Simplifying Matching Methods for Causal Inference

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(Talk at Stanford University, Department of Political Science, 1/14/2015)

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  - ⇝ "The Balance-Sample Size Frontier in Matching Methods for Causal Inference" (Gary King, Christopher Lucas and Richard Nielsen)

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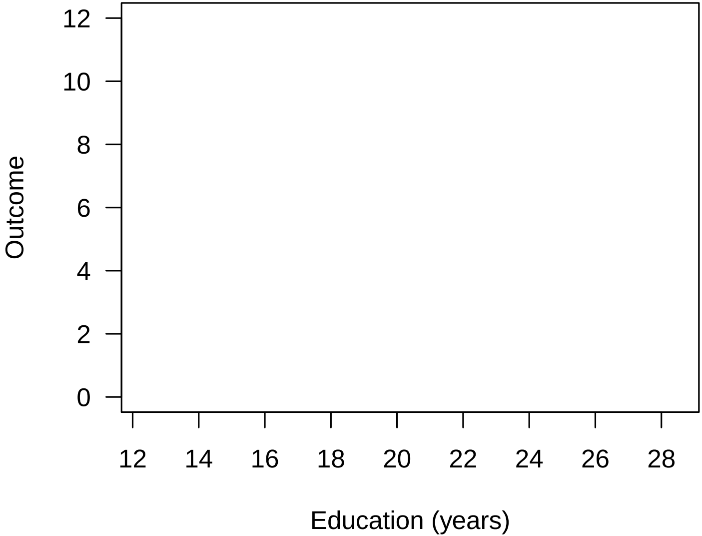

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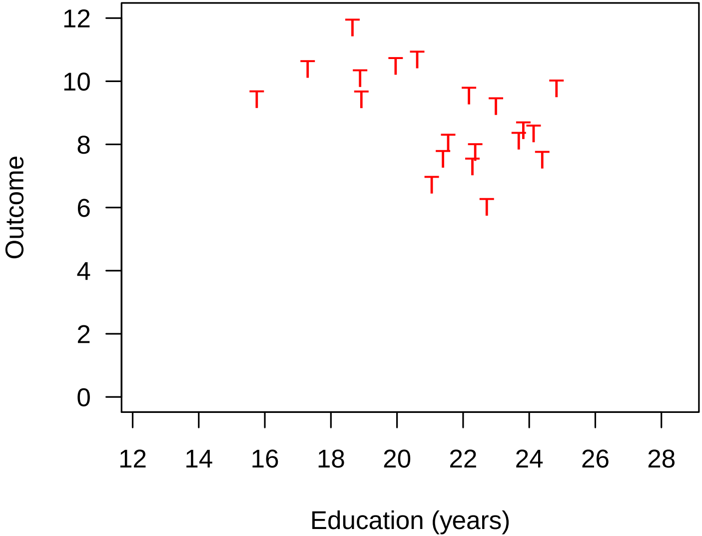

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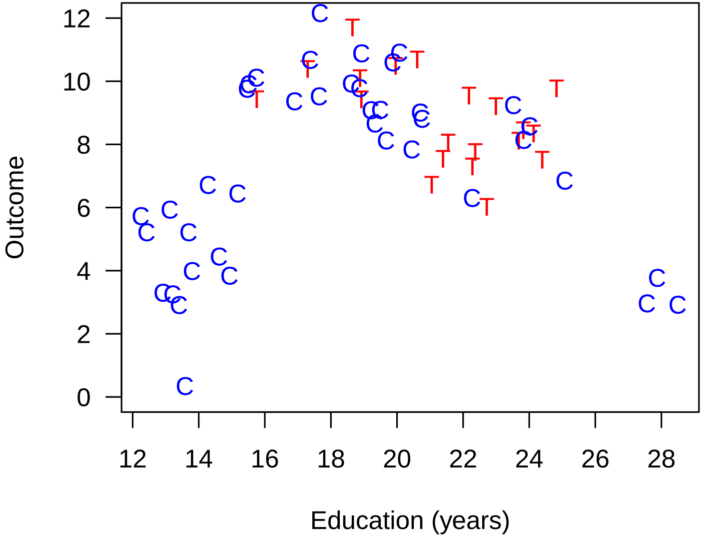

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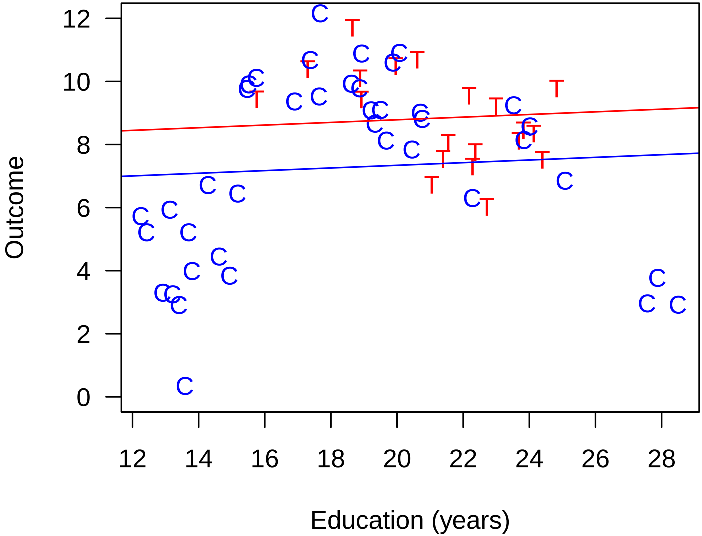

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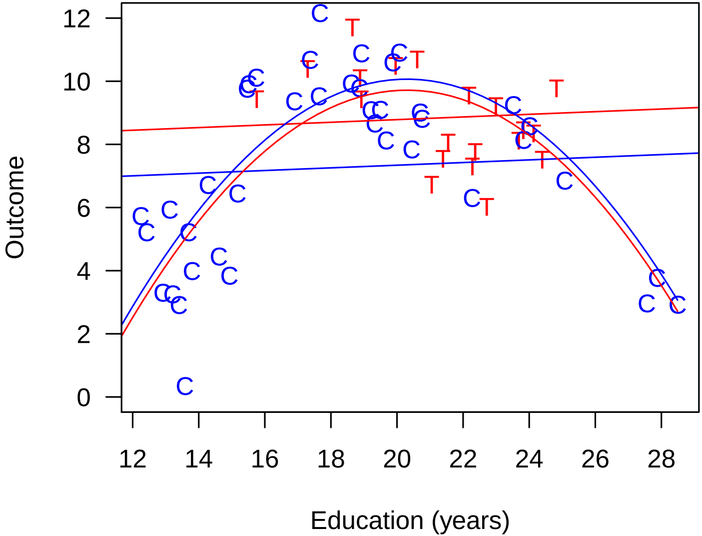

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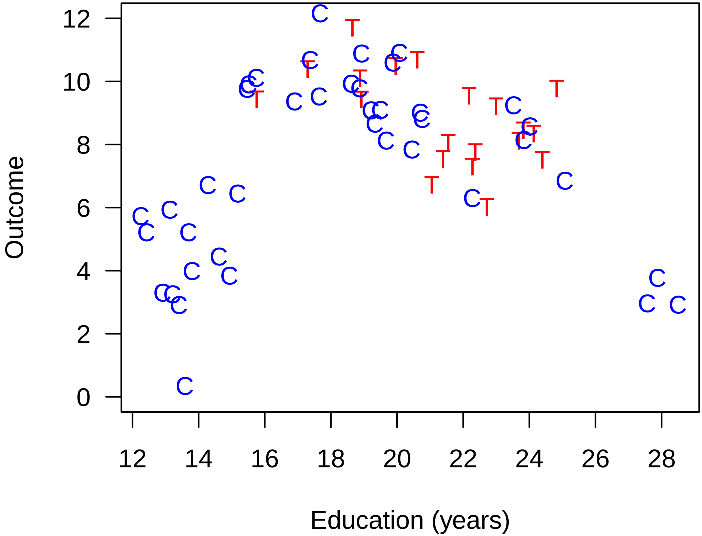

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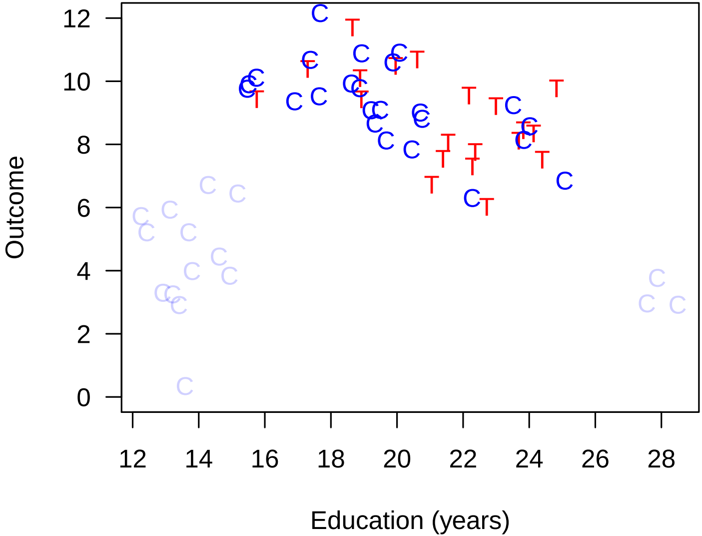

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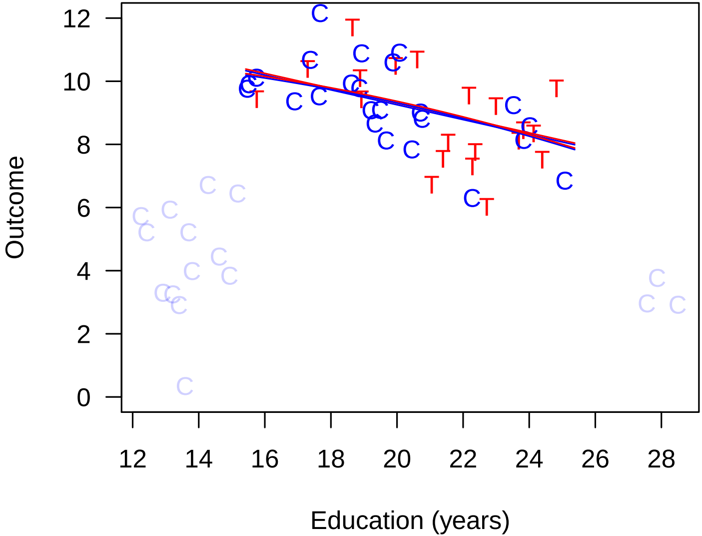

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- Big convenience: Follow preprocessing with whatever statistical method you'd have used without matching

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- Easy extensions for: multi-level, continuous, & mismeasured treatments; A too wide, n too small

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  - Other methods: fully blocked

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- Matching methods approximate which experiment?
  - PSM: complete randomization
  - Other methods: fully blocked
- $\Longrightarrow$ As we show, other methods usually dominate PSM

Approximating Randomized Experiments

- Types of experiments:
  1. Compete Randomization: Treatment assignment by coin flips
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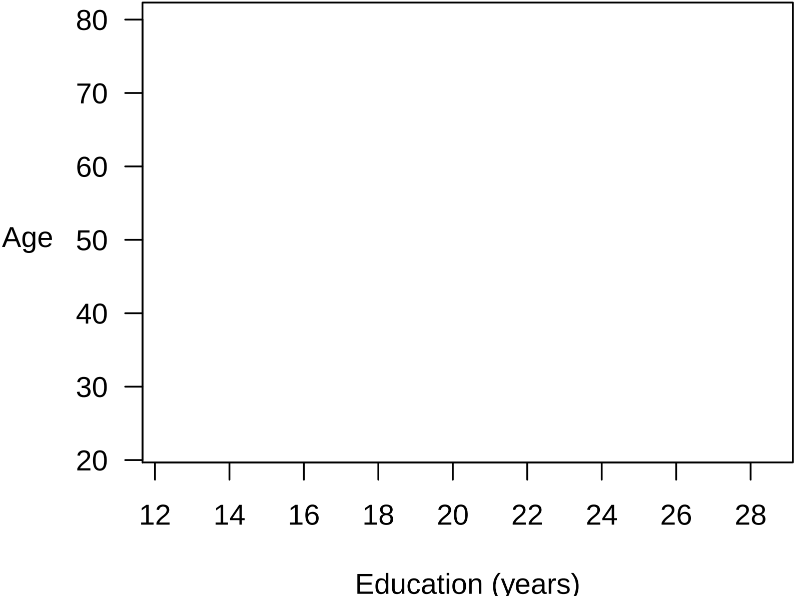

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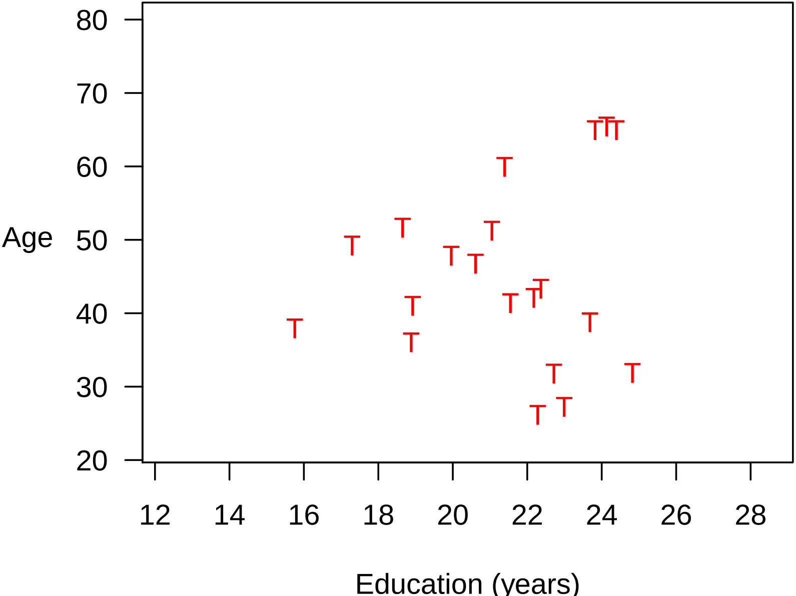

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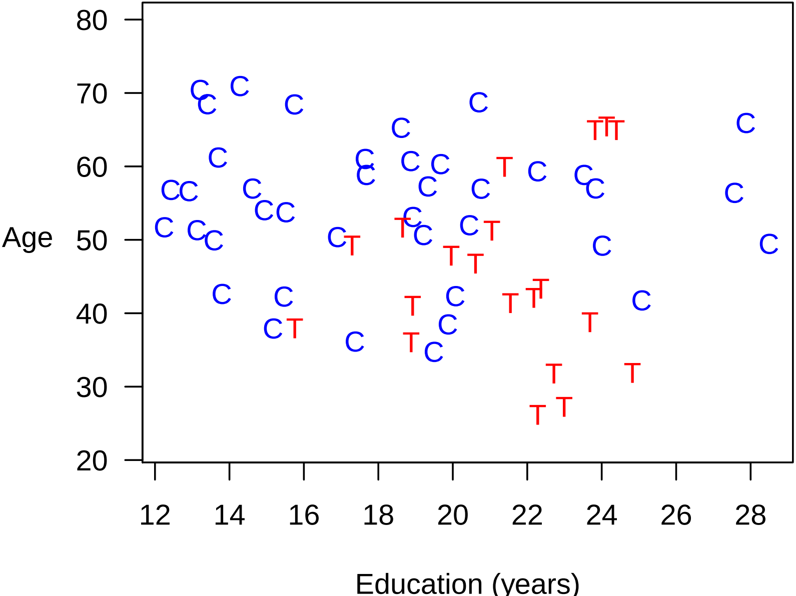

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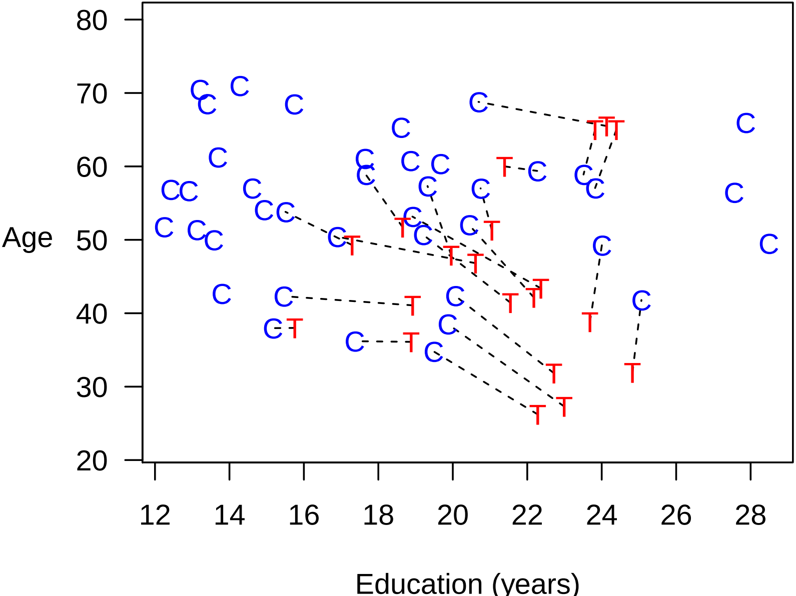

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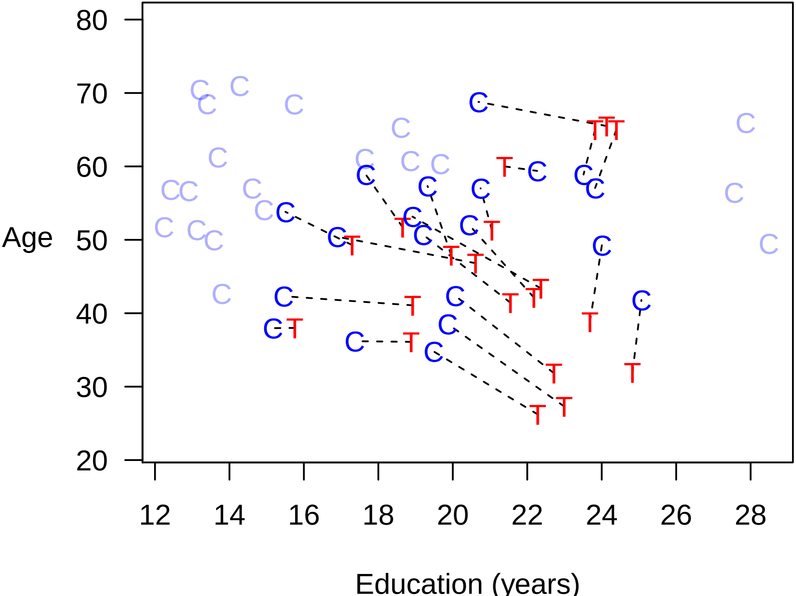

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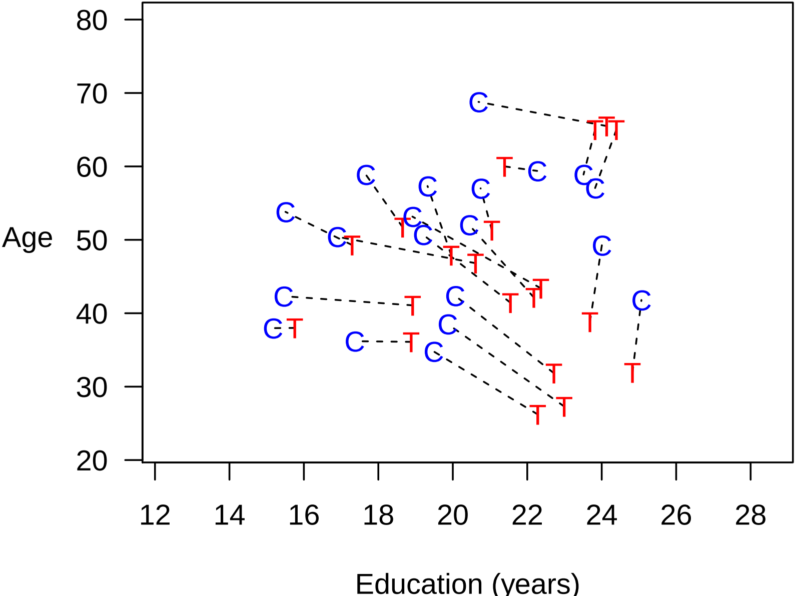

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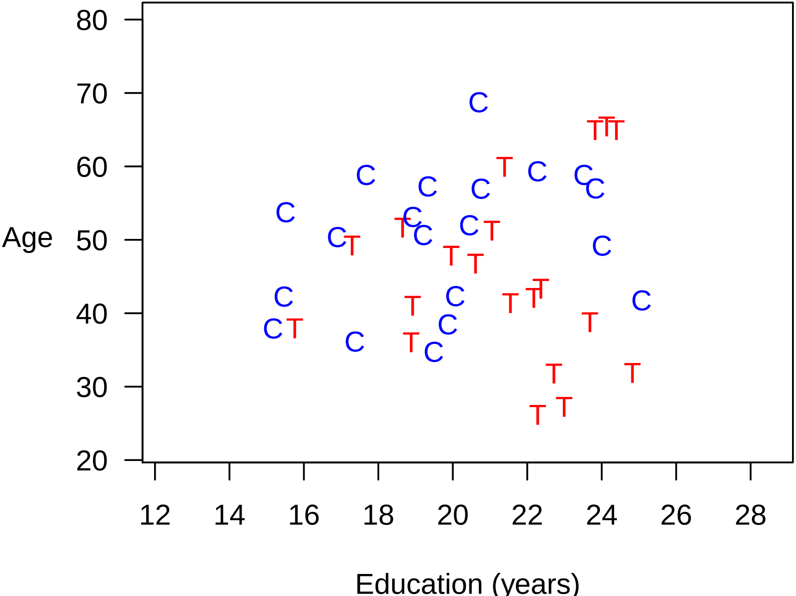

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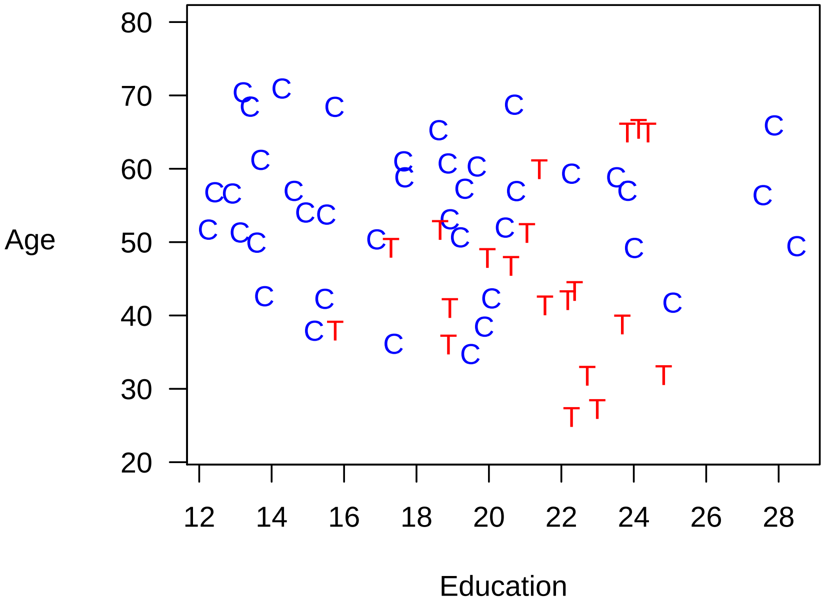

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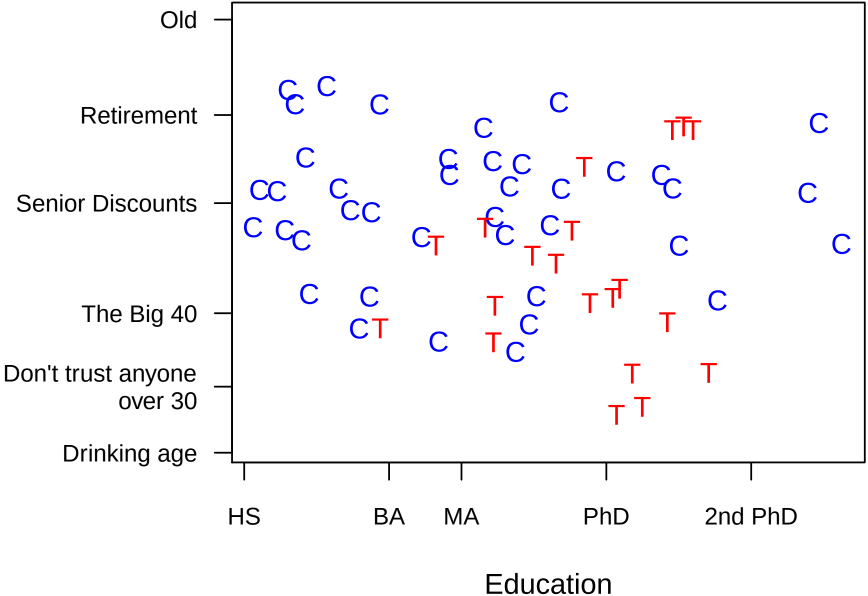

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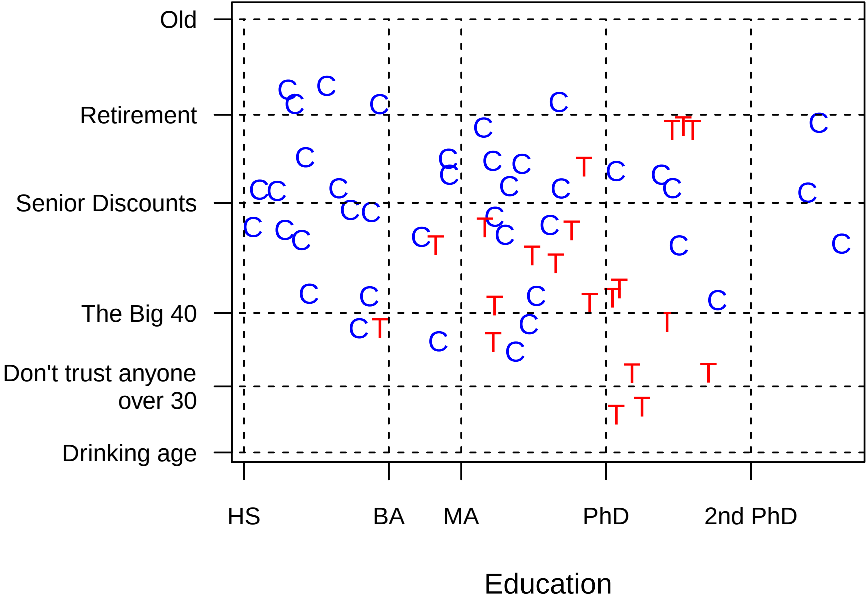

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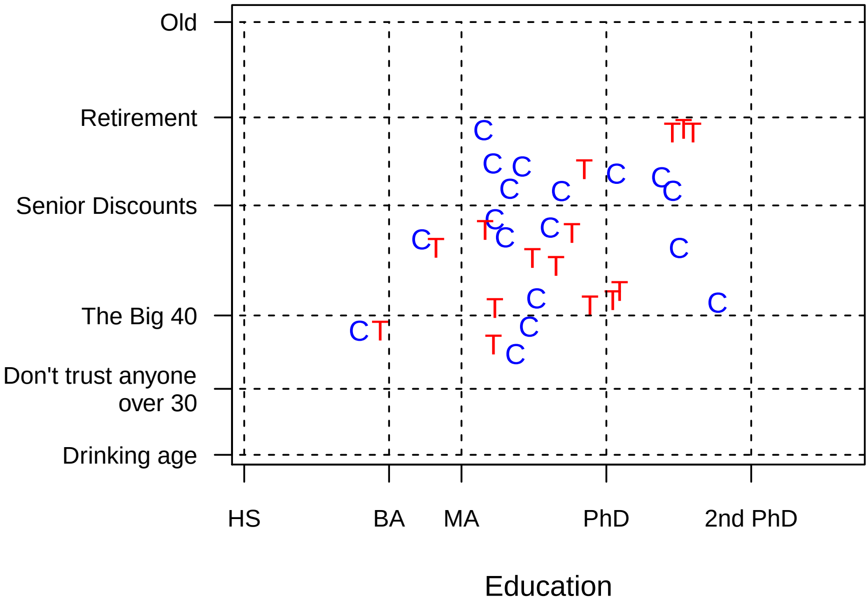

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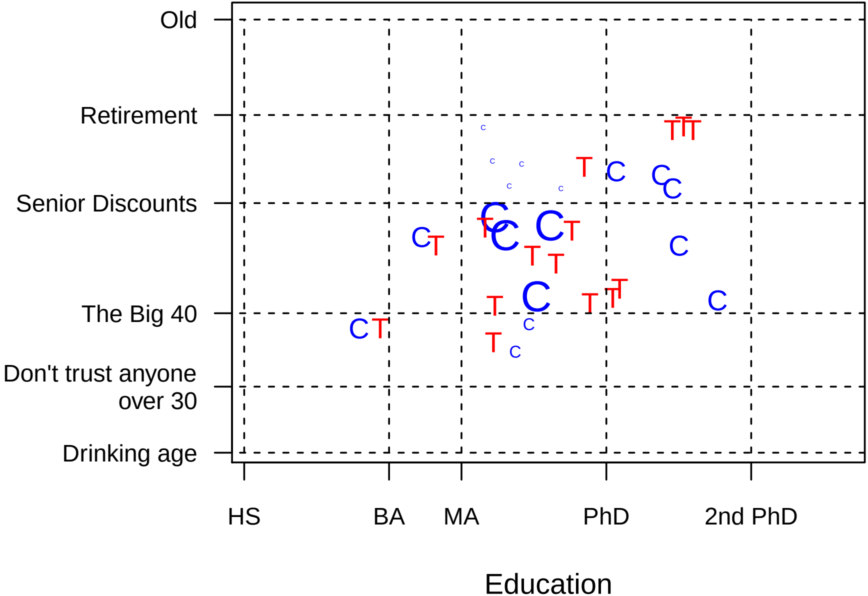

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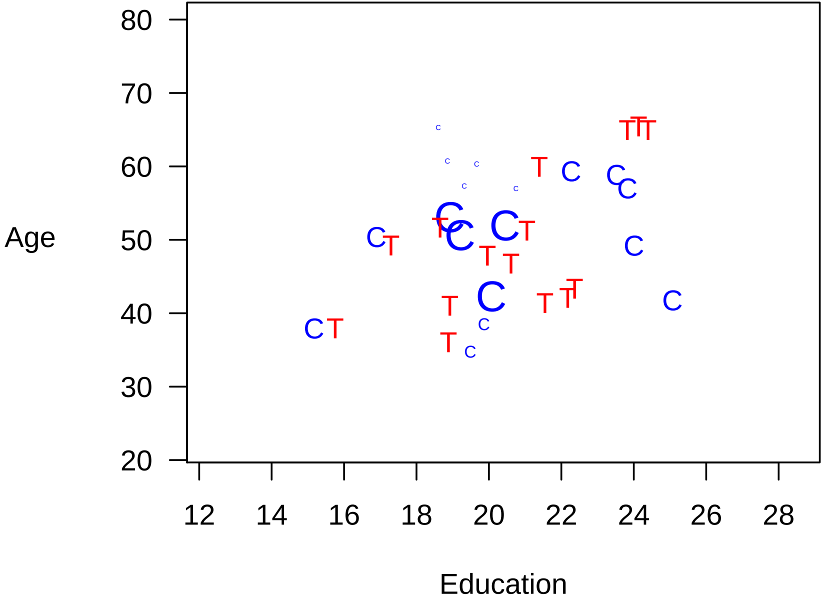

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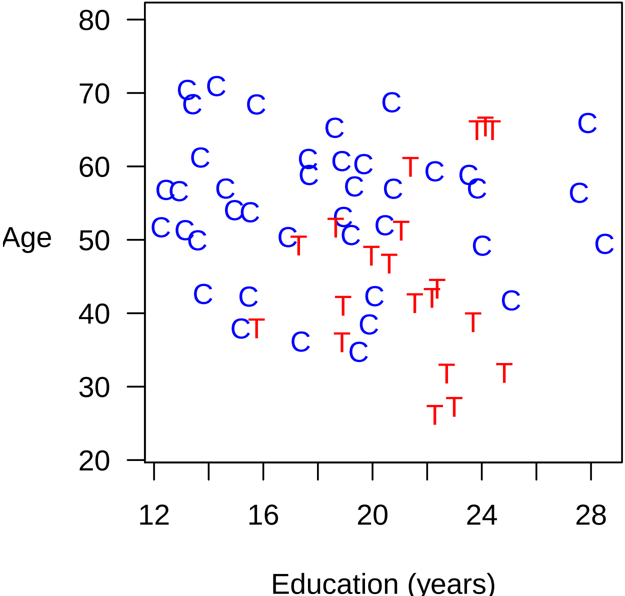

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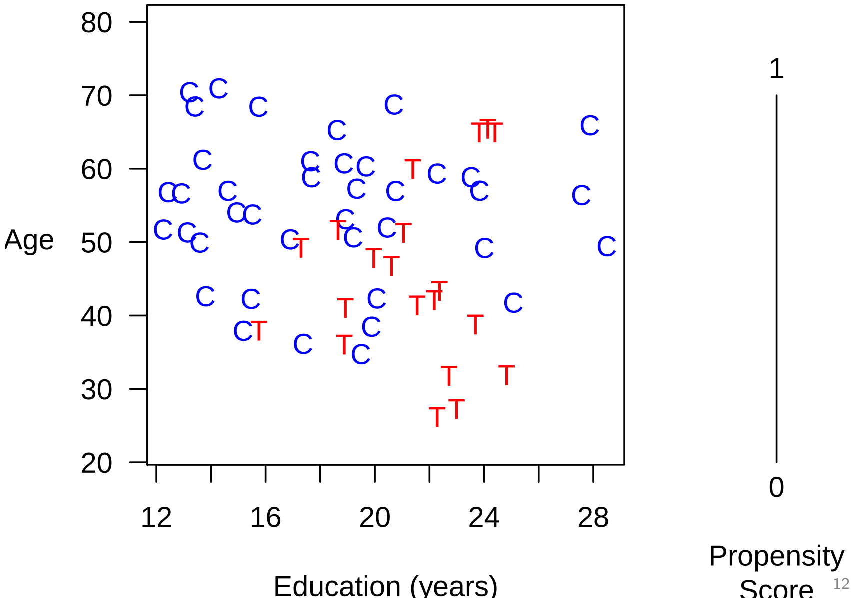

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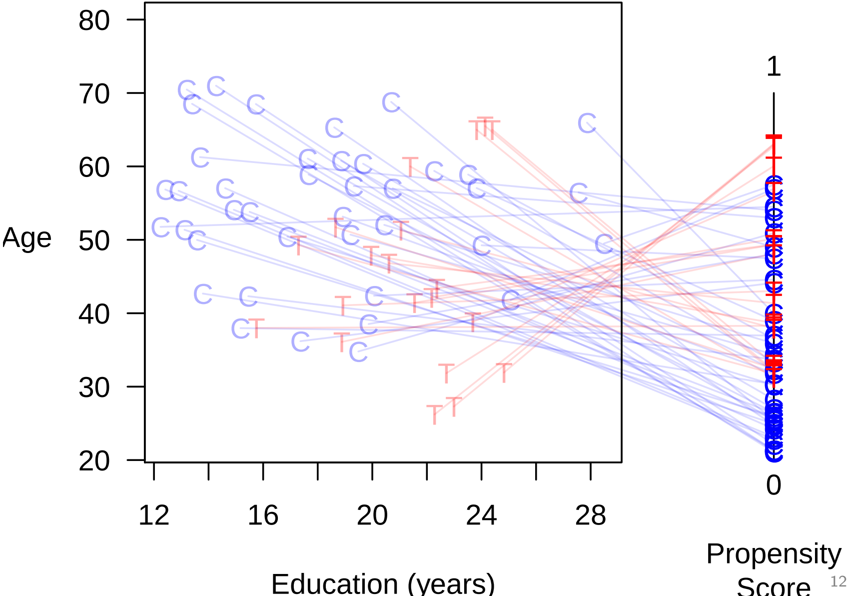

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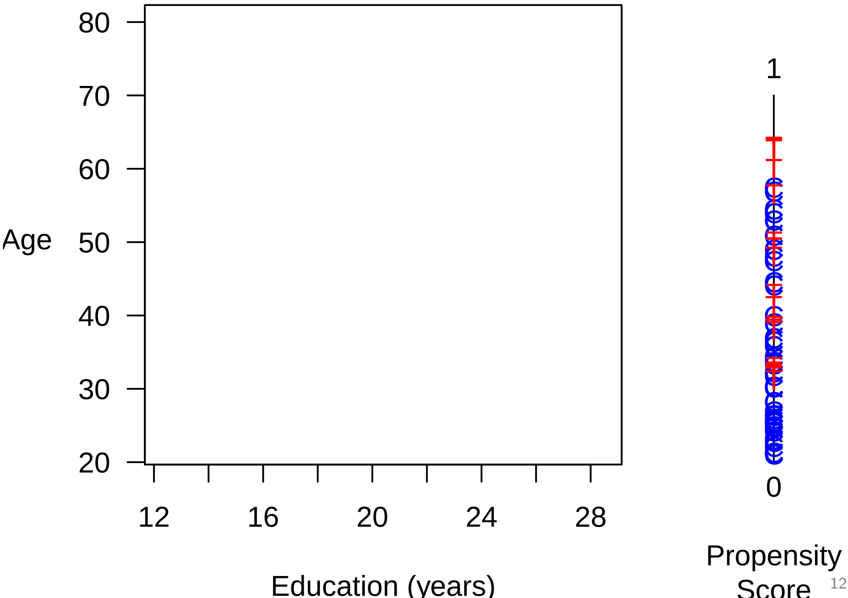

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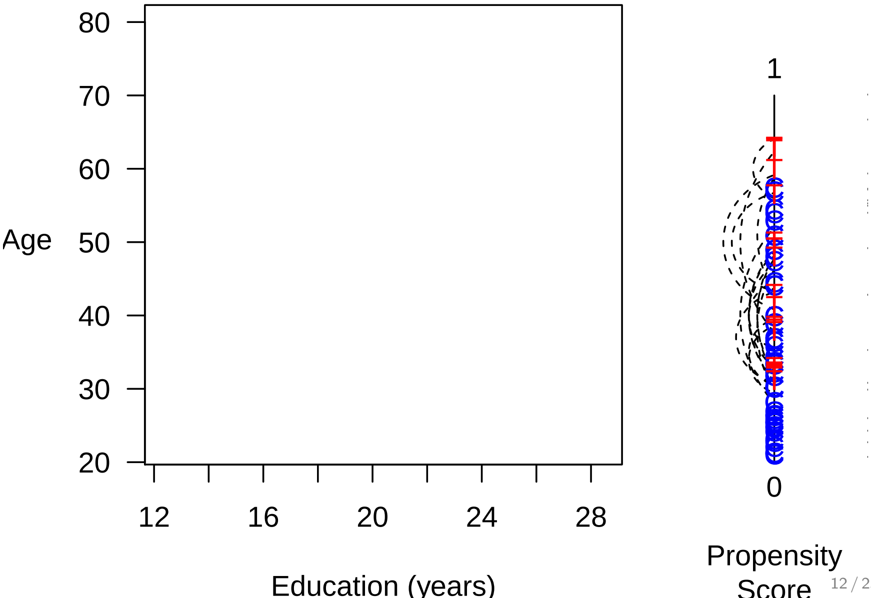

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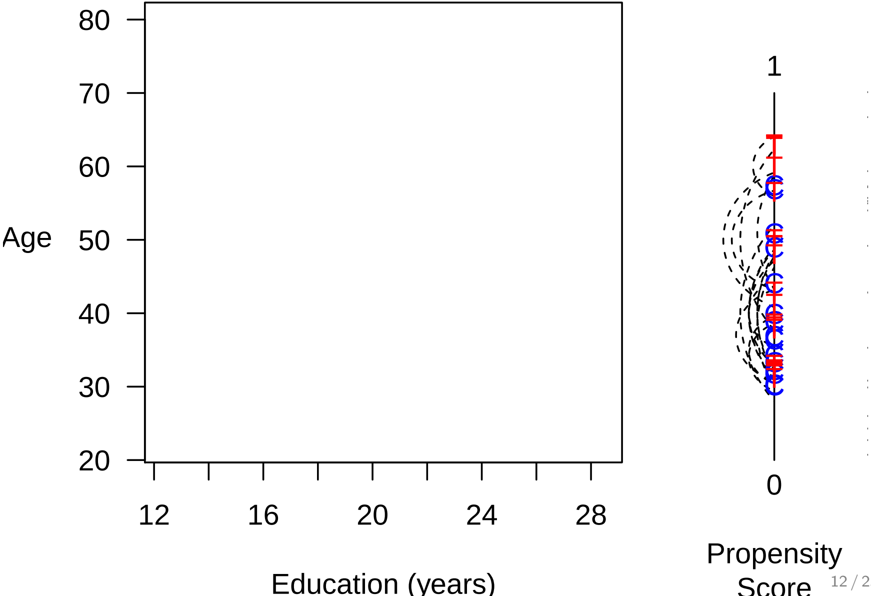

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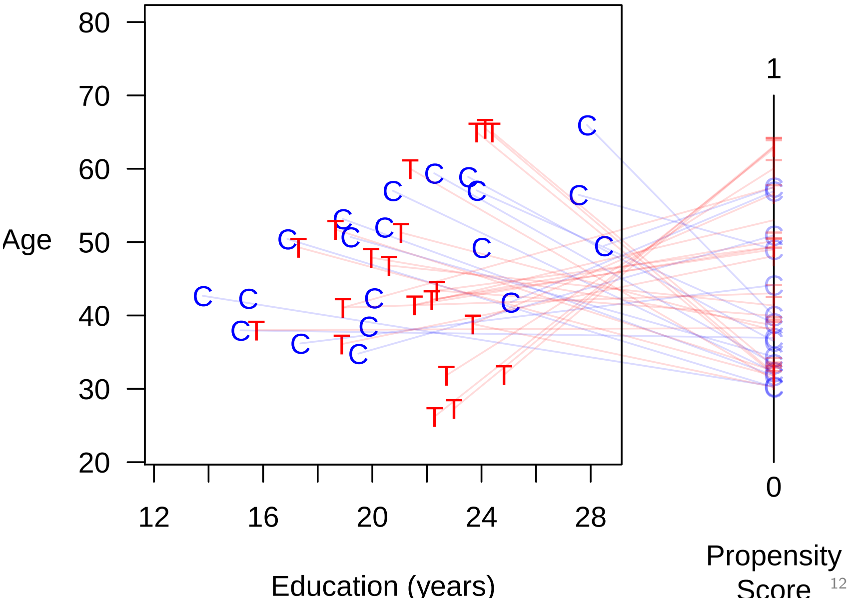

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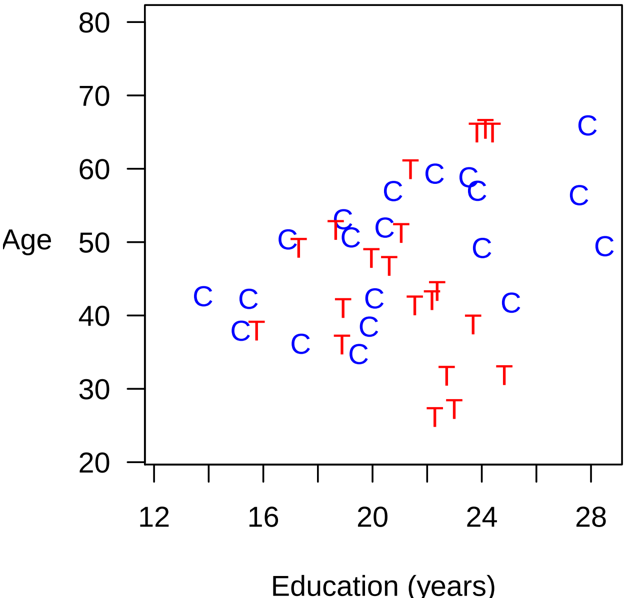

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  - The Reality: the paradox is worse with more covariates

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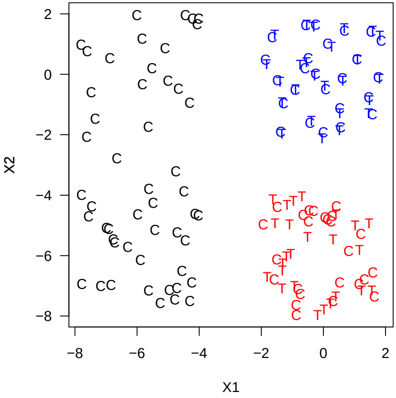

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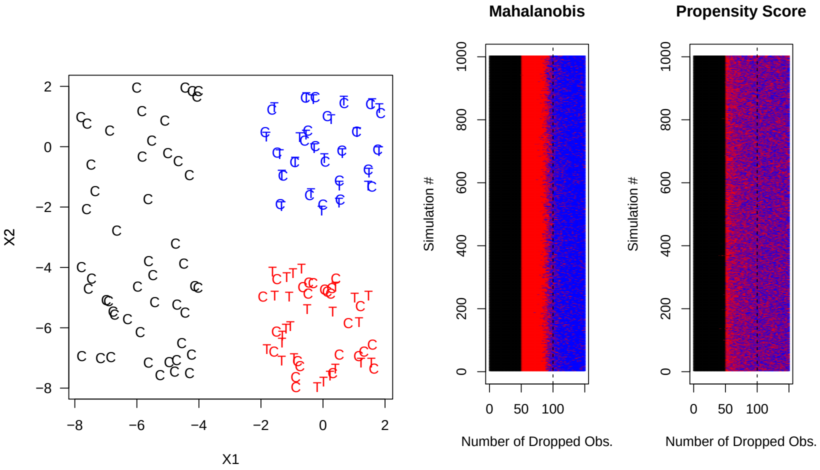
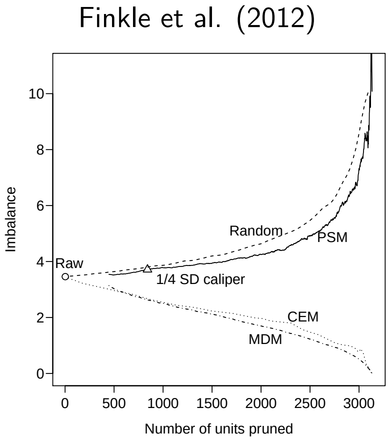
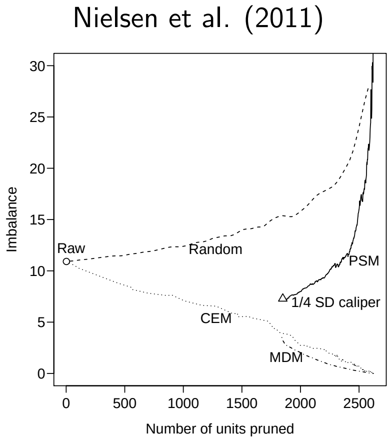
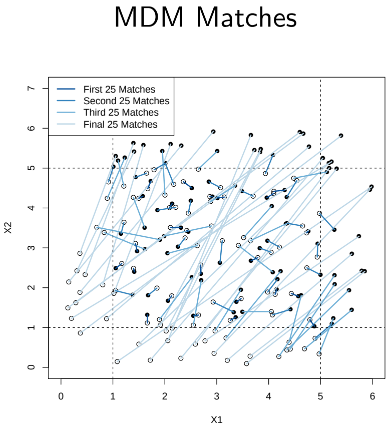


The Propensity Score Paradox



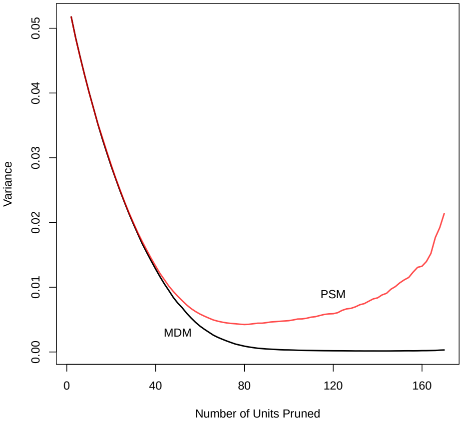


What Does PSM Match?



Controls: $X_1, X_2 \sim \text{Uniform}(0,5)$
Treateds: $X_1, X_2 \sim \text{Uniform}(1,6)$

PSM Increases Model Dependence



$$Y_i = 2T_i + X_{1i} + X_{2i} + \epsilon_i$$
$$\epsilon_i \sim N(0,1)$$

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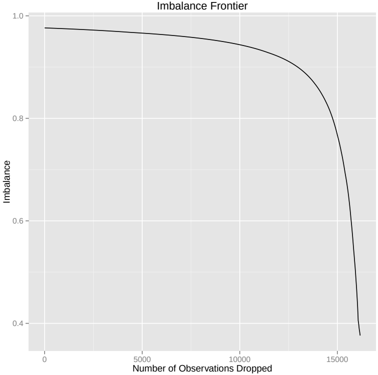
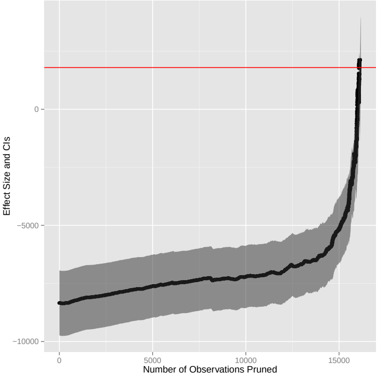
Job Training Data: Frontier and Causal Estimates





- 185 Ts; pruning most 16,252 Cs won't increase variance much
- Huge bias-variance trade-off after pruning most Cs
- Estimates converge to experiment after removing bias
- No mysteries: basis of inference clearly revealed

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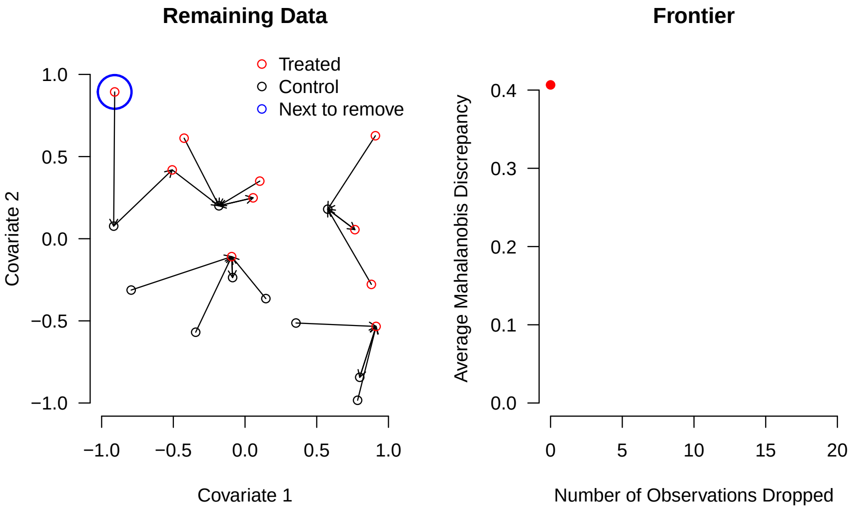

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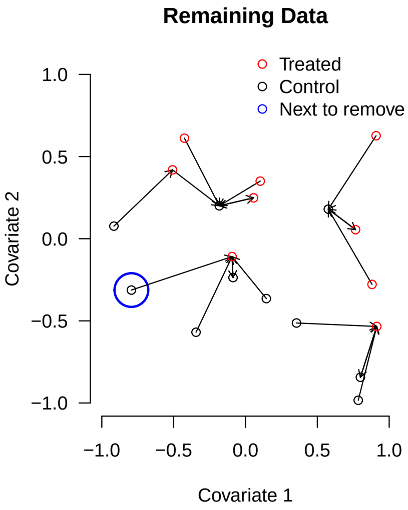

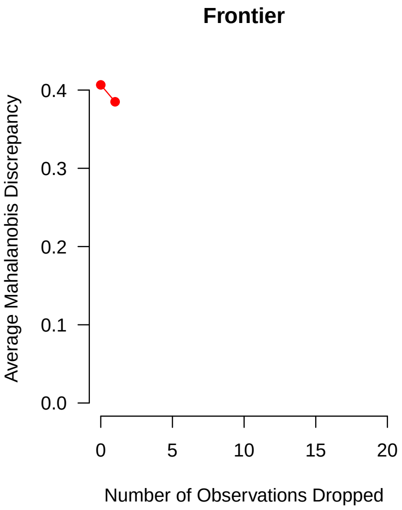

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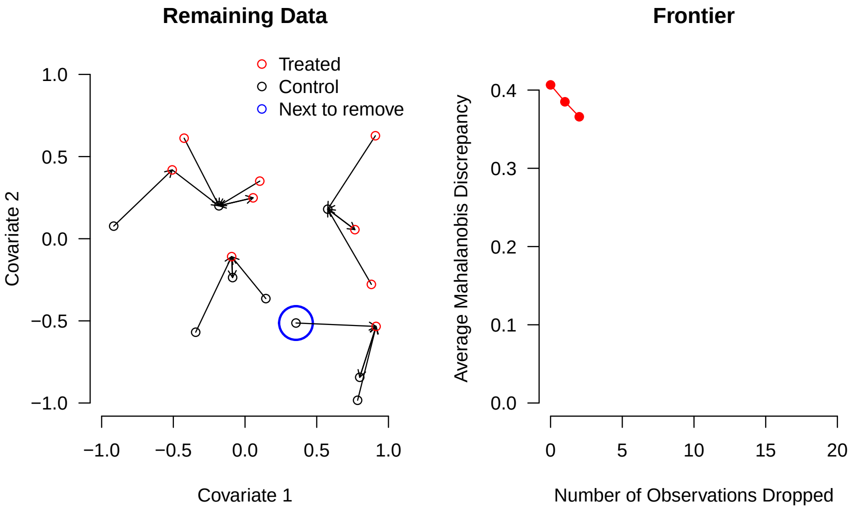

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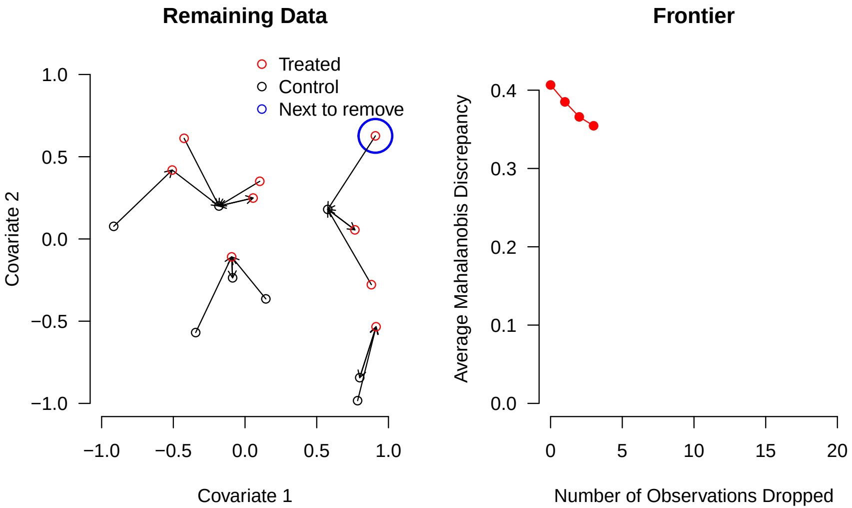

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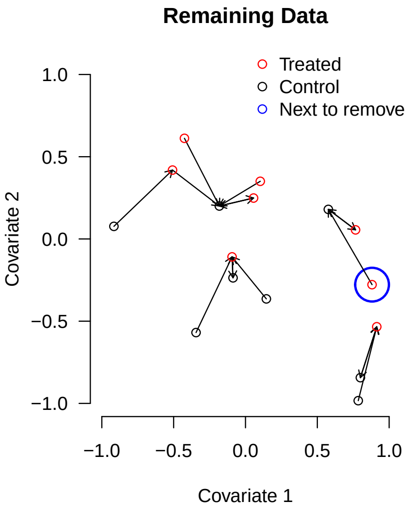

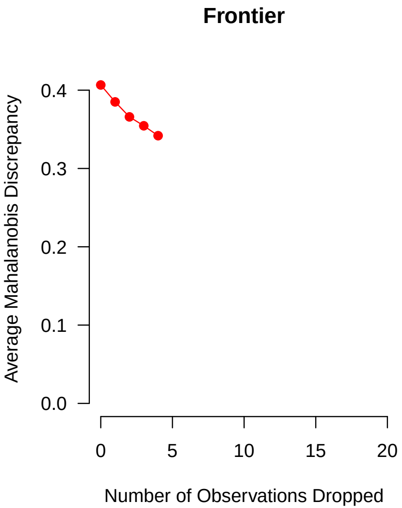

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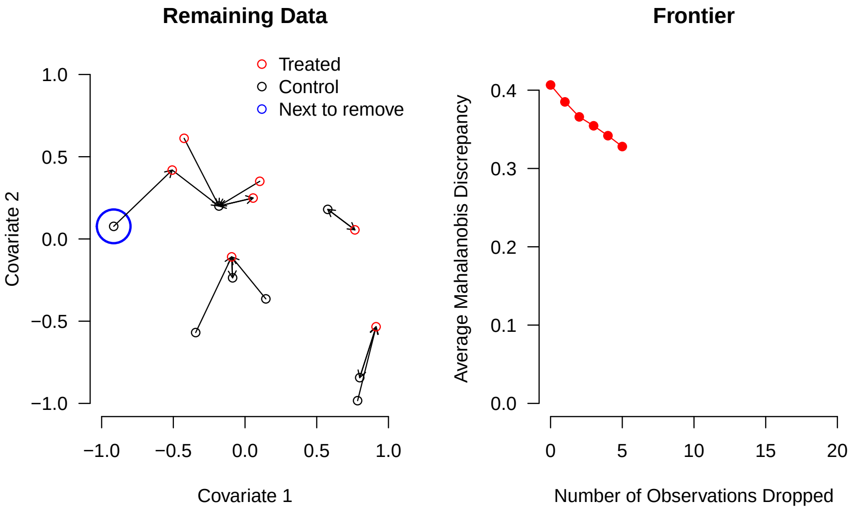

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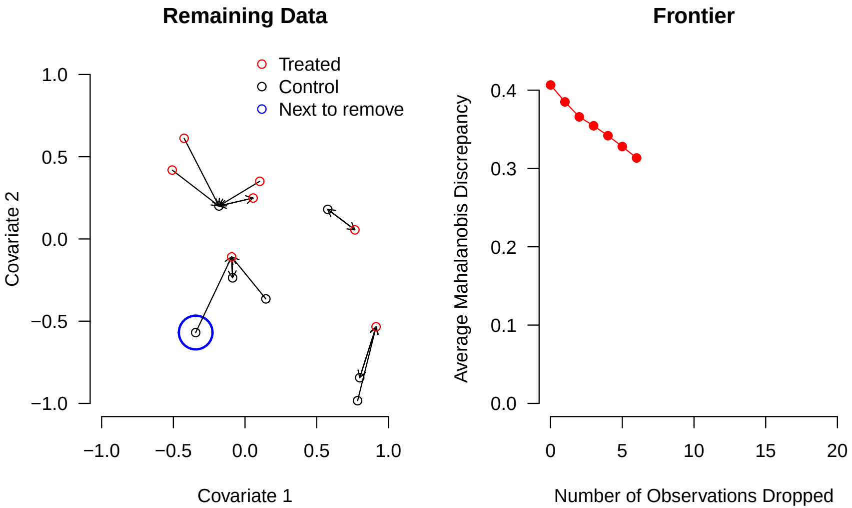

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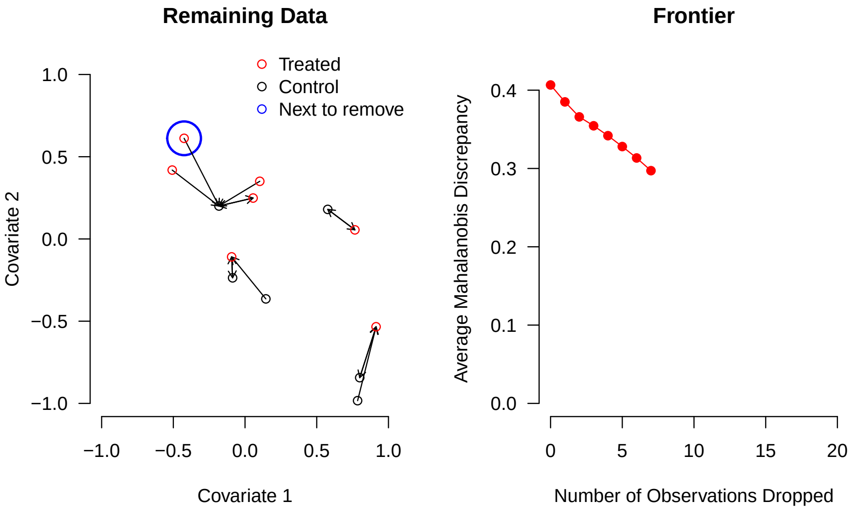

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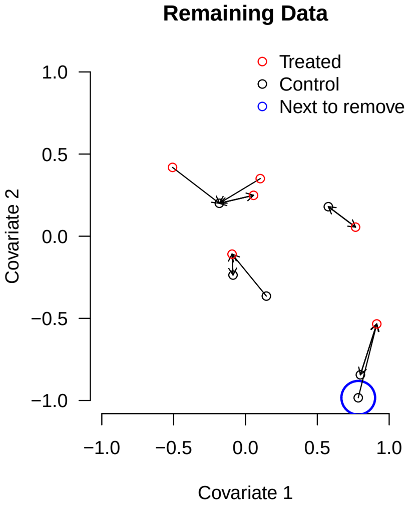

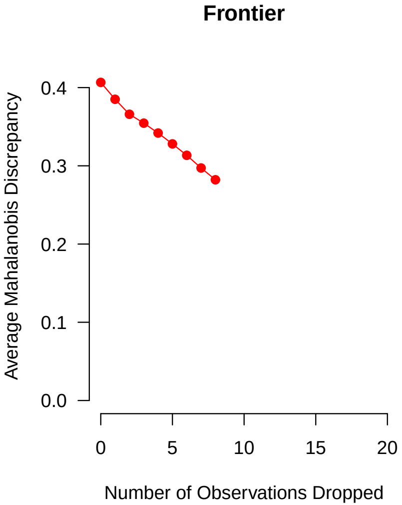

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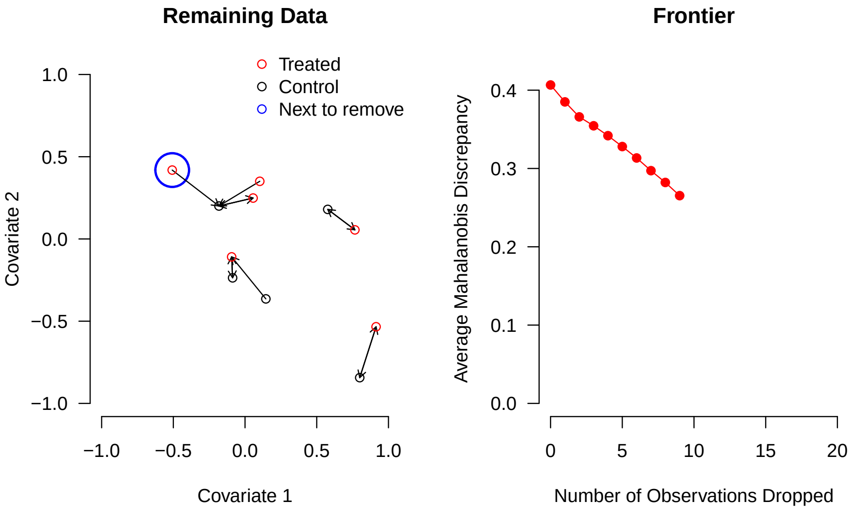

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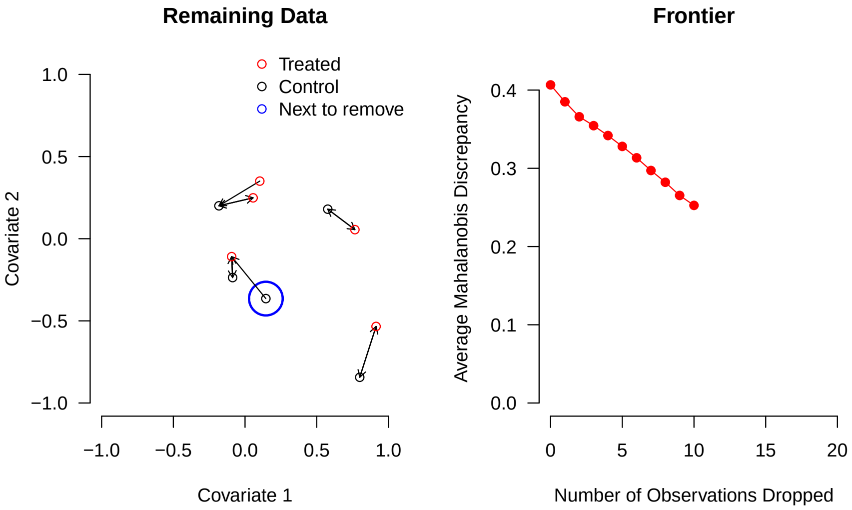

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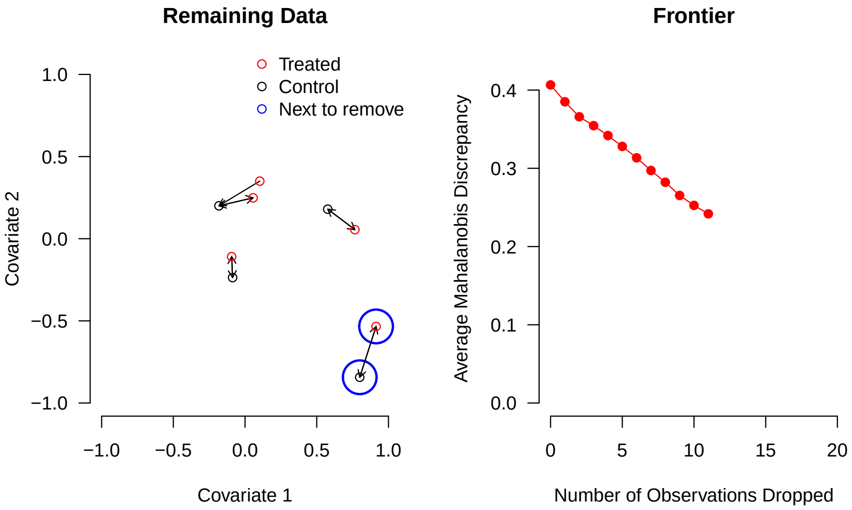

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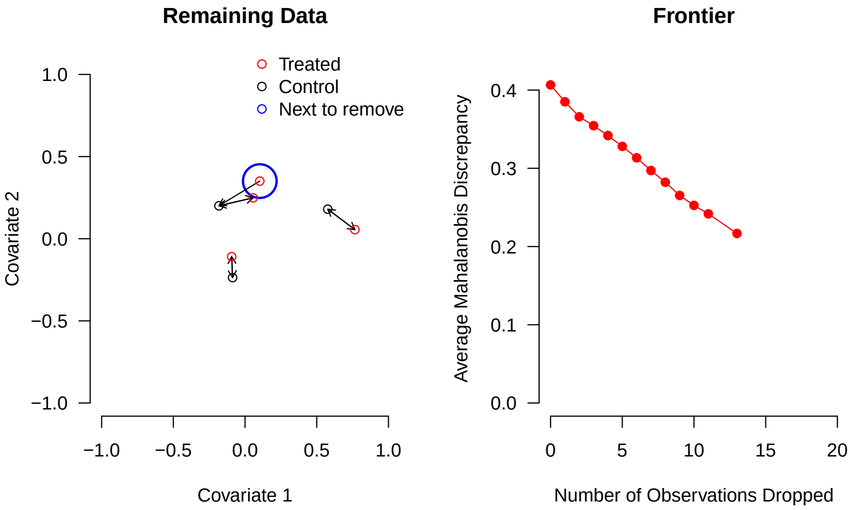

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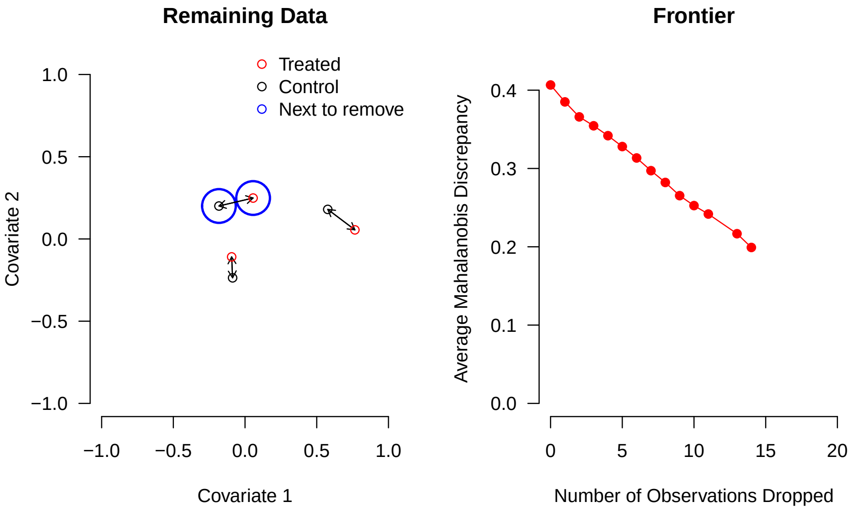

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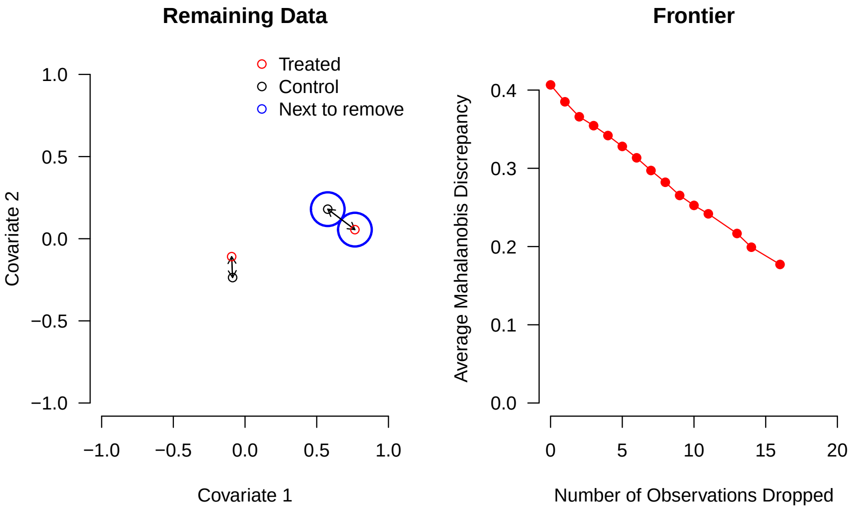

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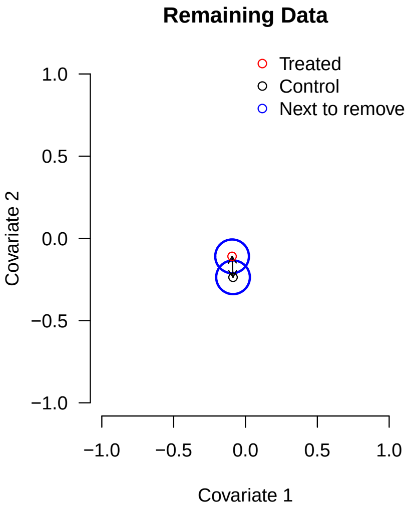

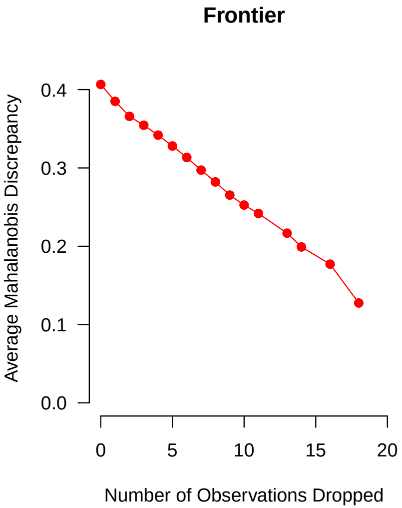

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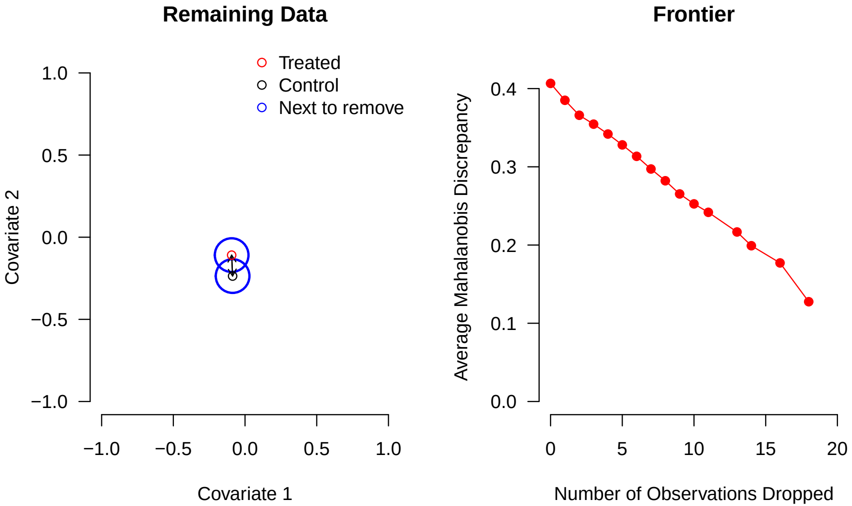

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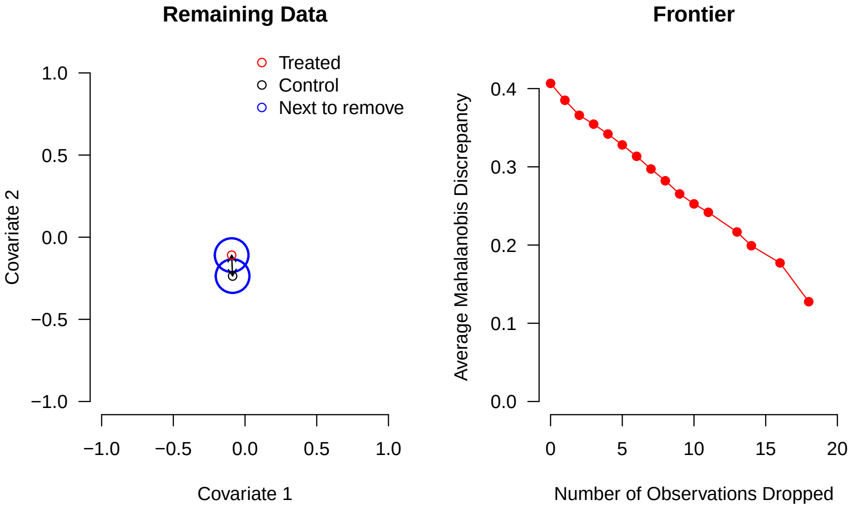

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- Very fast; works with any continuous imbalance metric

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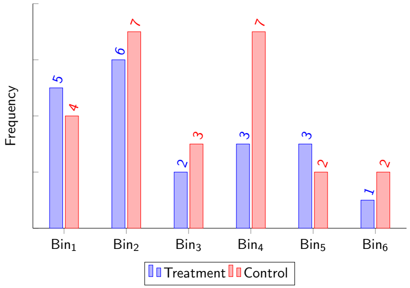

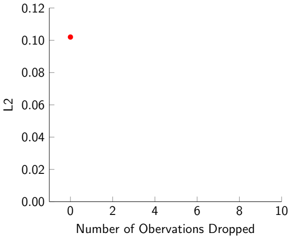

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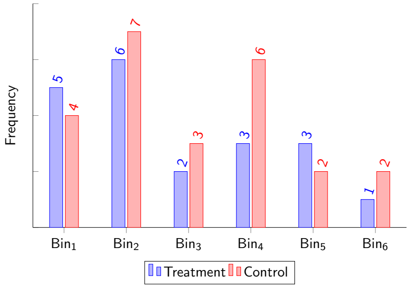

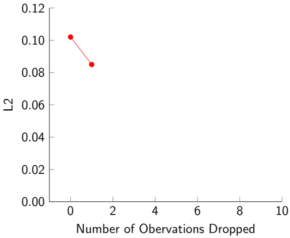

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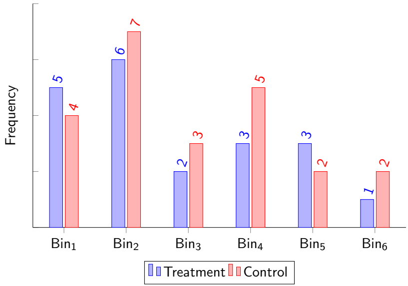

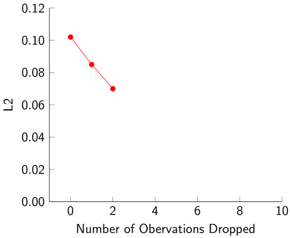

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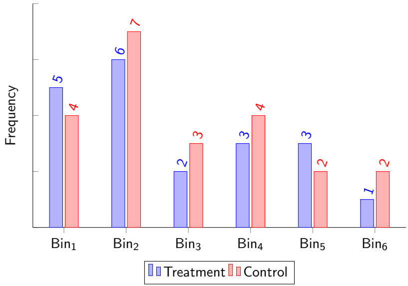

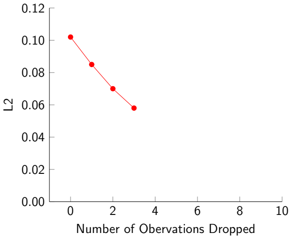

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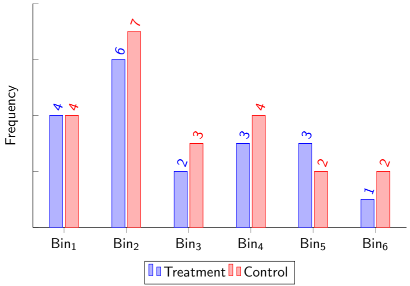

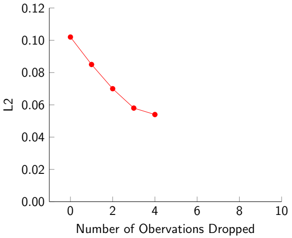

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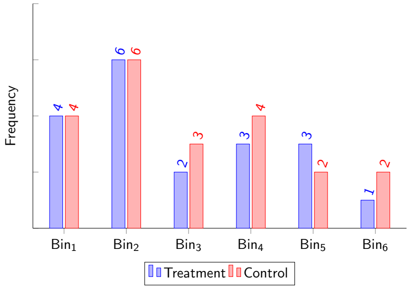

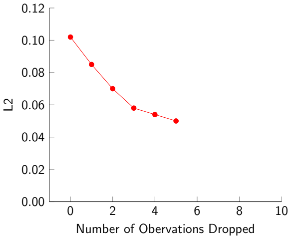

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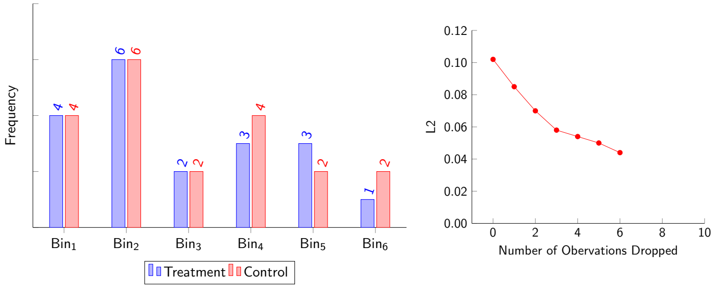

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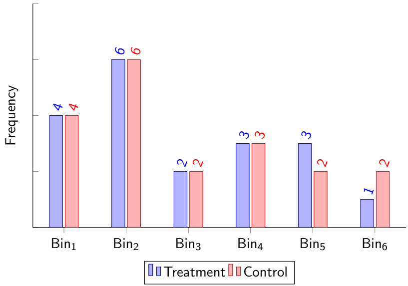

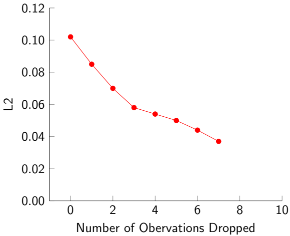

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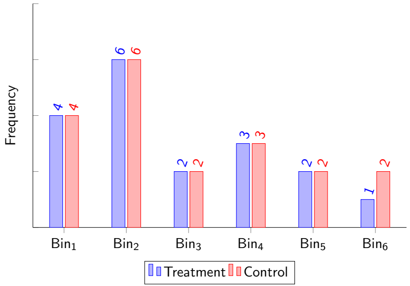

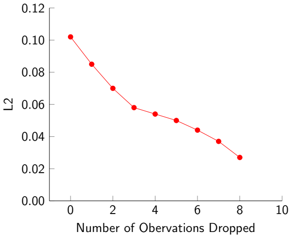

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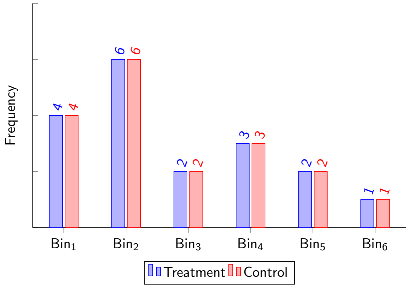

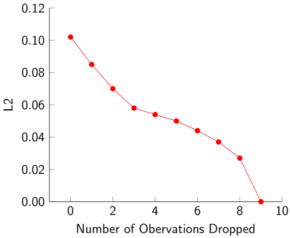

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For more information, papers, & software

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