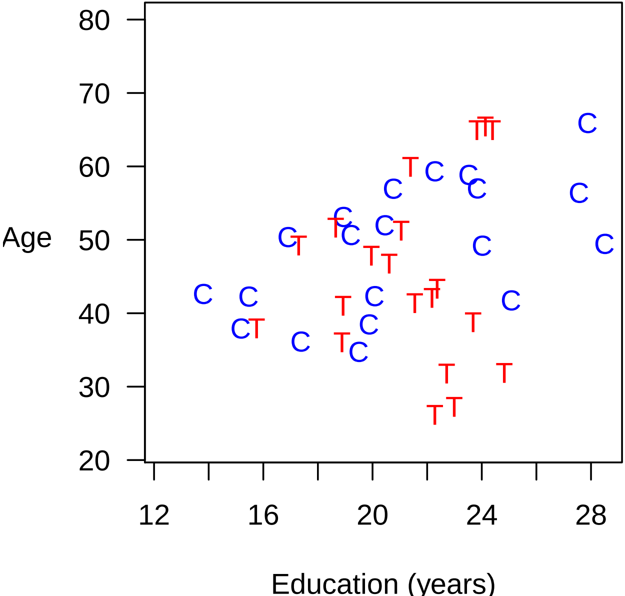

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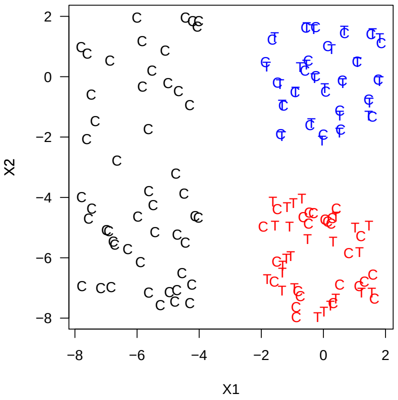

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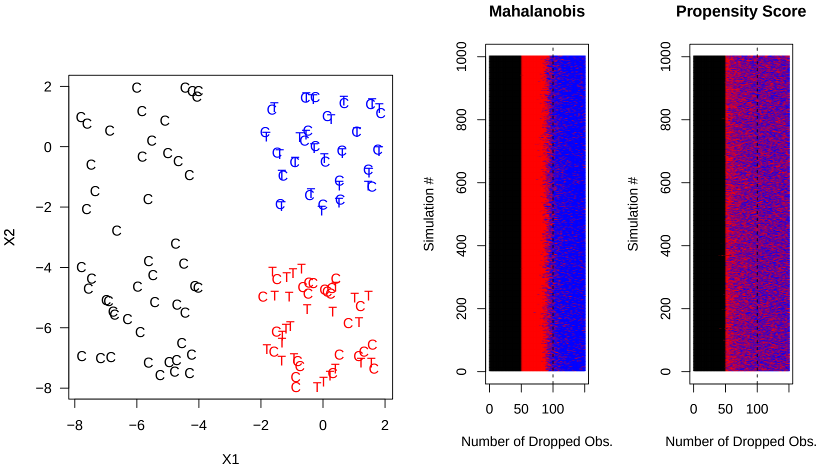
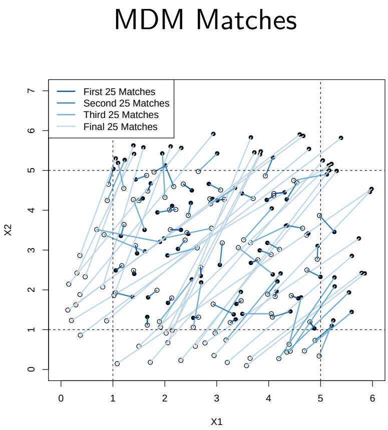
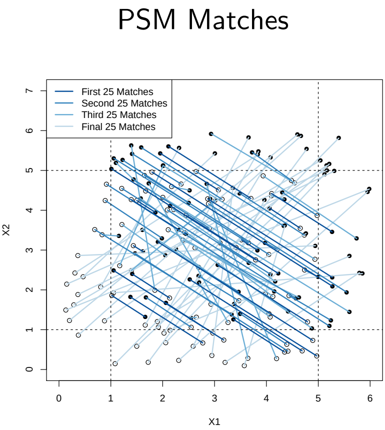


What Does PSM Match?

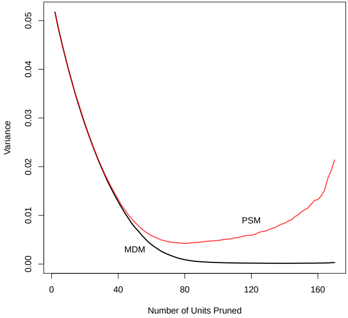




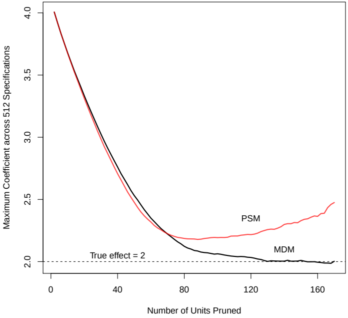
Controls: $X_1, X_2 \sim \text{Uniform}(0,5)$
Treateds: $X_1, X_2 \sim \text{Uniform}(1,6)$

PSM Increases Model Dependence & Bias

Model Dependence



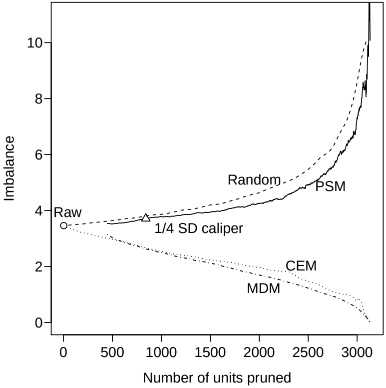
Bias



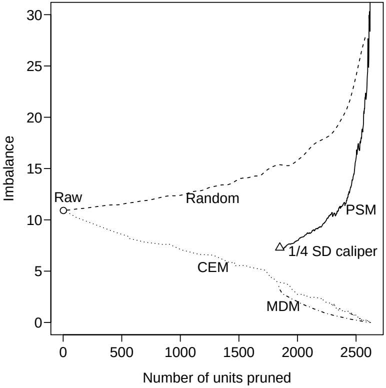
$$Y_i = 2T_i + X_{1i} + X_{2i} + \epsilon_i$$
$$\epsilon_i \sim N(0,1)$$

The Propensity Score Paradox

Finkle et al. (2012)



Nielsen et al. (2011)



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  - No cherry picking possible; you see everything optimal

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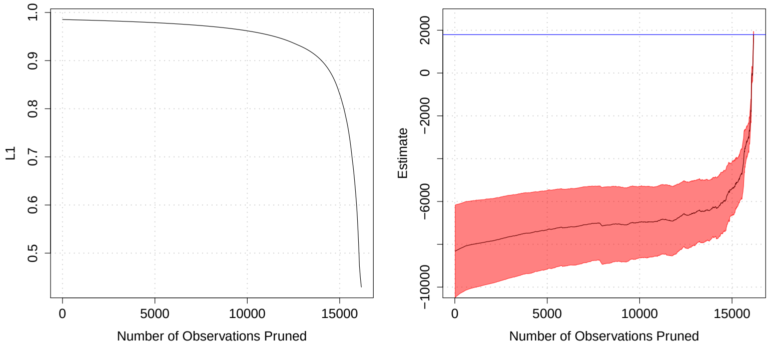
Job Training Data: Frontier and Causal Estimates



- 185 Ts; pruning most 16,252 Cs won't increase variance much
- Huge bias-variance trade-off after pruning most Cs
- Estimates converge to experiment after removing bias
- No mysteries: basis of inference clearly revealed

Constructing the FSATT Mahalanobis Frontier

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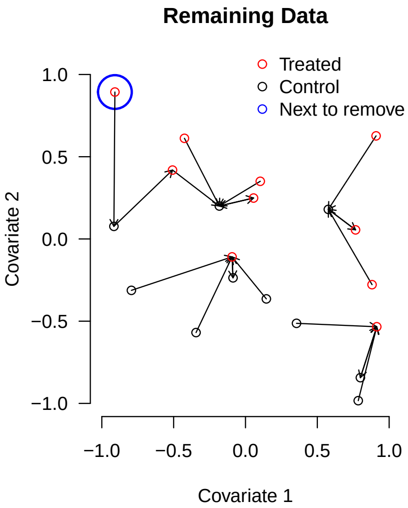

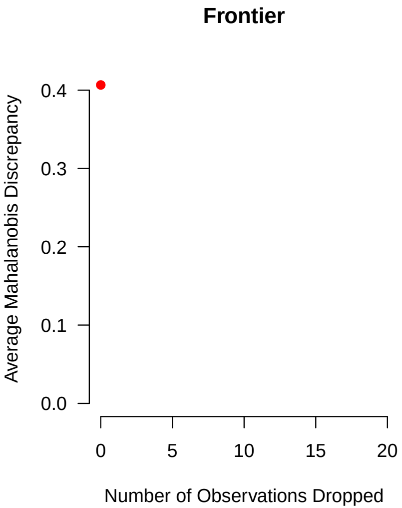

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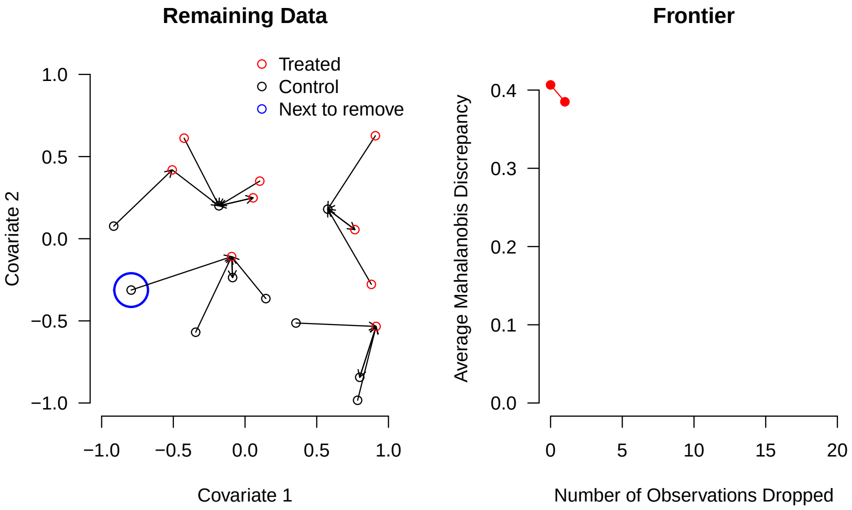

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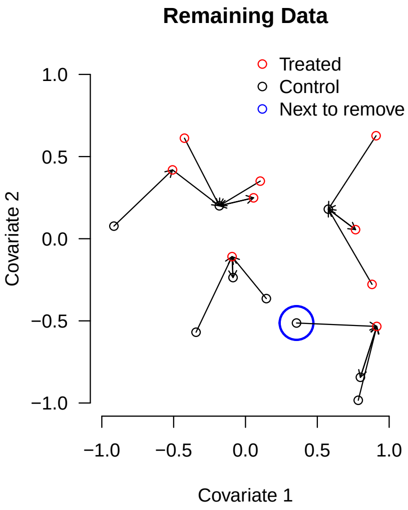

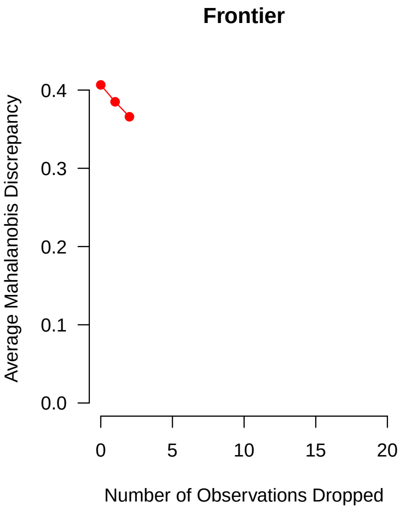

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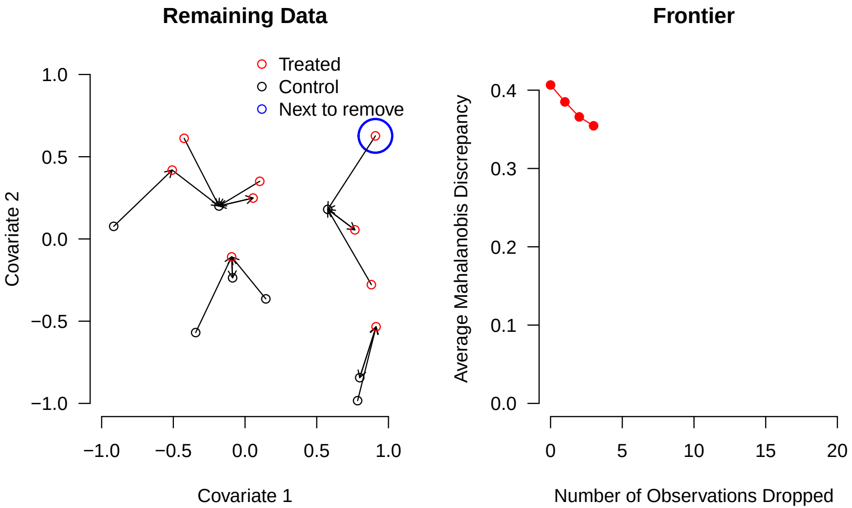

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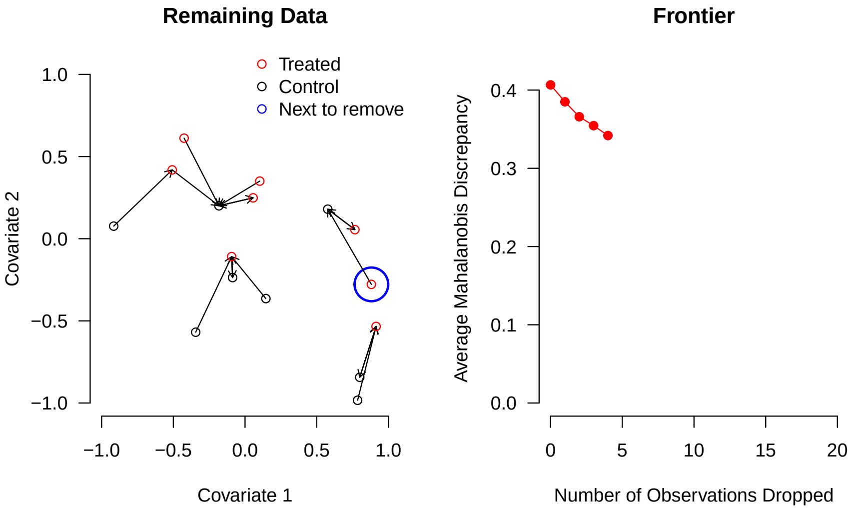

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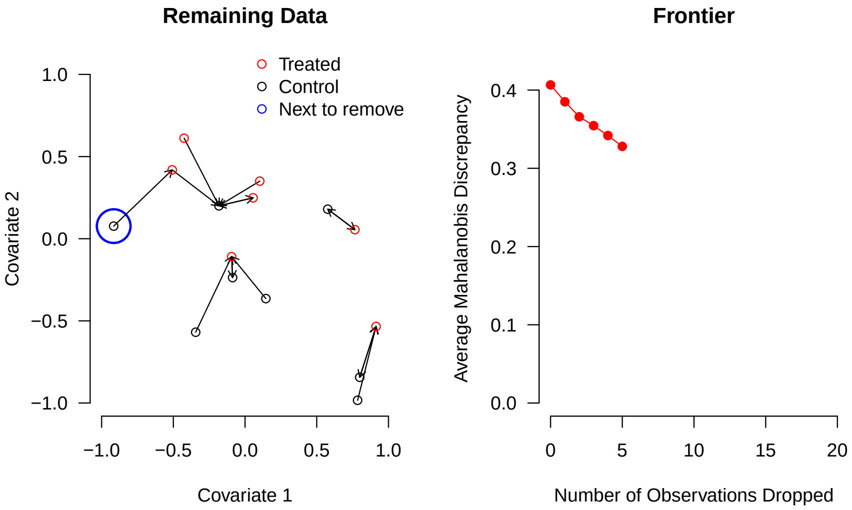

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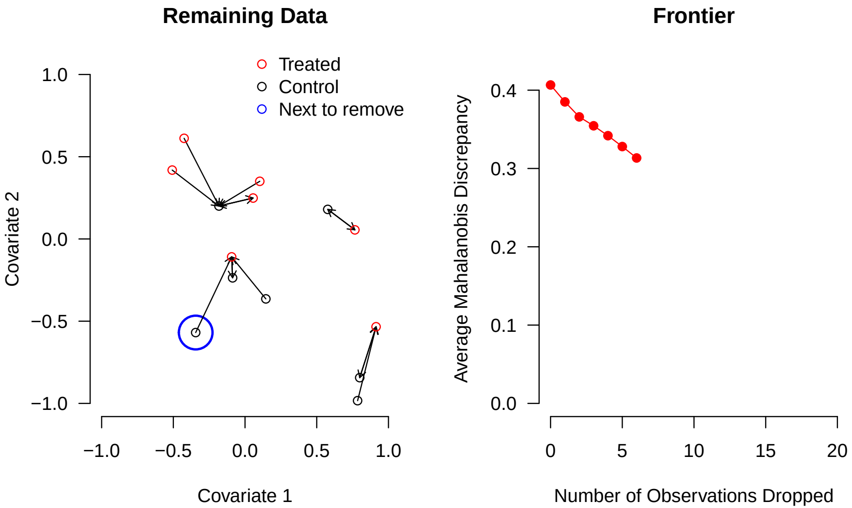

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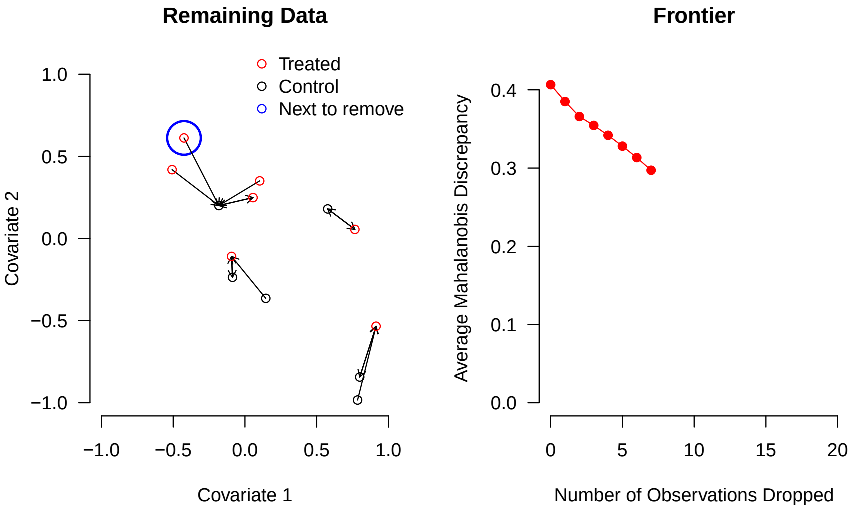

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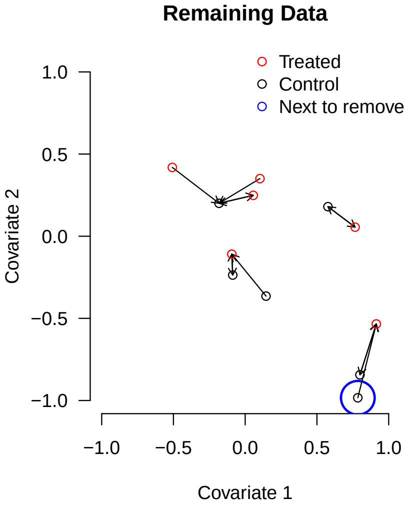

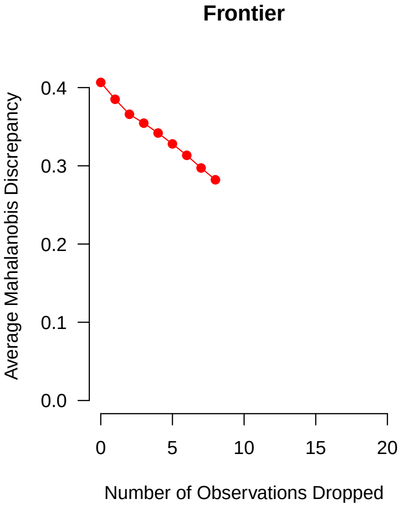

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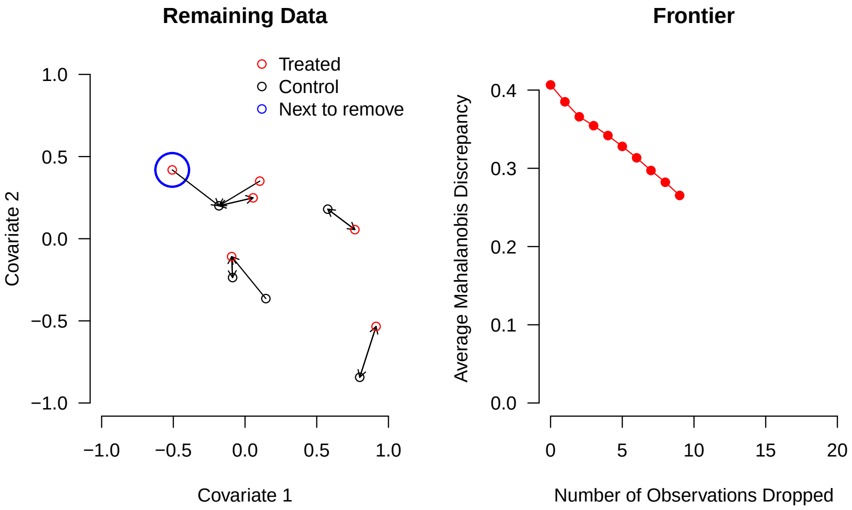

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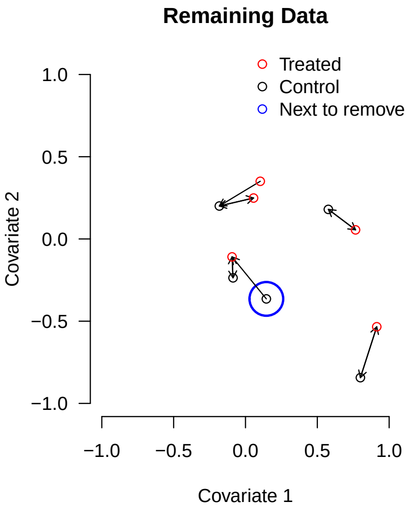

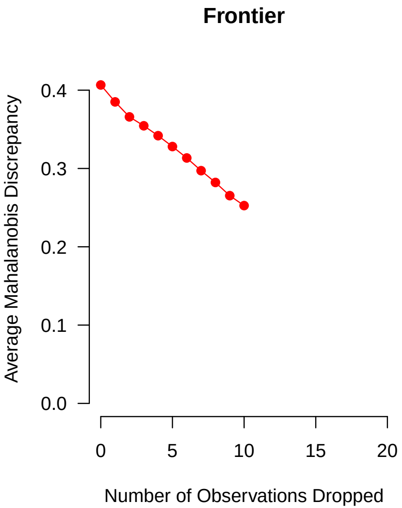

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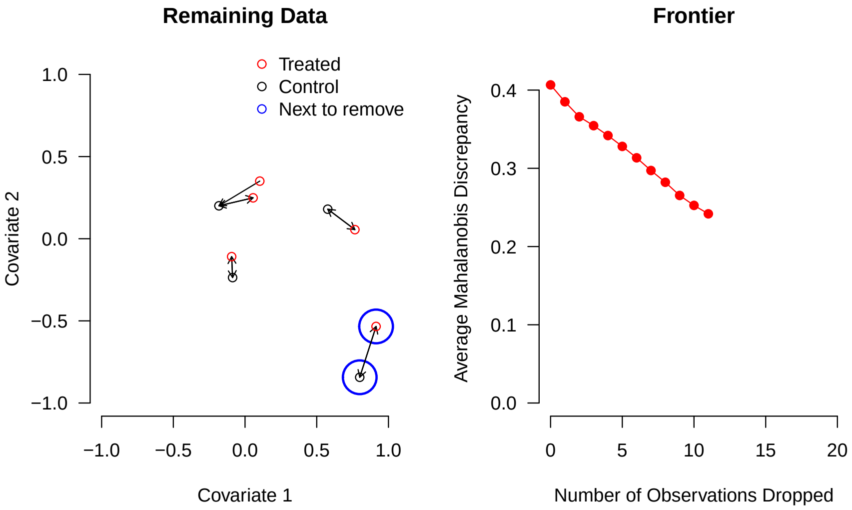

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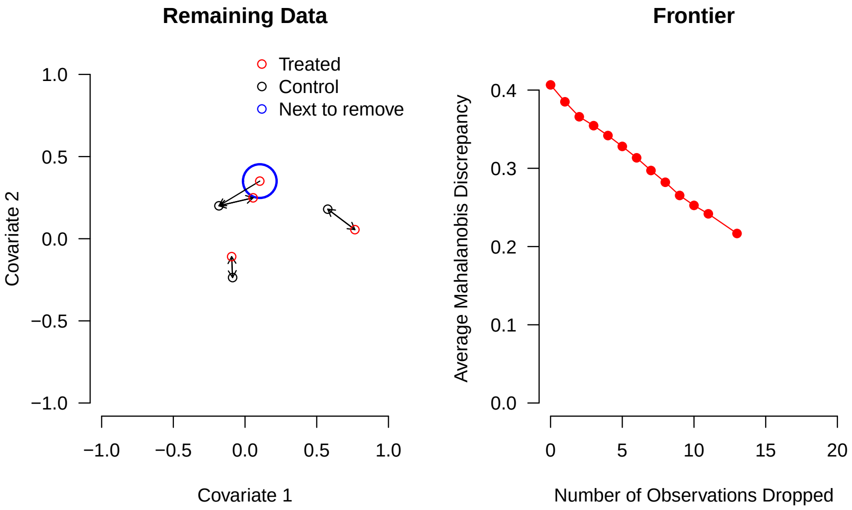

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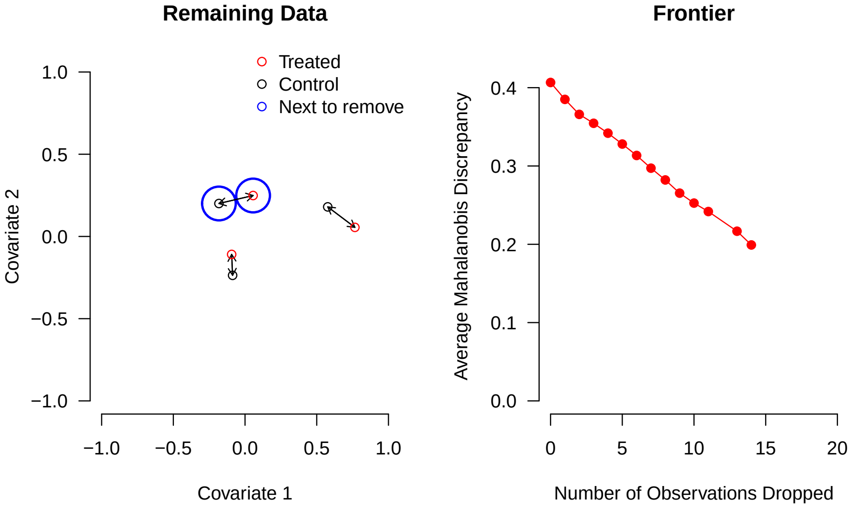

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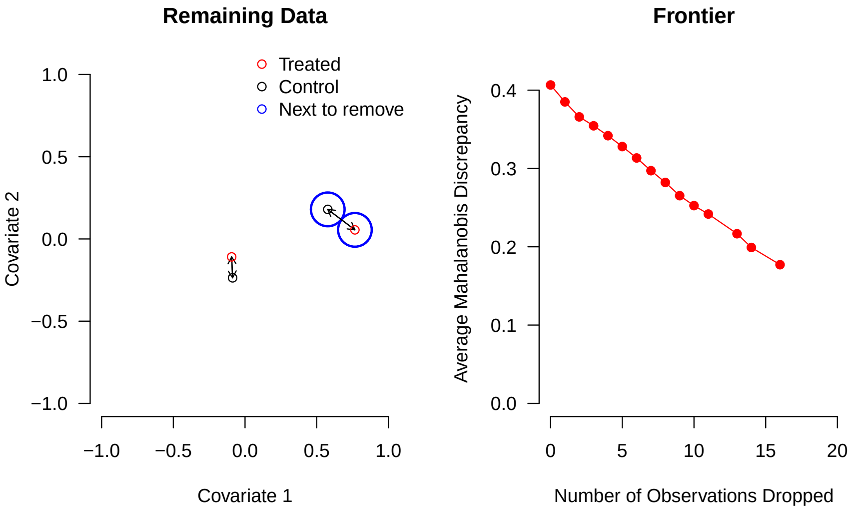

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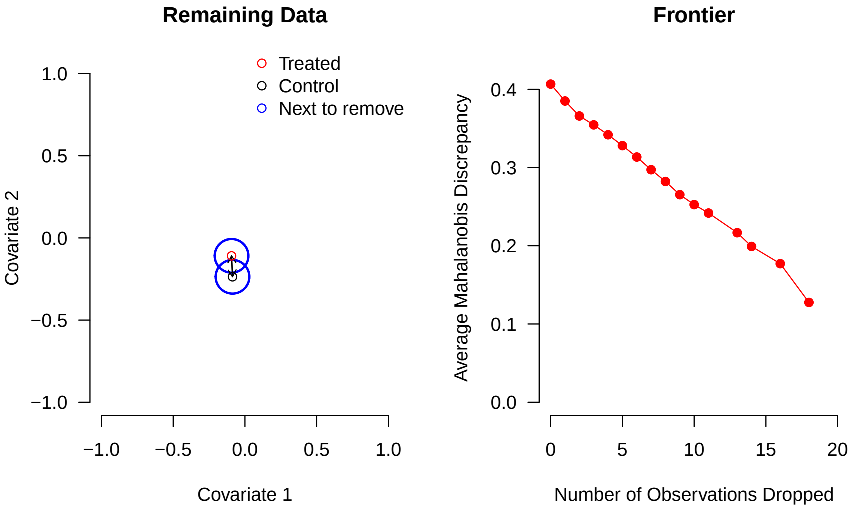

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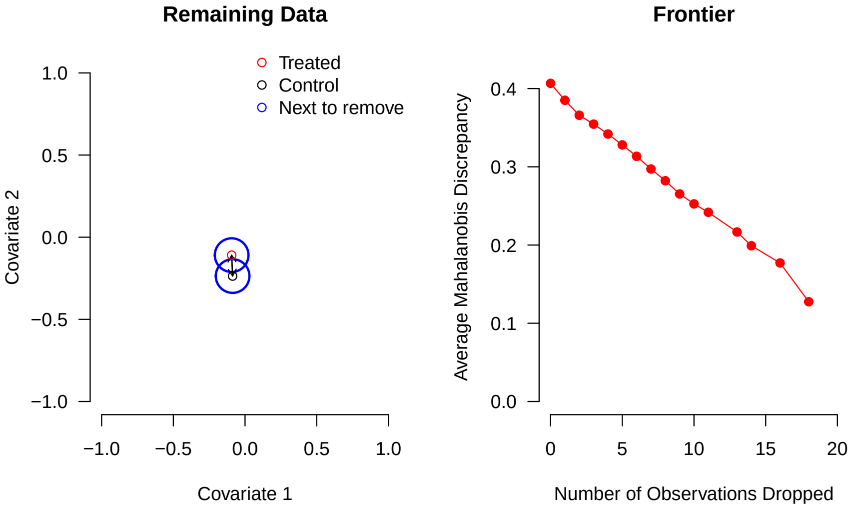

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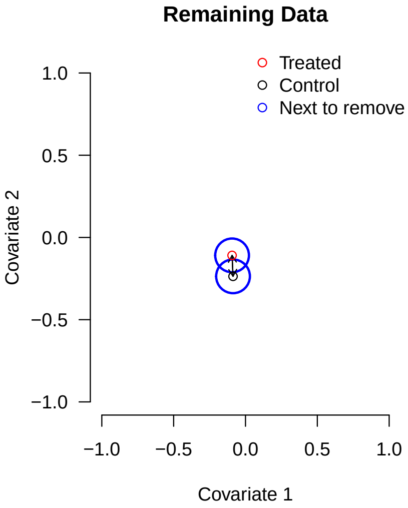

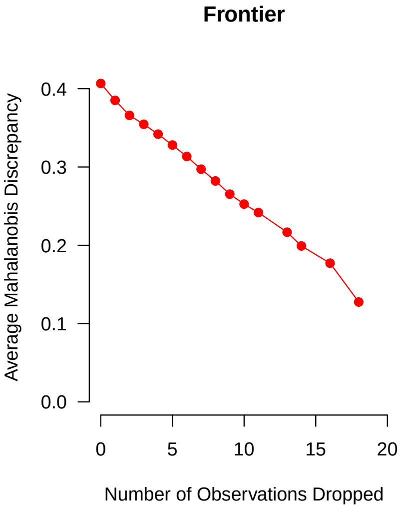

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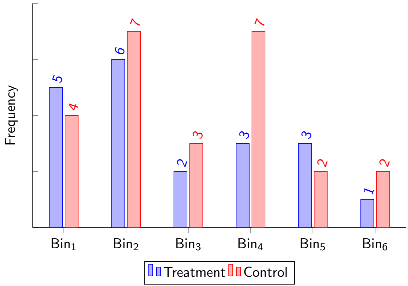
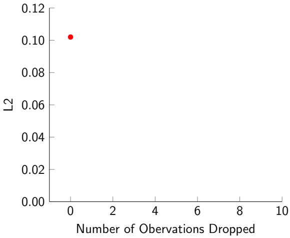
- Very fast; works with any continuous imbalance metric

Constructing the L1/L2 SATT Frontier





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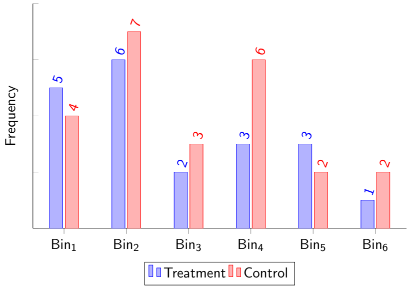

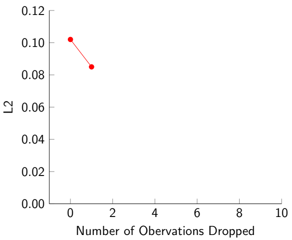

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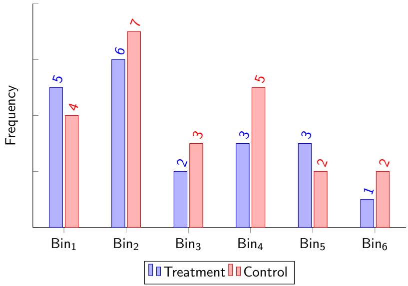

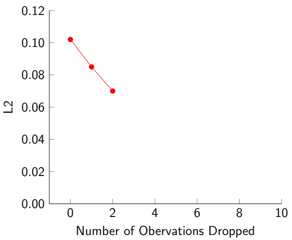

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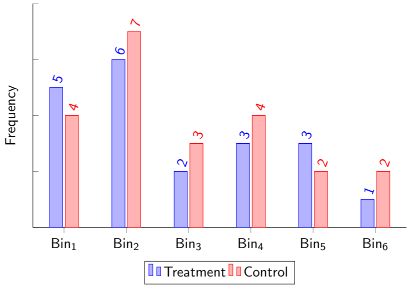

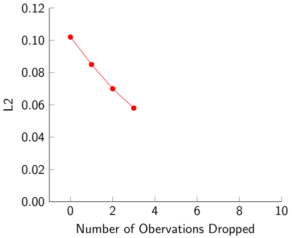

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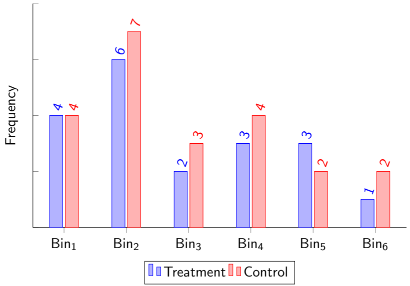

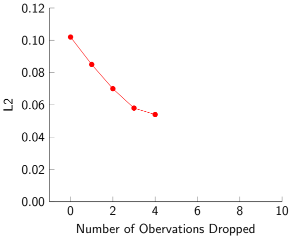

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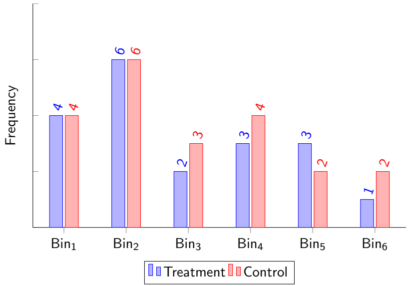

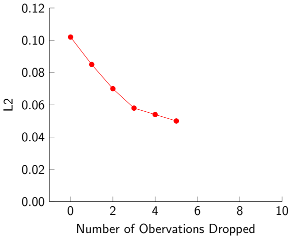

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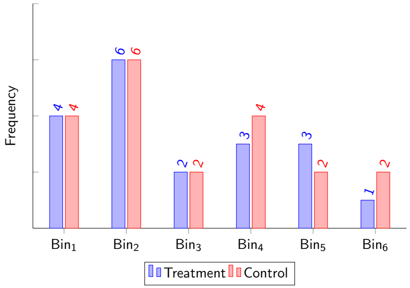

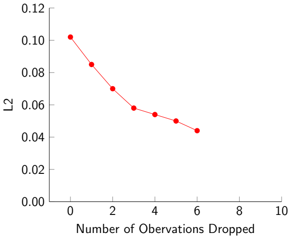

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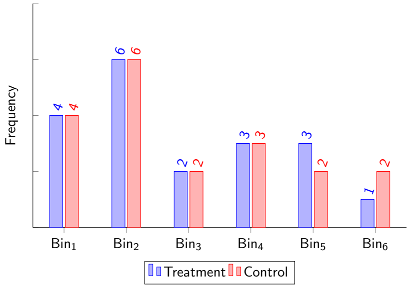

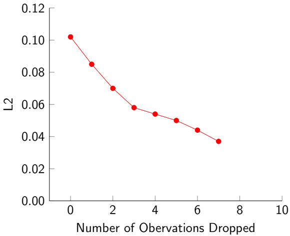

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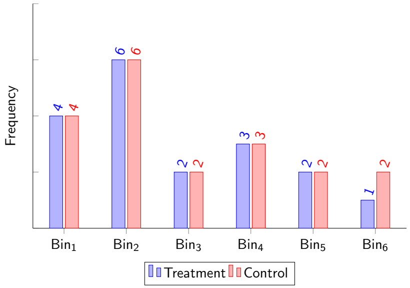

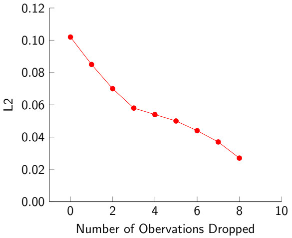

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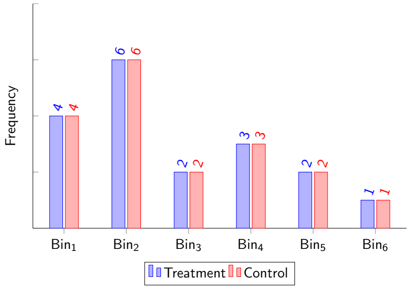

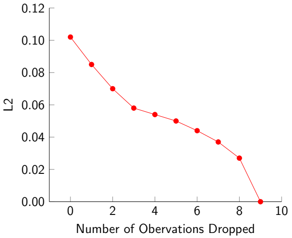

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- Warning: This figure omits some technical details too!

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For more information, papers, & software

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