Simplifying Matching Methods for Causal Inference

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3 Problems, 3 Solutions

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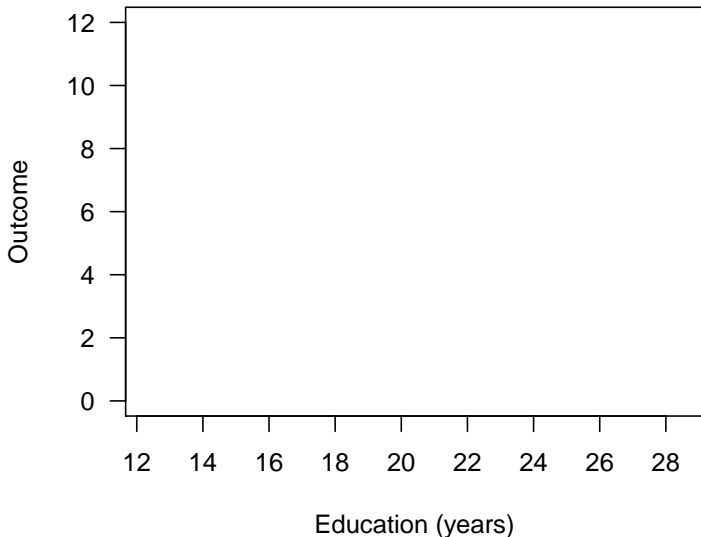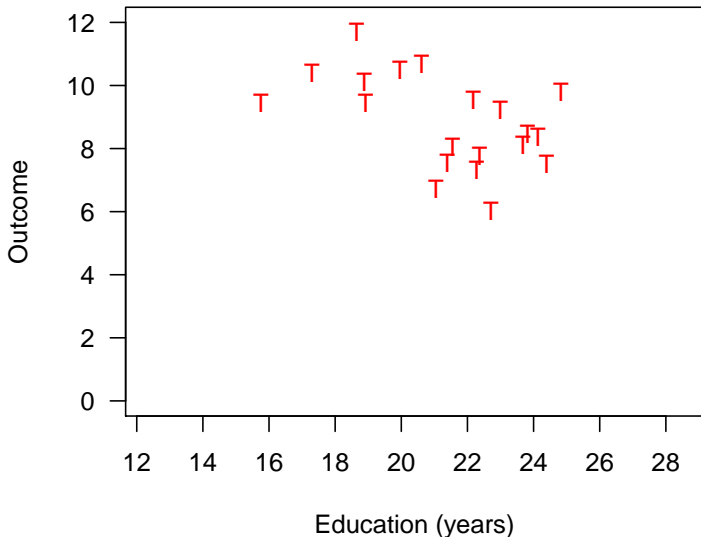
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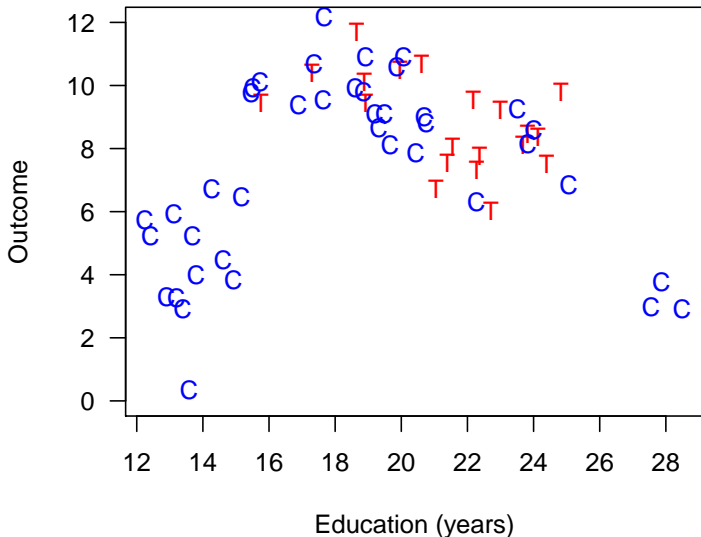
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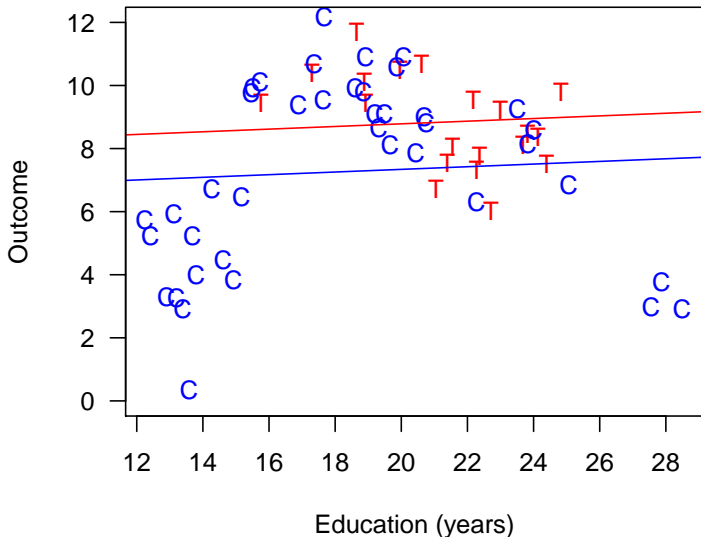
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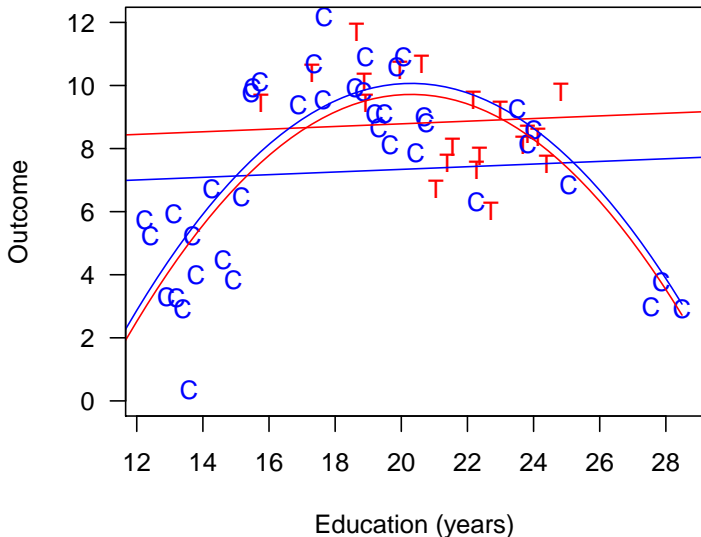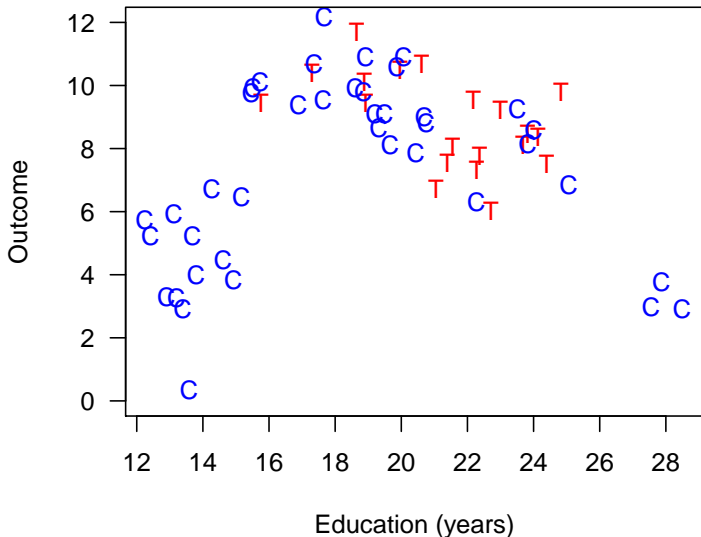
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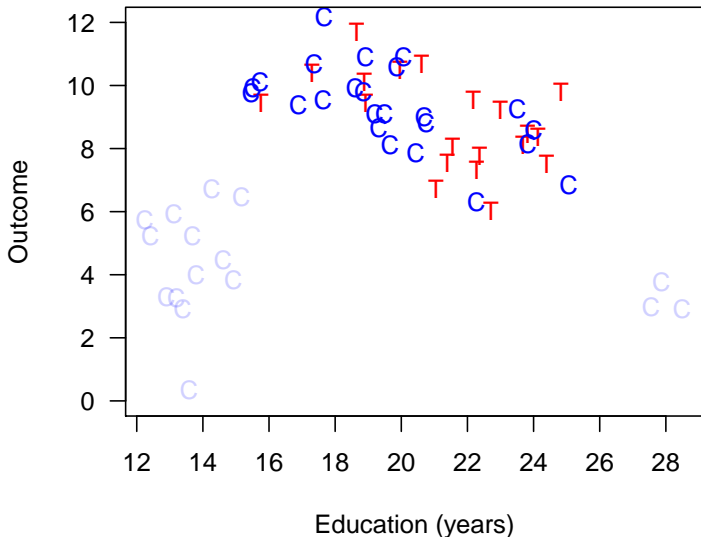
The Problems Matching Solves

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Without Matching:

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- "Teaching psychology is mostly a waste of time" (Kahneman 2011)

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A central project of statistics: Automating away human discretion

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- Pruning nonmatches makes control vars matter less: reduces imbalance, model dependence, researcher discretion, & bias

Matching: Finding Hidden Randomized Experiments

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Types of Experiments

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Complete Randomization

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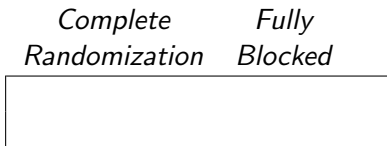
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- Other methods: *fully blocked*
- Other matching methods dominate PSM (wait, it gets worse)

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(Approximates Fully Blocked Experiment)

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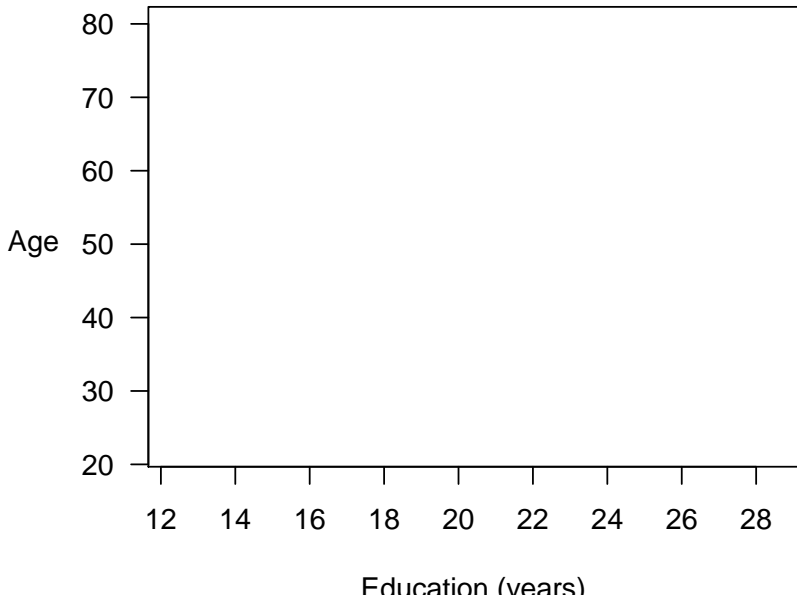
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   - (Many adjustments available to this basic method)
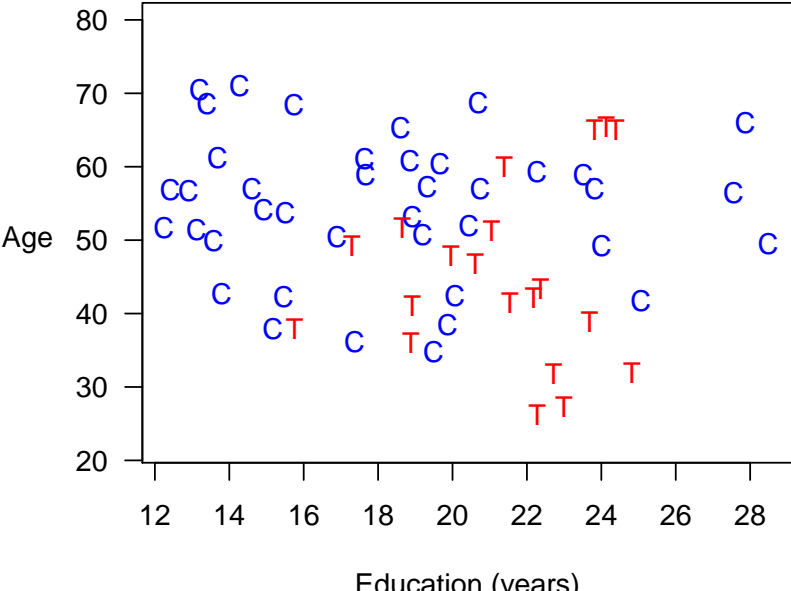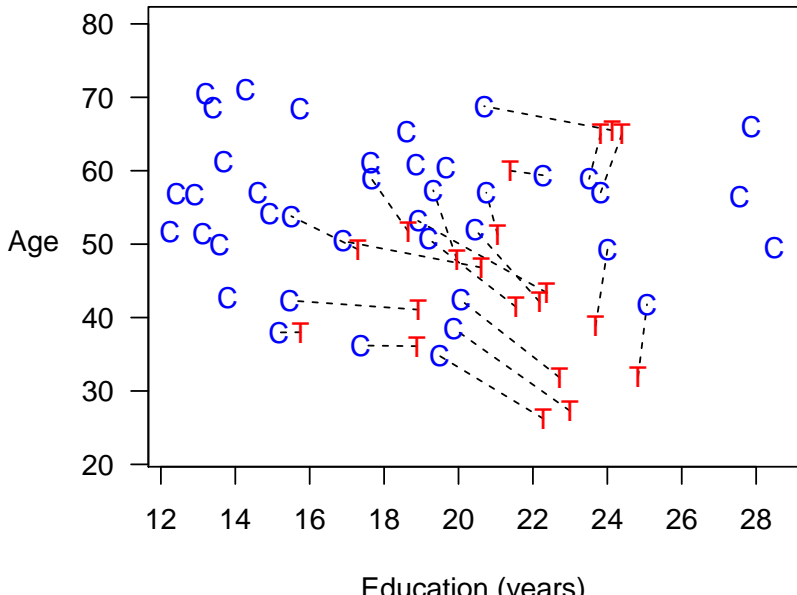2. Estimation Difference in means or a model

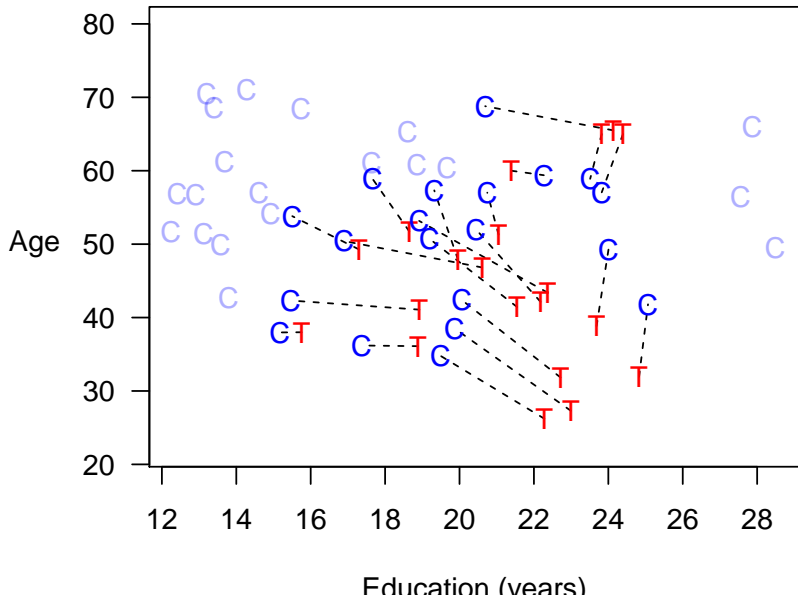## Mahalanobis Distance Matching
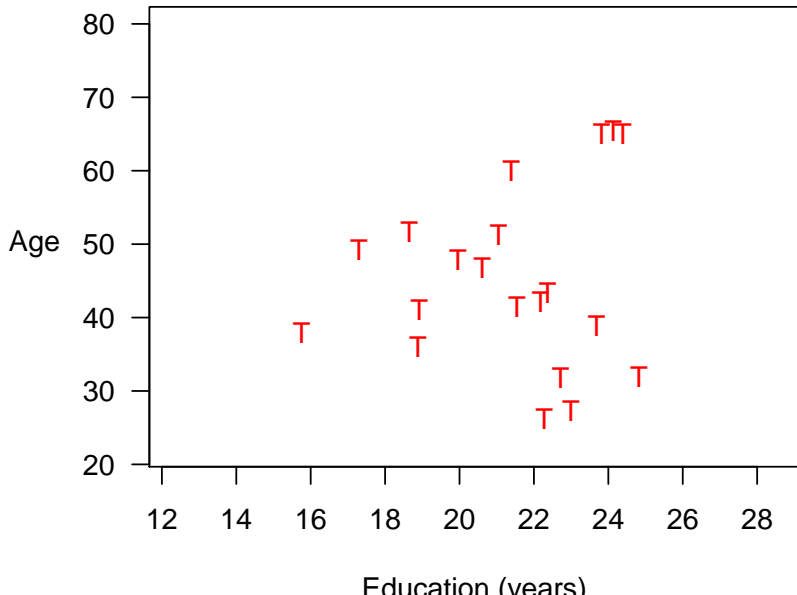
Mahalanobis Distance Matching

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Age vs. Education (years)

Mahalanobis Distance Matching

Age / Education (years)

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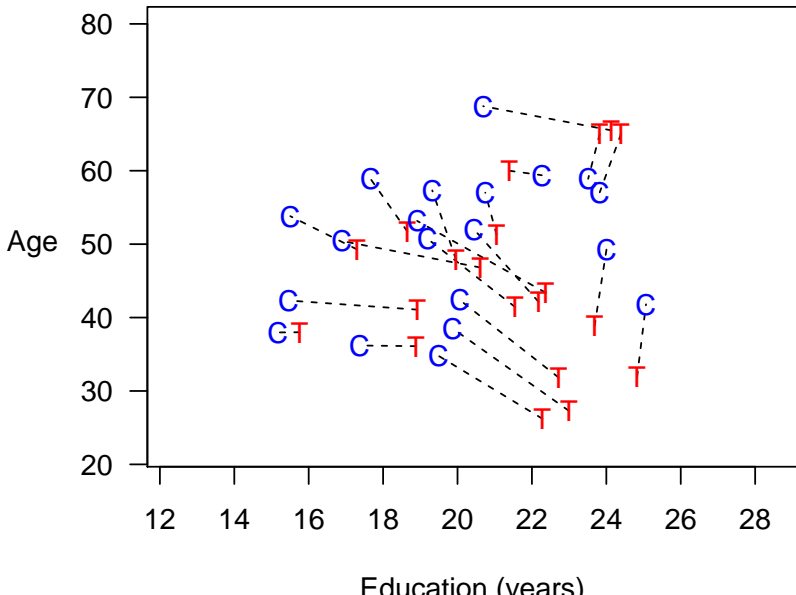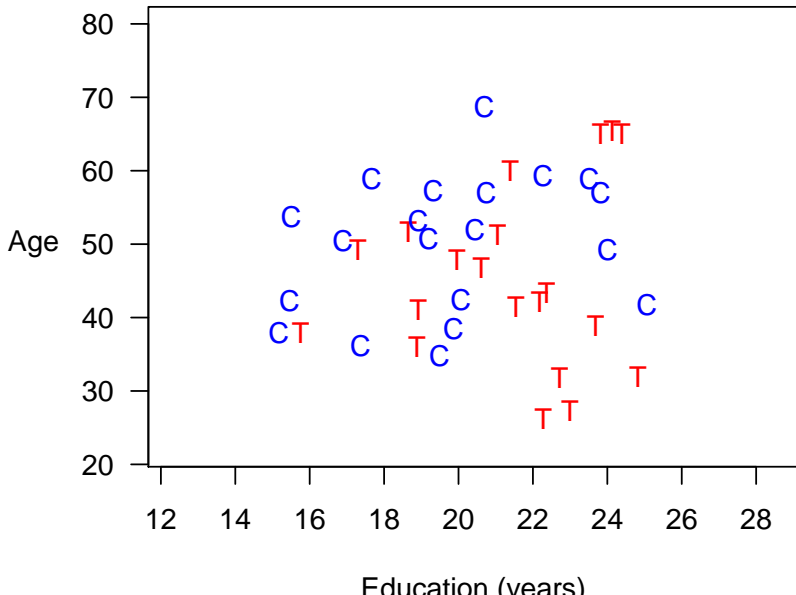
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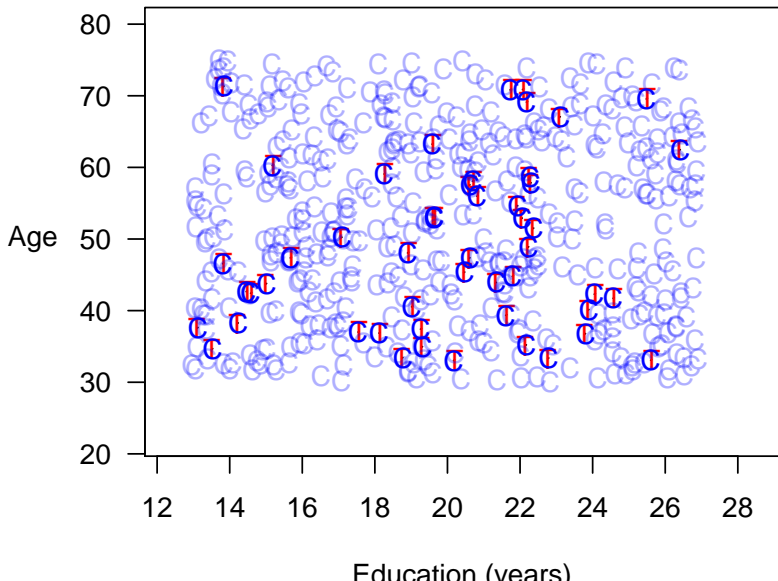
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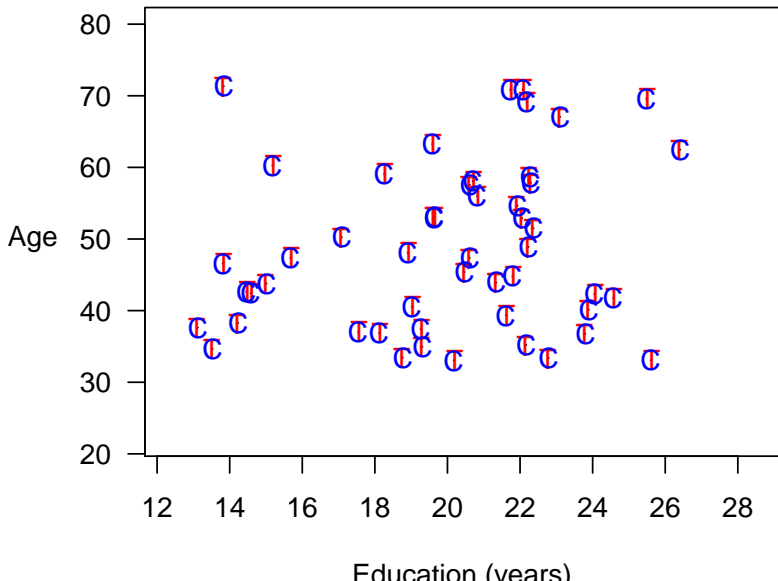
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(Most powerful easy-to-use approach)
(Approximates Fully Blocked Experiment)

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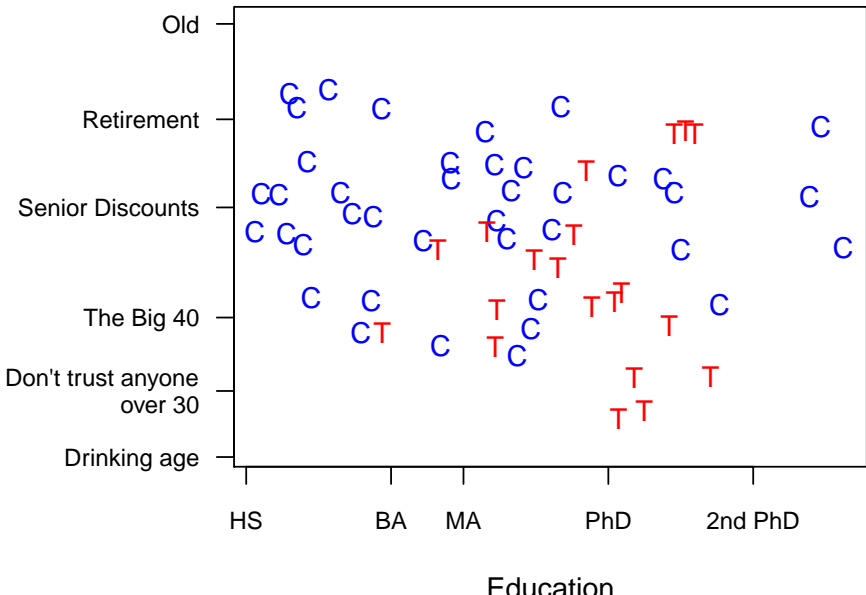   - Weight controls in each stratum to equal treateds
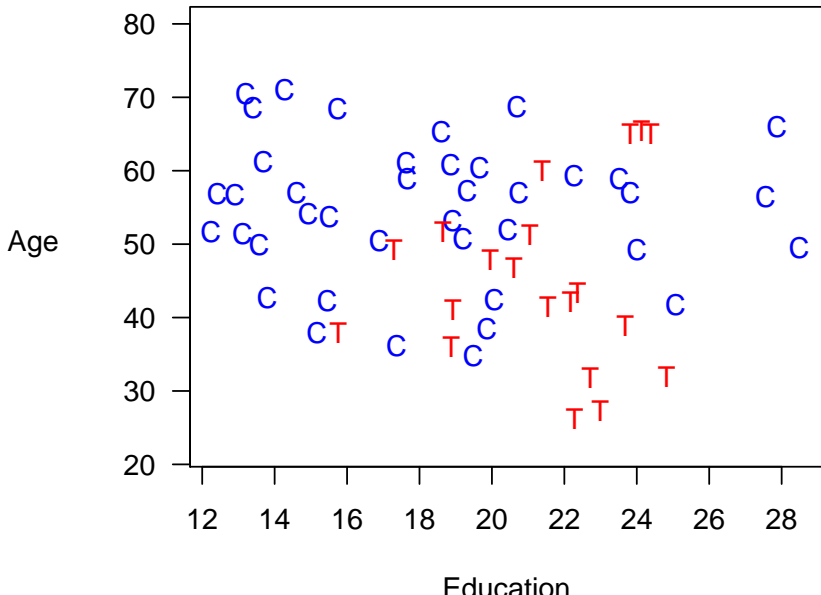
Coarsened Exact Matching

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Age

Education

Coarsened Exact Matching

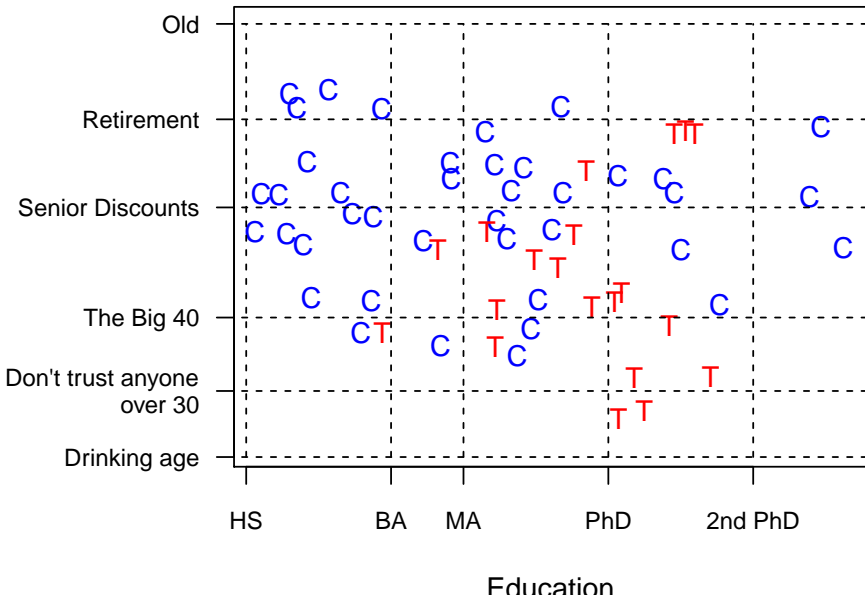**Coarsened Exact Matching**

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Best Case: Coarsened Exact Matching

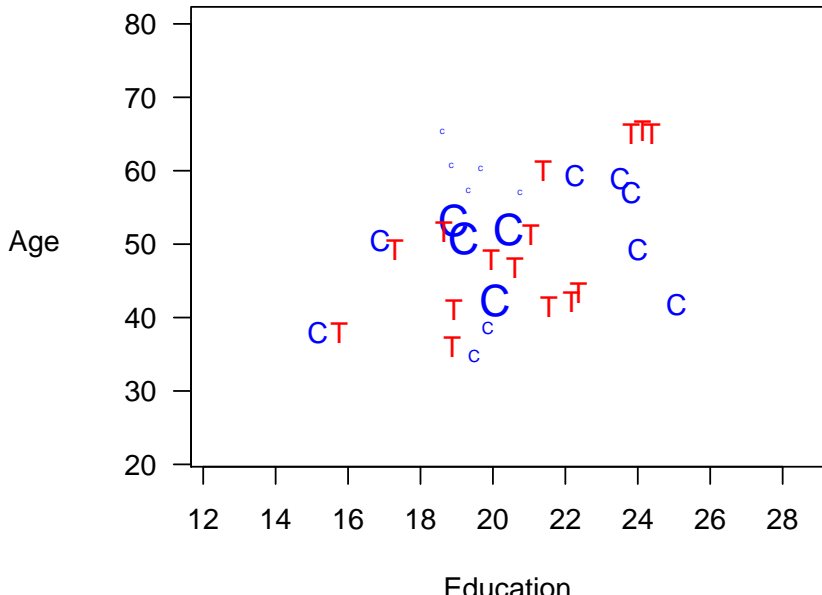# Best Case: Coarsened Exact Matching
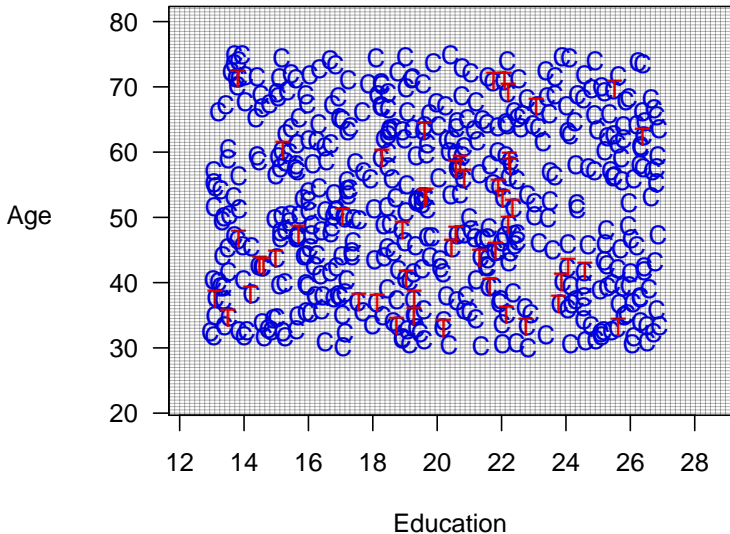
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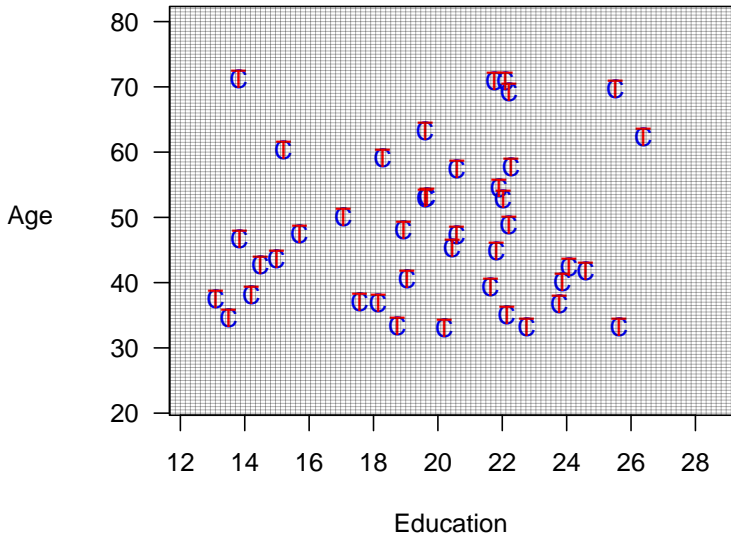
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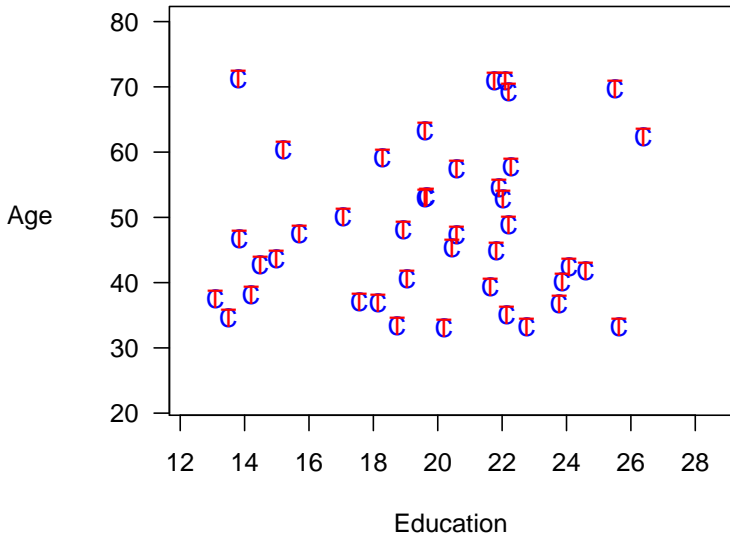
Method 3: Propensity Score Matching

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   - Match each treated unit to the nearest control unit
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   - Prune matches if Distance$>$caliper
   - (Many adjustments available to this basic method)
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Propensity Score Matching
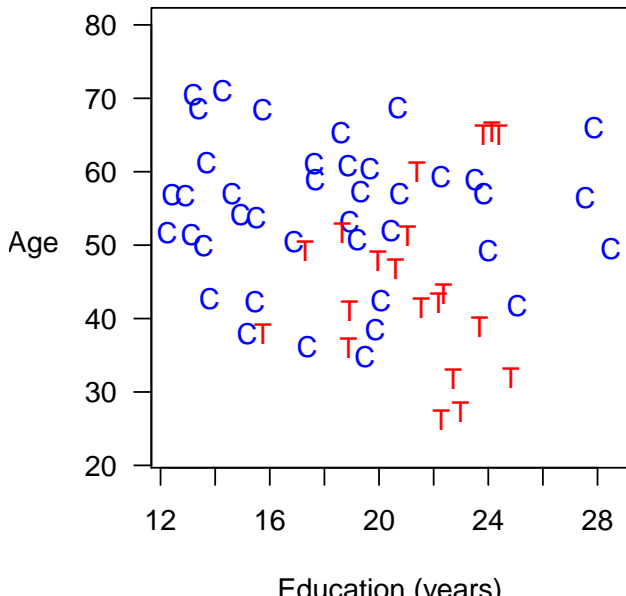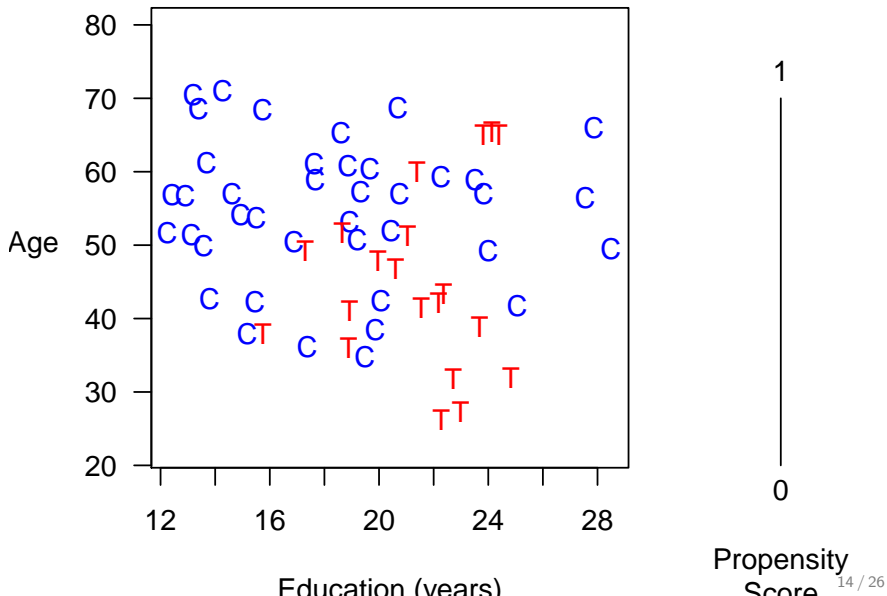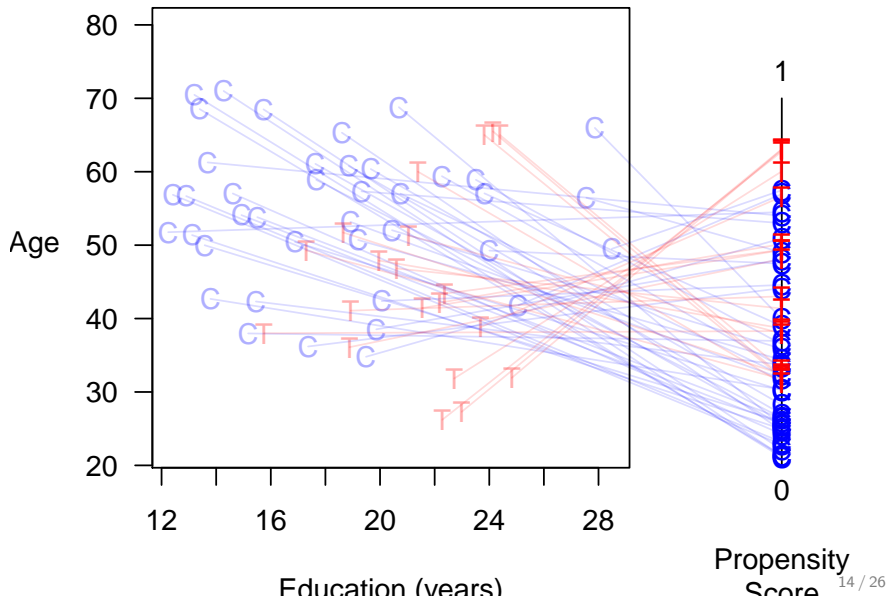
Age vs Education (years)

Propensity Score Matching

Propensity Score Matching

Propensity Score Matching

Propensity Score Matching

Age

Education (years)

Propensity
Score

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Best Case: Propensity Score Matching

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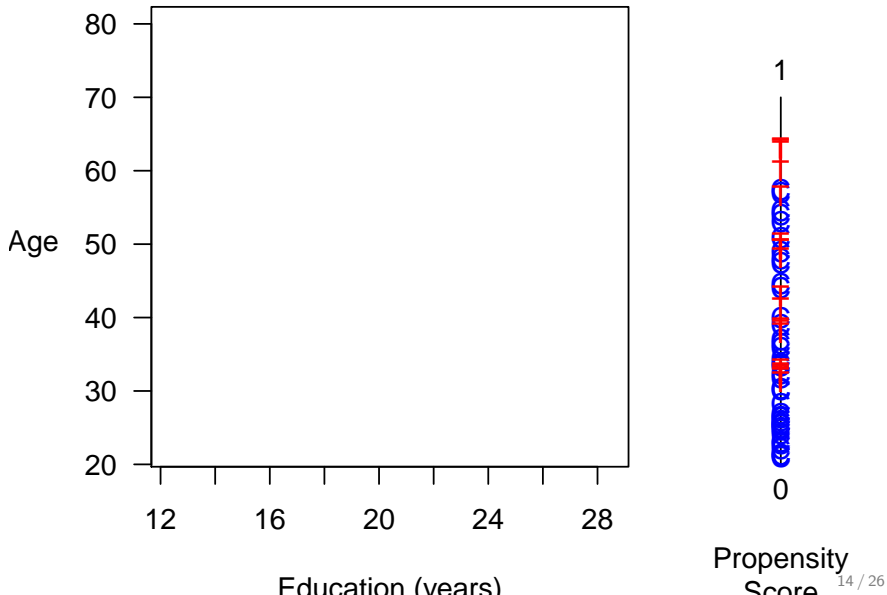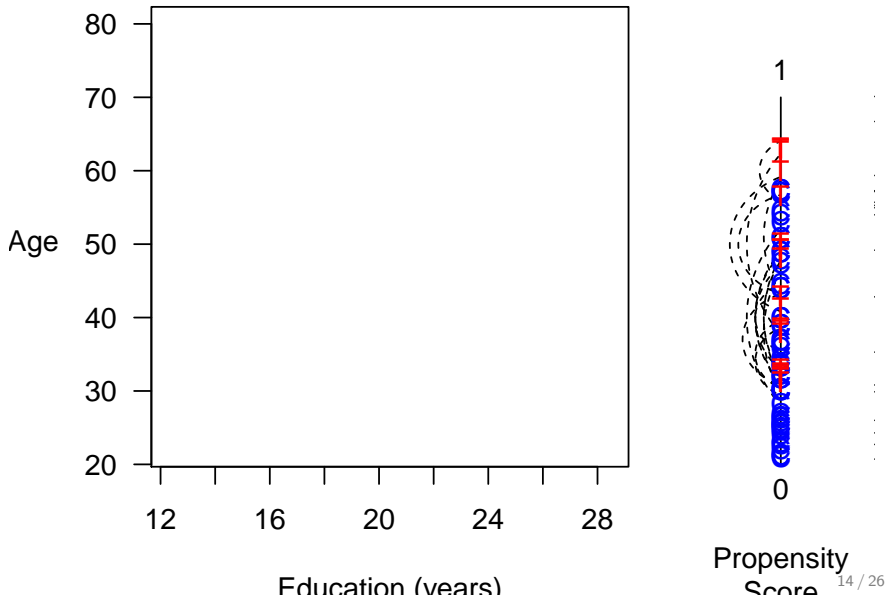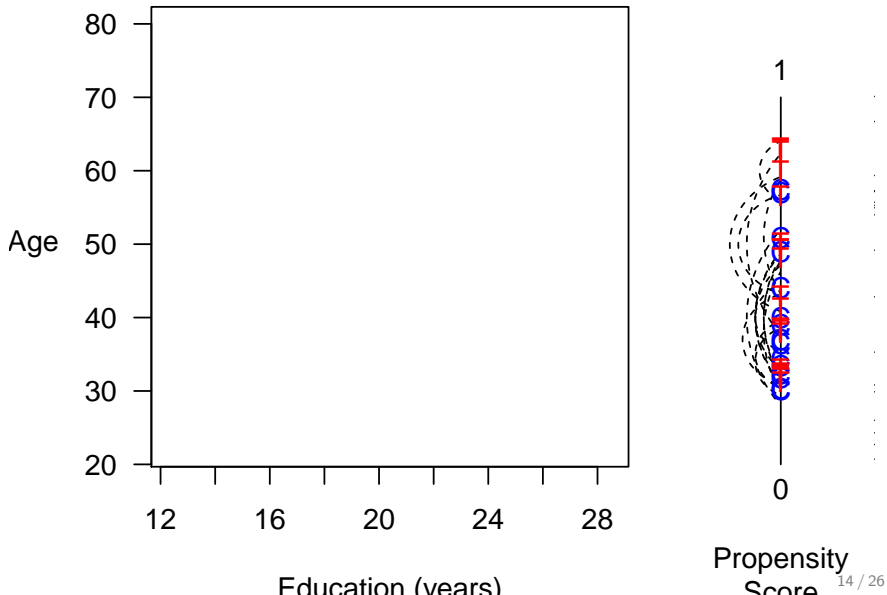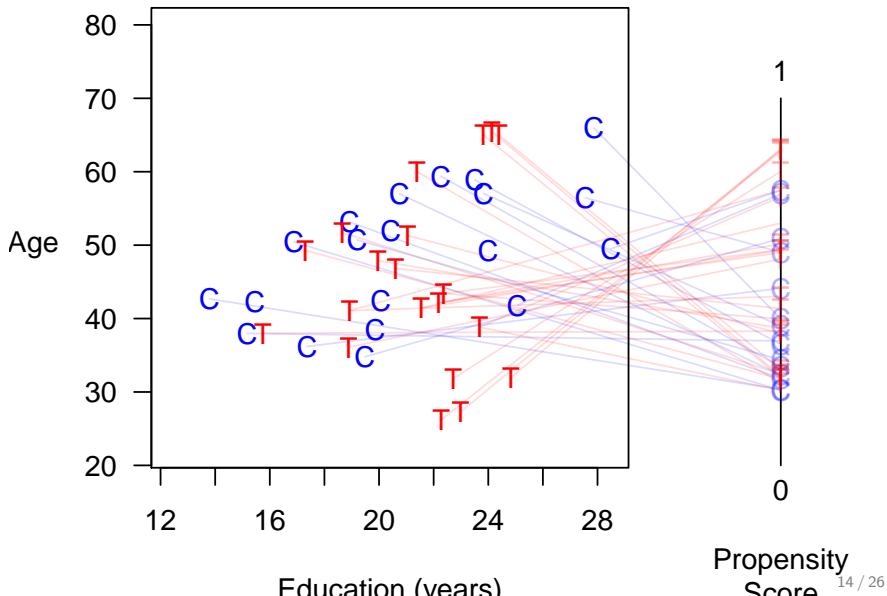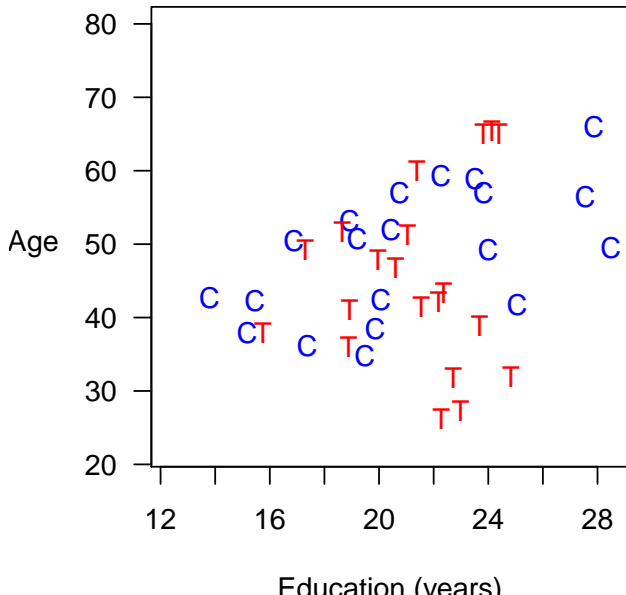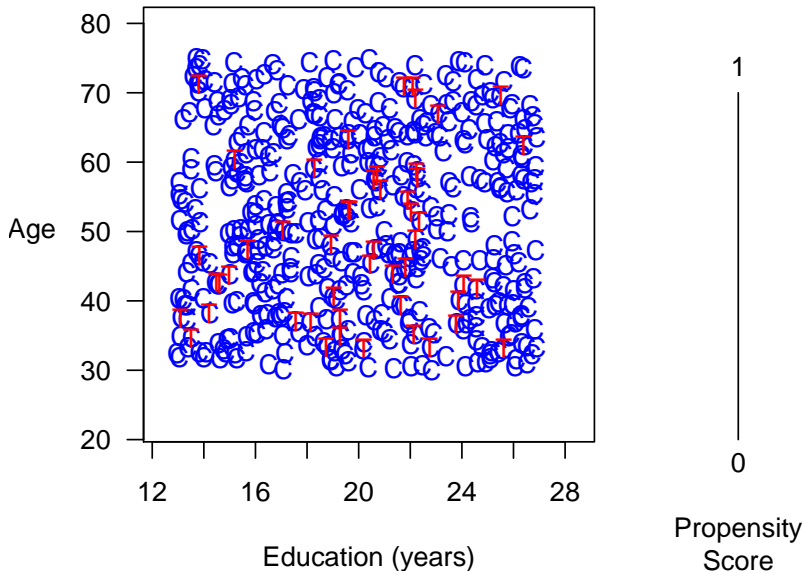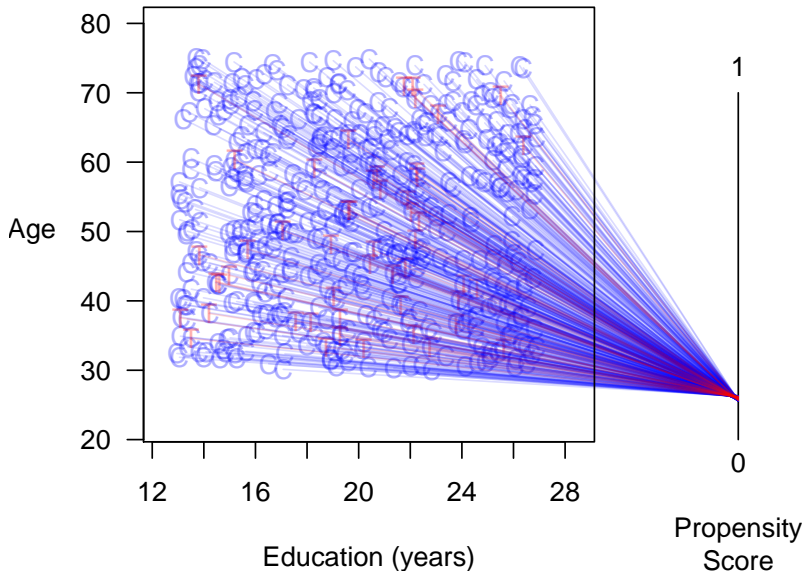
Propensity Score

Best Case: Propensity Score Matching
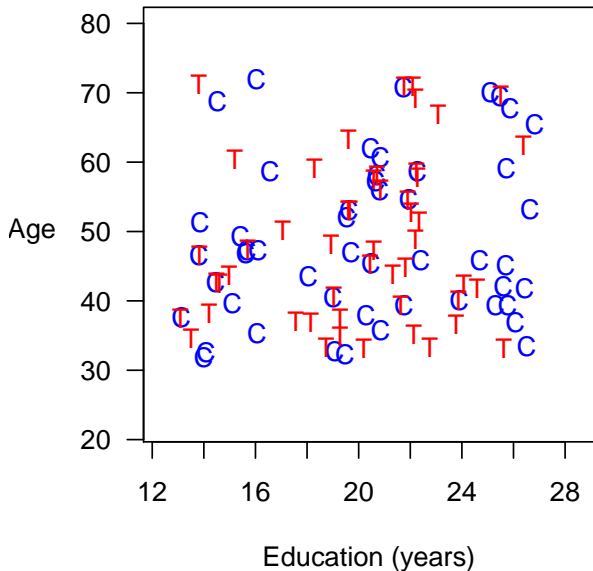
Best Case: Propensity Score Matching is Suboptimal
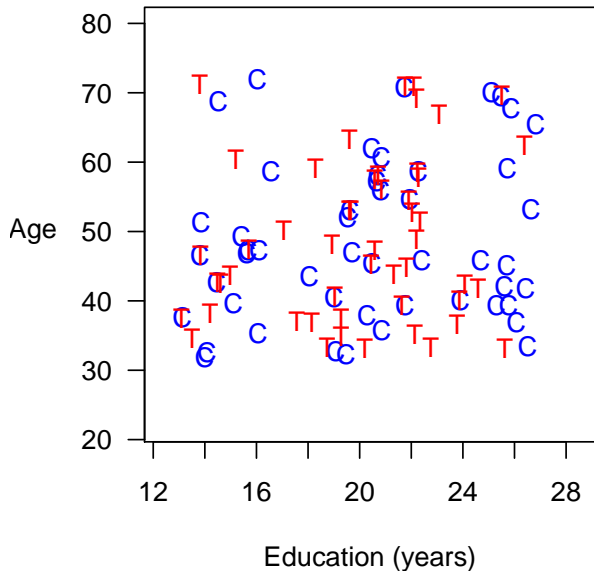
Random Pruning Increases Imbalance

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- Result is completely general (see math in the paper)

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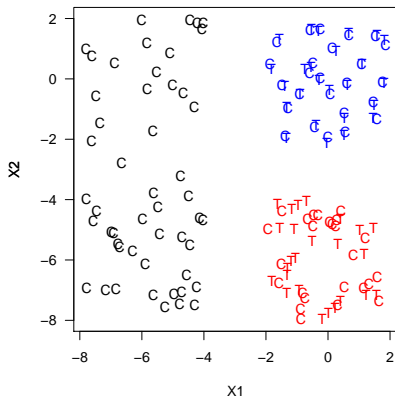
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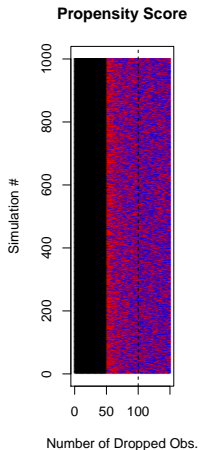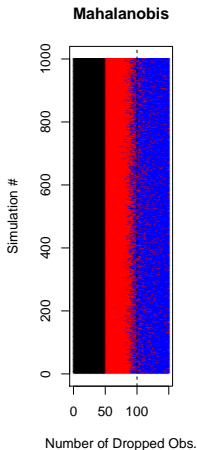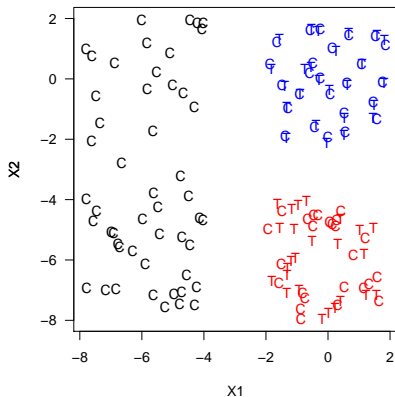
What Does PSM Match?
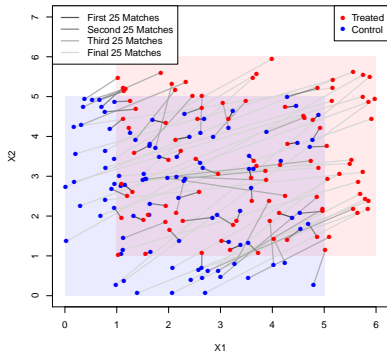
MDM Matches   PSM Matches
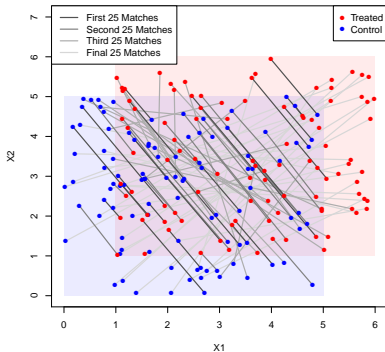
Controls: $X_1, X_2 \sim$ Uniform(0,5)
Treateds: $X_1, X_2 \sim$ Uniform(1,6)

PSM Increases Model Dependence & Bias

Model Dependence
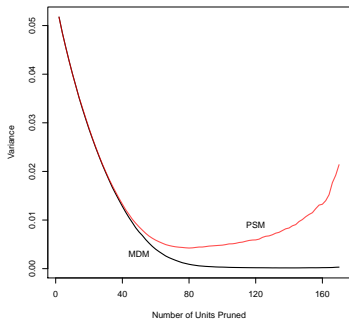
Bias
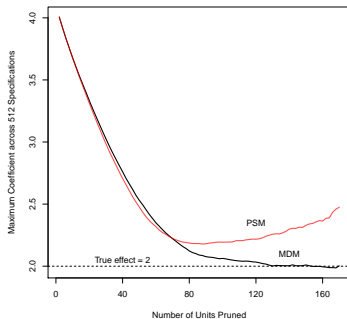
$$Y_i = 2T_i + X_{1i} + X_{2i} + \epsilon_i$$
$$\epsilon_i \sim N(0, 1)$$

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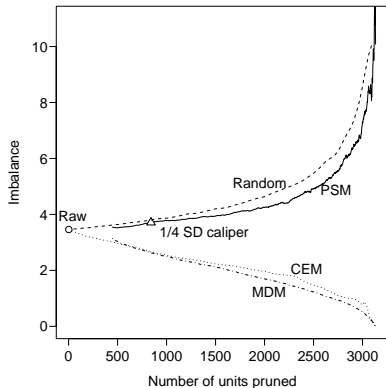
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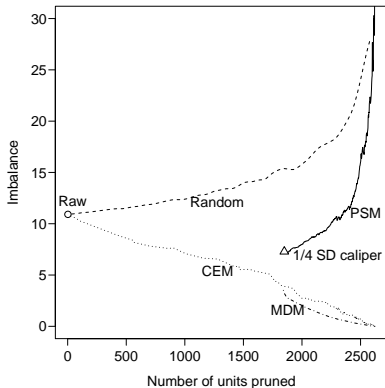
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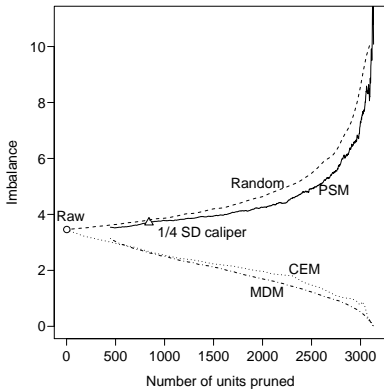
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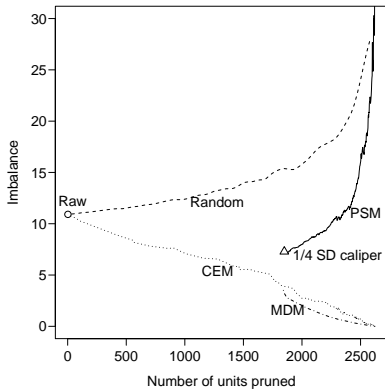
Similar pattern for > 20 other real data sets we checked

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- Choose an imbalance metric, then run.

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Job Training Data: Frontier and Causal Estimates
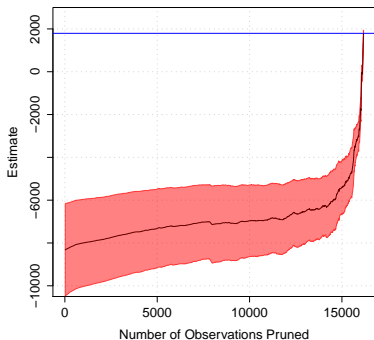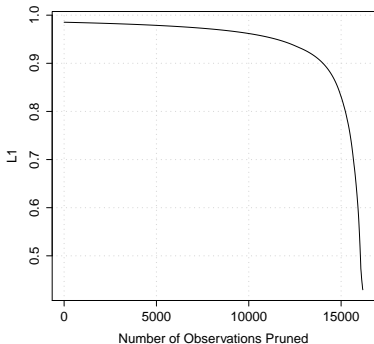
- 185 Ts; pruning most 16,252 Cs won't increase variance much
- Huge bias-variance trade-off after pruning most Cs
- Estimates converge to experiment after removing bias
- No mysteries: basis of inference clearly revealed

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For more information, articles, & software

GaryKing.org