# Simplifying Matching Methods for Causal Inference

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1. The most popular method (propensity score matching, used in 103,000 articles!) sounds magical:

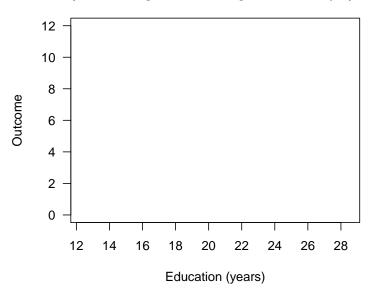
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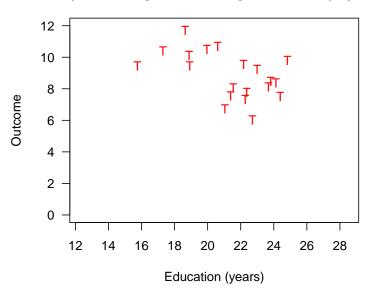
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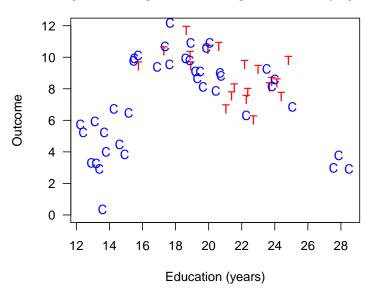
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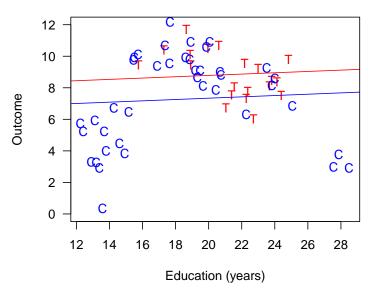
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  - "Causal Inference Without Balance Checking: Coarsened Exact Matching" (PA, 2011. Stefano Iacus, Gary King, and Giuseppe Porro)
- 3. Matching methods optimize either imbalance ( $\approx$  bias) or # units pruned ( $\approx$  variance); users need both simultaneously:

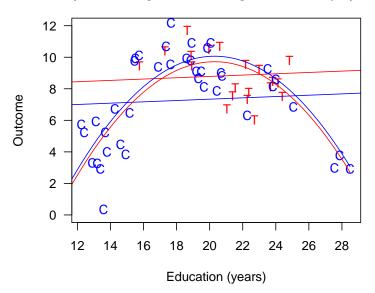
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  - "The Balance-Sample Size Frontier in Matching Methods for Causal Inference" (In press, AJPS; Gary King, Christopher Lucas and Richard Nielsen)

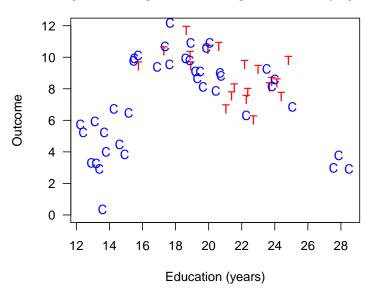


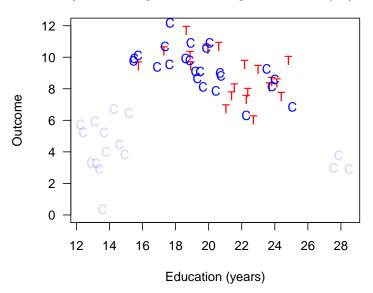


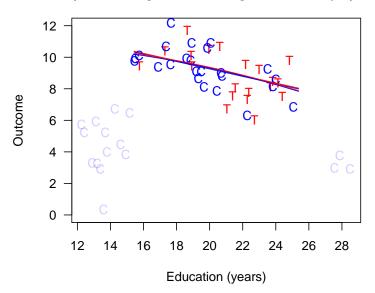












Without Matching:

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**Imbalance** 

Without Matching:

Imbalance → Model Dependence

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- "Teaching psychology is mostly a waste of time" (Kahneman 2011)

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### The Problems Matching Solves

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```
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A central project of statistics: Automating away human discretion

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- Pruning nonmatches makes control vars matter less: reduces imbalance, model dependence, researcher discretion, & bias

### Matching: Finding Hidden Randomized Experiments

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Complete Randomization

Complete Fully Randomization Blocked

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- Other matching methods dominate PSM

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- Other matching methods dominate PSM (wait, it gets worse)

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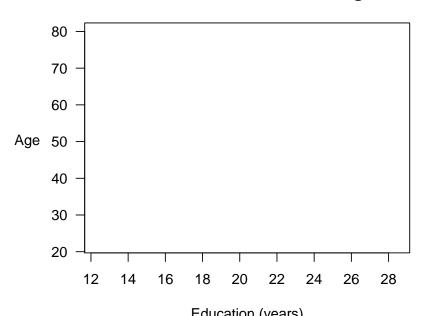
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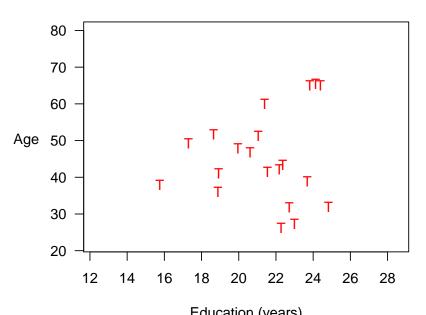
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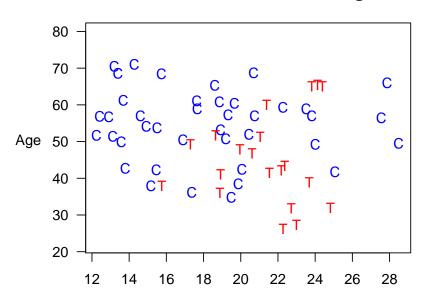
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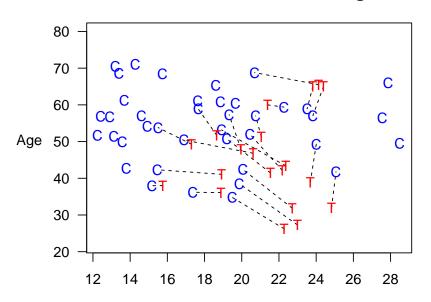
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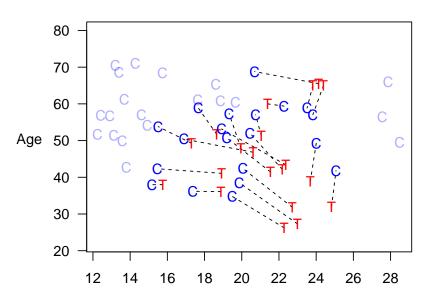
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  - (Many adjustments available to this basic method)
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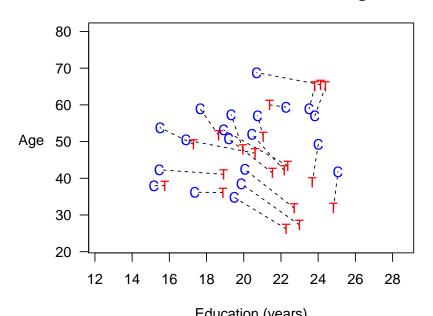


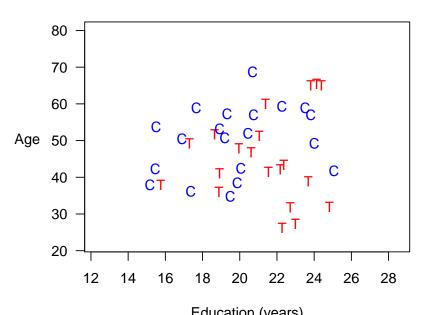






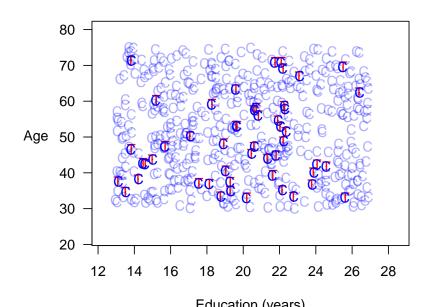




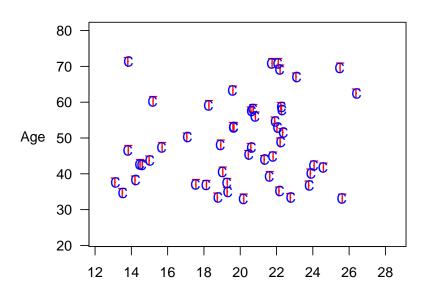


## Best Case: Mahalanobis Distance Matching

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# Method 2: Coarsened Exact Matching (Most powerful easy-to-use approach) (Approximates Fully Blocked Experiment)

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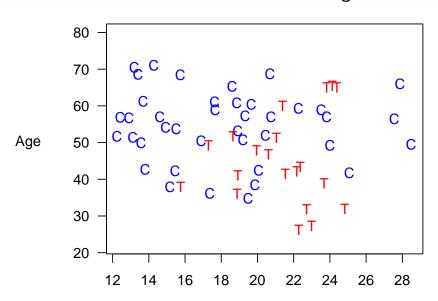
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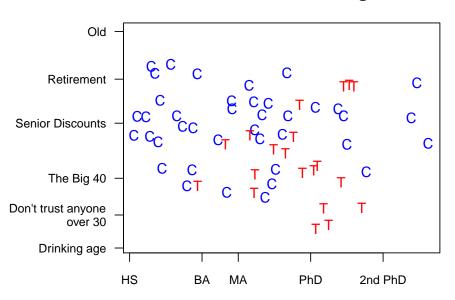
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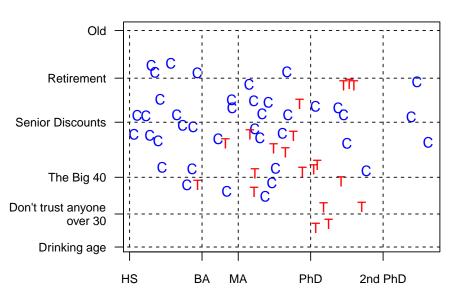
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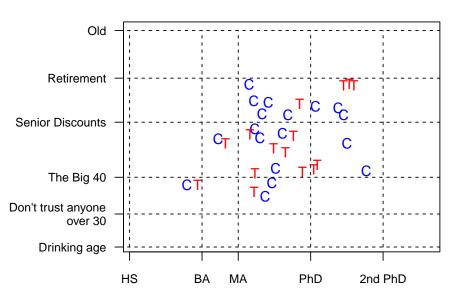
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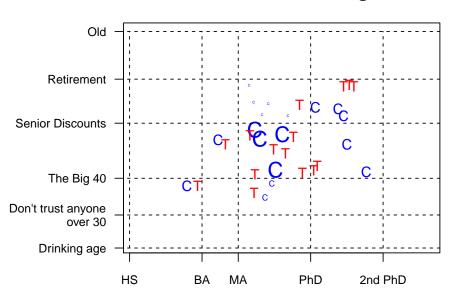






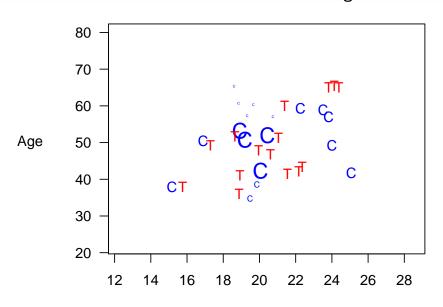
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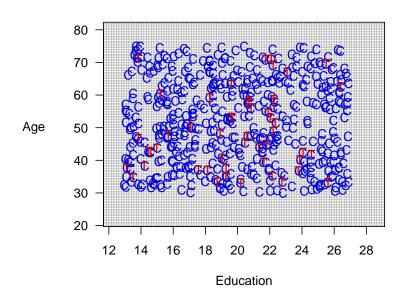
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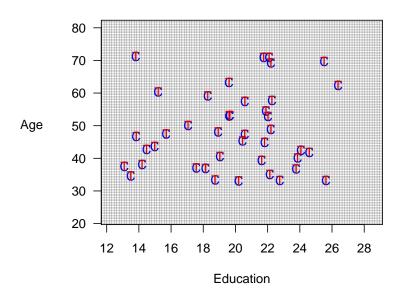


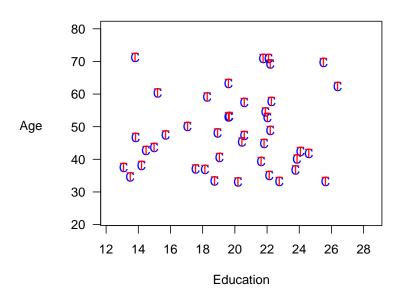
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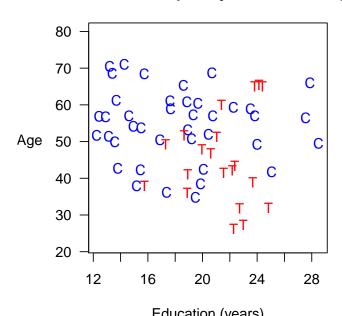
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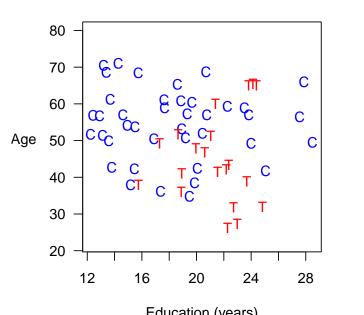
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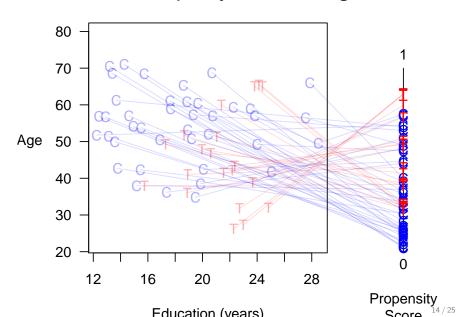
(Approximates Completely Randomized Experiment)

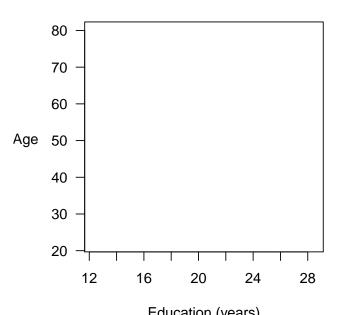
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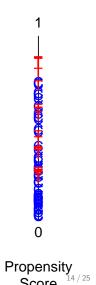


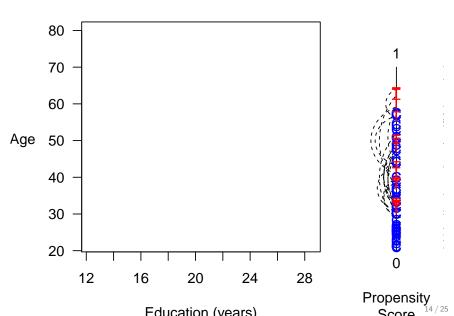


Propensity Score 14/25

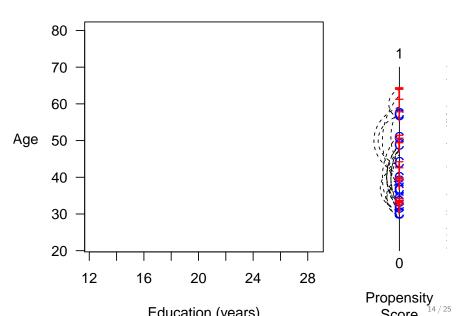




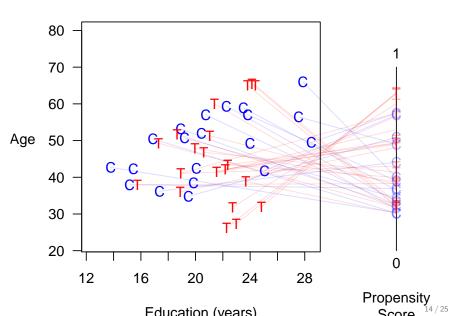


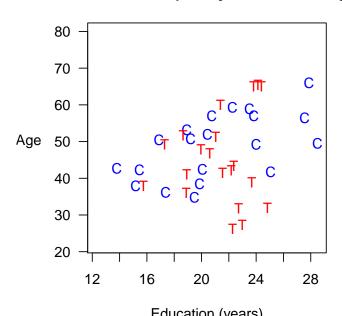


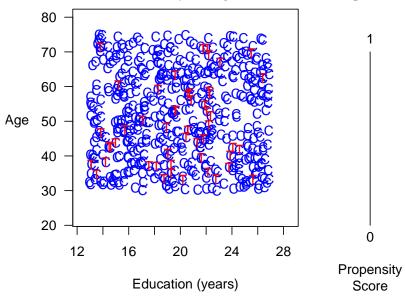
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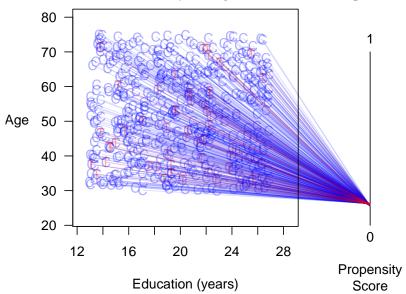


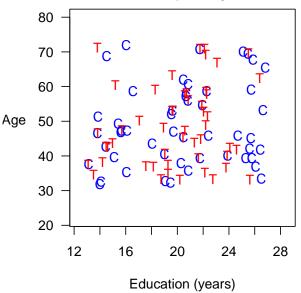
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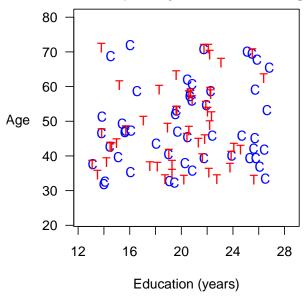








### Best Case: Propensity Score Matching is Suboptimal



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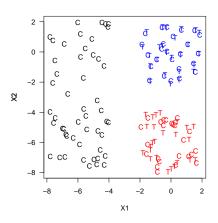
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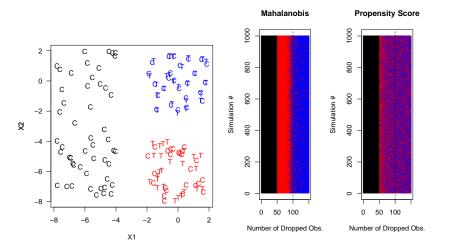
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## PSM is Blind Where Other Methods Can See

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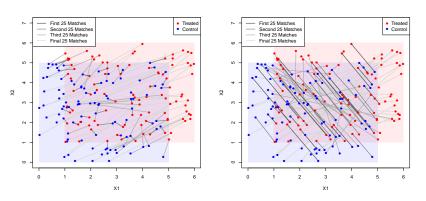
### PSM is Blind Where Other Methods Can See



### What Does PSM Match?

#### MDM Matches

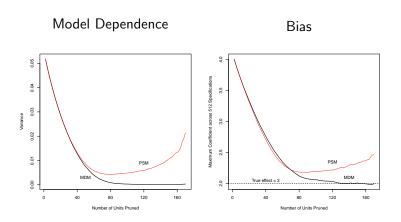
#### PSM Matches



Controls:  $X_1, X_2 \sim \text{Uniform}(0,5)$ 

Treateds:  $X_1, X_2 \sim \text{Uniform}(1,6)$ 

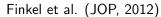
# PSM Increases Model Dependence & Bias

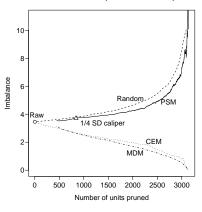


$$Y_i = 2T_i + X_{1i} + X_{2i} + \epsilon_i$$
  
$$\epsilon_i \sim N(0, 1)$$

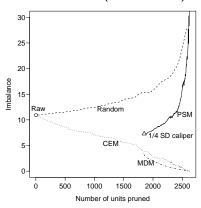
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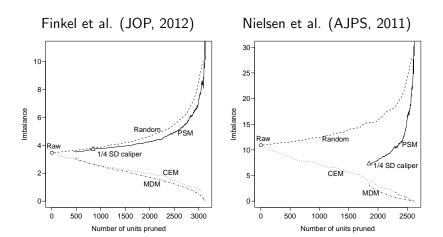




### Nielsen et al. (AJPS, 2011)



# The Propensity Score Paradox in Real Data



Similar pattern for > 20 other real data sets we checked

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- Choose an imbalance metric, then run.

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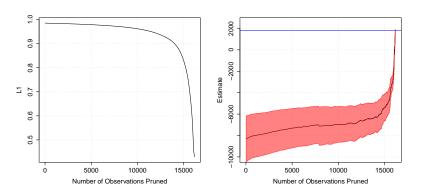
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# Job Training Data: Frontier and Causal Estimates



- 185 Ts; pruning most 16,252 Cs won't increase variance much
- Huge bias-variance trade-off after pruning most Cs
- Estimates converge to experiment after removing bias
- No mysteries: basis of inference clearly revealed

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# For more information, articles, & software

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