

# Detecting Model Dependence

Gary King

Institute for Quantitative Social Science  
Harvard University, <http://GKing.Harvard.edu>

Talk at Washington University, St. Louis, 1/22/2010

- King, Gary and Langche Zeng. “The Dangers of Extreme Counterfactuals,” *Political Analysis*, 14, 2, (2007): 131-159.

- King, Gary and Langche Zeng. “The Dangers of Extreme Counterfactuals,” *Political Analysis*, 14, 2, (2007): 131-159.
- King, Gary and Langche Zeng. “When Can History be Our Guide? The Pitfalls of Counterfactual Inference,” *International Studies Quarterly*, 2006, 51 (March, 2007): 183–210.

- King, Gary and Langche Zeng. “**The Dangers of Extreme Counterfactuals,**” *Political Analysis*, 14, 2, (2007): 131-159.
- King, Gary and Langche Zeng. “**When Can History be Our Guide? The Pitfalls of Counterfactual Inference,**” *International Studies Quarterly*, 2006, 51 (March, 2007): 183–210.
- Related Software: **WhatIf, MatchIt, Zelig, CEM**

- King, Gary and Langche Zeng. “The Dangers of Extreme Counterfactuals,” *Political Analysis*, 14, 2, (2007): 131-159.
- King, Gary and Langche Zeng. “When Can History be Our Guide? The Pitfalls of Counterfactual Inference,” *International Studies Quarterly*, 2006, 51 (March, 2007): 183–210.
- Related Software: WhatIf, MatchIt, Zelig, CEM

<http://GKing.Harvard.edu/projects/cause.shtml>

# Counterfactuals

# Counterfactuals

- Three types:

- Three types:
  - 1 **Forecasts** Will the U.S. be in Iraq in 2008?



- Three types:
  - 1 **Forecasts** Will the U.S. be in Iraq in 2008?
  - 2 **Whatif Questions** What would have happened if the U.S. had not invaded Iraq?

- Three types:
  - 1 **Forecasts** Will the U.S. be in Iraq in 2008?
  - 2 **Whatif Questions** What would have happened if the U.S. had not invaded Iraq?
  - 3 **Causal Effects** What is the causal effect of the Iraq war on U.S. Supreme Court decision making? (a factual minus a counterfactual)

- Three types:
  - 1 **Forecasts** Will the U.S. be in Iraq in 2008?
  - 2 **Whatif Questions** What would have happened if the U.S. had not invaded Iraq?
  - 3 **Causal Effects** What is the causal effect of the Iraq war on U.S. Supreme Court decision making? (a factual minus a counterfactual)
- Counterfactuals are part of almost all research questions.

# Model Dependence in Practice

# Model Dependence in Practice

- How do you conduct empirical analyses?

# Model Dependence in Practice

- How do you conduct empirical analyses?
  - collect the data over many months or years.

# Model Dependence in Practice

- How do you conduct empirical analyses?
  - collect the data over many months or years.
  - finish recording and merging.

# Model Dependence in Practice

- How do you conduct empirical analyses?
  - collect the data over many months or years.
  - finish recording and merging.
  - sit in front of your computer with nobody to bother you.



# Model Dependence in Practice

- How do you conduct empirical analyses?
  - collect the data over many months or years.
  - finish recording and merging.
  - sit in front of your computer with nobody to bother you.
  - run one regression.

# Model Dependence in Practice

- How do you conduct empirical analyses?
  - collect the data over many months or years.
  - finish recording and merging.
  - sit in front of your computer with nobody to bother you.
  - run one regression.
  - run another regression with different control variables.

# Model Dependence in Practice

- How do you conduct empirical analyses?
  - collect the data over many months or years.
  - finish recording and merging.
  - sit in front of your computer with nobody to bother you.
  - run one regression.
  - run another regression with different control variables.
  - run another regression with different functional forms.

# Model Dependence in Practice

- How do you conduct empirical analyses?
  - collect the data over many months or years.
  - finish recording and merging.
  - sit in front of your computer with nobody to bother you.
  - run one regression.
  - run another regression with different control variables.
  - run another regression with different functional forms.
  - run another regression with different measures.

# Model Dependence in Practice

- How do you conduct empirical analyses?
  - collect the data over many months or years.
  - finish recording and merging.
  - sit in front of your computer with nobody to bother you.
  - run one regression.
  - run another regression with different control variables.
  - run another regression with different functional forms.
  - run another regression with different measures.
  - run yet another regression with a subset of the data.

# Model Dependence in Practice

- How do you conduct empirical analyses?
  - collect the data over many months or years.
  - finish recording and merging.
  - sit in front of your computer with nobody to bother you.
  - run one regression.
  - run another regression with different control variables.
  - run another regression with different functional forms.
  - run another regression with different measures.
  - run yet another regression with a subset of the data.
  - end up with 100 or 1000 *different* estimates.

# Model Dependence in Practice

- How do you conduct empirical analyses?
  - collect the data over many months or years.
  - finish recording and merging.
  - sit in front of your computer with nobody to bother you.
  - run one regression.
  - run another regression with different control variables.
  - run another regression with different functional forms.
  - run another regression with different measures.
  - run yet another regression with a subset of the data.
  - end up with 100 or 1000 *different* estimates.
  - put 1 or maybe 5 regression results in the paper.

# Model Dependence in Practice

- How do you conduct empirical analyses?
  - collect the data over many months or years.
  - finish recording and merging.
  - sit in front of your computer with nobody to bother you.
  - run one regression.
  - run another regression with different control variables.
  - run another regression with different functional forms.
  - run another regression with different measures.
  - run yet another regression with a subset of the data.
  - end up with 100 or 1000 *different* estimates.
  - put 1 or maybe 5 regression results in the paper.
- What's the problem?



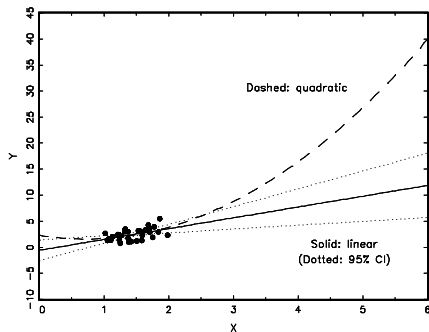
# Model Dependence in Practice

- How do you conduct empirical analyses?
  - collect the data over many months or years.
  - finish recording and merging.
  - sit in front of your computer with nobody to bother you.
  - run one regression.
  - run another regression with different control variables.
  - run another regression with different functional forms.
  - run another regression with different measures.
  - run yet another regression with a subset of the data.
  - end up with 100 or 1000 *different* estimates.
  - put 1 or maybe 5 regression results in the paper.
- What's the problem?
  - Some specification is designated as the “correct” one, only after looking at the estimates.

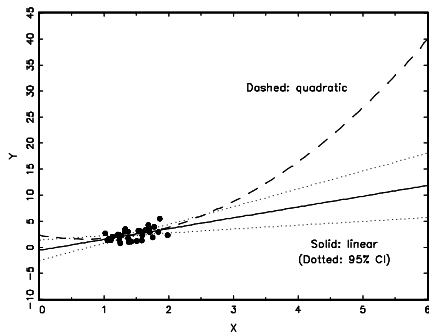
# Model Dependence in Practice

- How do you conduct empirical analyses?
  - collect the data over many months or years.
  - finish recording and merging.
  - sit in front of your computer with nobody to bother you.
  - run one regression.
  - run another regression with different control variables.
  - run another regression with different functional forms.
  - run another regression with different measures.
  - run yet another regression with a subset of the data.
  - end up with 100 or 1000 *different* estimates.
  - put 1 or maybe 5 regression results in the paper.
- What's the problem?
  - Some specification is designated as the “correct” one, only after looking at the estimates.
  - Is this a true test of an ex ante hypothesis or merely a demonstration that it is *possible* to find results consistent with your favorite hypothesis?

# Which model would you choose? (Both fit the data well.)

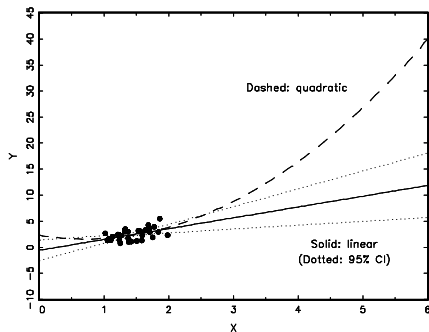


# Which model would you choose? (Both fit the data well.)



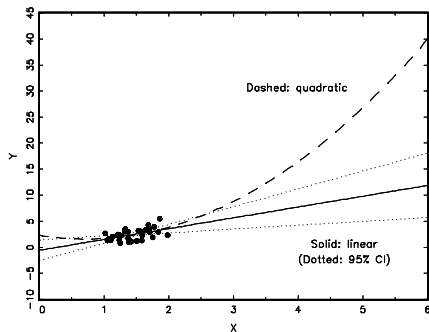
- Compare prediction at  $x = 1.5$  to prediction at  $x = 5$

# Which model would you choose? (Both fit the data well.)



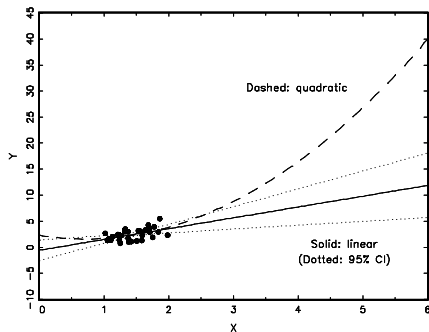
- Compare prediction at  $x = 1.5$  to prediction at  $x = 5$
- How do you choose a model?

# Which model would you choose? (Both fit the data well.)



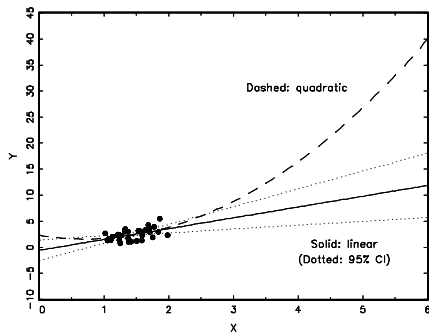
- Compare prediction at  $x = 1.5$  to prediction at  $x = 5$
- How do you choose a model?  $R^2$ ?

# Which model would you choose? (Both fit the data well.)



- Compare prediction at  $x = 1.5$  to prediction at  $x = 5$
- How do you choose a model?  $R^2$ ? Some “test”?

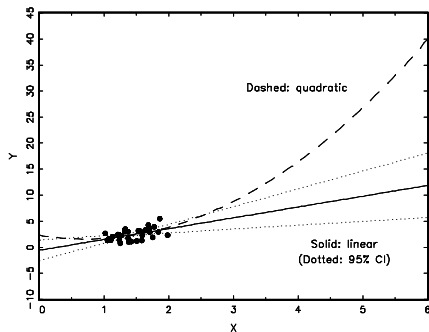
# Which model would you choose? (Both fit the data well.)



- Compare prediction at  $x = 1.5$  to prediction at  $x = 5$
- How do you choose a model?  $R^2$ ? Some “test”? “Theory”?

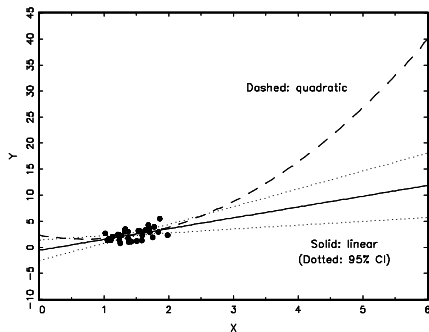


# Which model would you choose? (Both fit the data well.)



- Compare prediction at  $x = 1.5$  to prediction at  $x = 5$
- How do you choose a model?  $R^2$ ? Some “test”? “Theory”?
- The bottom line: answers to some questions don't exist in the data.

# Which model would you choose? (Both fit the data well.)



- Compare prediction at  $x = 1.5$  to prediction at  $x = 5$
- How do you choose a model?  $R^2$ ? Some “test”? “Theory”?
- The bottom line: answers to some questions don't exist in the data.
- Same for what if questions, predictions, and causal inferences

# Model Dependence Proof

# Model Dependence Proof

## Model Free Inference

# Model Dependence Proof

## Model Free Inference

To estimate  $E(Y|X = x)$  at  $x$ , average many observed  $Y$  with value  $x$

# Model Dependence Proof

## Model Free Inference

To estimate  $E(Y|X = x)$  at  $x$ , average many observed  $Y$  with value  $x$

## Assumptions (Model-Based Inference)

# Model Dependence Proof

## Model Free Inference

To estimate  $E(Y|X = x)$  at  $x$ , average many observed  $Y$  with value  $x$

## Assumptions (Model-Based Inference)

- 1 Definition: model dependence at  $x$  is the difference between predicted outcomes for any two models that fit about equally well.

# Model Dependence Proof

## Model Free Inference

To estimate  $E(Y|X = x)$  at  $x$ , average many observed  $Y$  with value  $x$

## Assumptions (Model-Based Inference)

- 1 Definition: model dependence at  $x$  is the difference between predicted outcomes for any two models that fit about equally well.
- 2 The functional form follows strong continuity (think smoothness, although it is less restrictive)



# Model Dependence Proof

## Model Free Inference

To estimate  $E(Y|X = x)$  at  $x$ , average many observed  $Y$  with value  $x$

## Assumptions (Model-Based Inference)

- 1 Definition: model dependence at  $x$  is the difference between predicted outcomes for any two models that fit about equally well.
- 2 The functional form follows strong continuity (think smoothness, although it is less restrictive)

## Result

# Model Dependence Proof

## Model Free Inference

To estimate  $E(Y|X = x)$  at  $x$ , average many observed  $Y$  with value  $x$

## Assumptions (Model-Based Inference)

- 1 Definition: model dependence at  $x$  is the difference between predicted outcomes for any two models that fit about equally well.
- 2 The functional form follows strong continuity (think smoothness, although it is less restrictive)

## Result

The maximum degree of model dependence: solely a function of the **distance from the counterfactual to the data**

# Detecting Model Dependence

# Detecting Model Dependence

## A (Hypothetical) Research Design

# Detecting Model Dependence

## A (Hypothetical) Research Design

- Randomly select a large number of infants

# Detecting Model Dependence

## A (Hypothetical) Research Design

- Randomly select a large number of infants
- Randomly assign them to **0,6,8,10,12,16** years of education

# Detecting Model Dependence

## A (Hypothetical) Research Design

- Randomly select a large number of infants
- Randomly assign them to **0,6,8,10,12,16** years of education
- Assume 100% compliance, and no measurement error, omitted variables, or missing data

# Detecting Model Dependence

## A (Hypothetical) Research Design

- Randomly select a large number of infants
- Randomly assign them to **0,6,8,10,12,16** years of education
- Assume 100% compliance, and no measurement error, omitted variables, or missing data
- Regress cumulative salary in year 17 on education

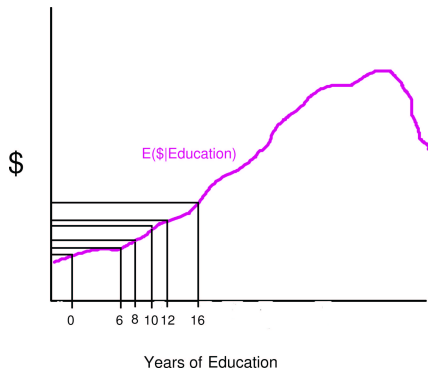


# Detecting Model Dependence

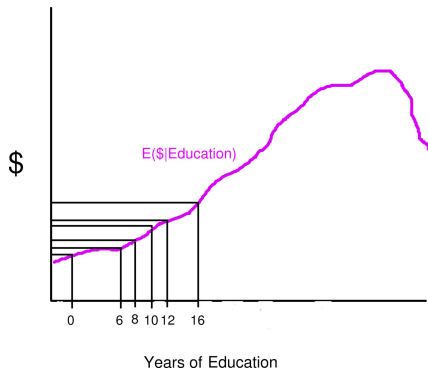
## A (Hypothetical) Research Design

- Randomly select a large number of infants
- Randomly assign them to **0,6,8,10,12,16** years of education
- Assume 100% compliance, and no measurement error, omitted variables, or missing data
- Regress cumulative salary in year 17 on education
- We find a coefficient of  $\hat{\beta} = \$1,000$ , big t-statistics, narrow confidence intervals, and pass every test for auto-correlation, fit, normality, linearity, homoskedasticity, etc.

# What Inferences Would You Be Willing to Make?

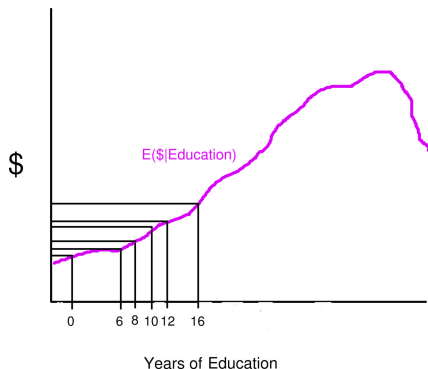


# What Inferences Would You Be Willing to Make?



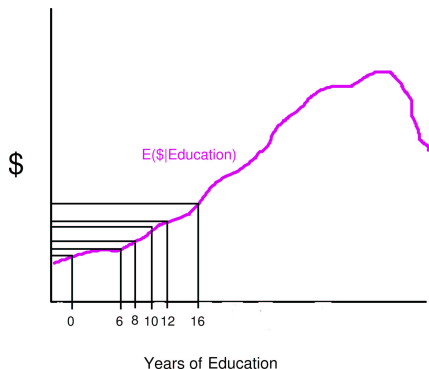
- A Factual Question: How much salary would someone receive with 12 years of education (a high school degree)?

# What Inferences Would You Be Willing to Make?



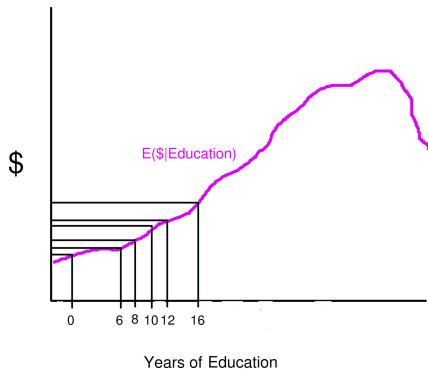
- A Factual Question: How much salary would someone receive with **12** years of education (a high school degree)?
- The **model-free** estimate:  $\text{mean}(Y)$  among those with  $X = 12$ .

# What Inferences Would You Be Willing to Make?

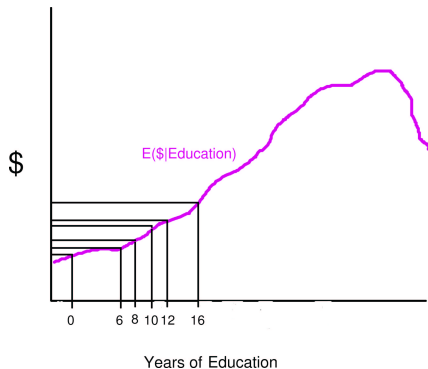


- A Factual Question: How much salary would someone receive with 12 years of education (a high school degree)?
- The **model-free** estimate:  $\text{mean}(Y)$  among those with  $X = 12$ .
- The **model-based** linear estimate:  $\hat{Y} = X\hat{\beta} = 12 \times \$1,000 = \$12,000$

# Counterfactual Inferences with Interpolation

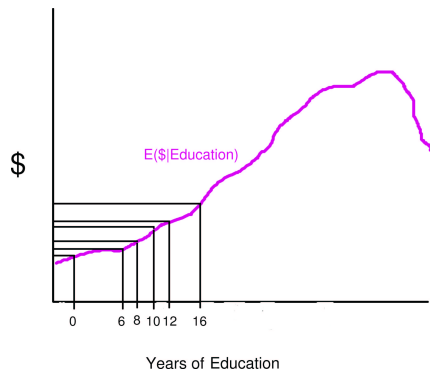


# Counterfactual Inferences with Interpolation



- How much salary would someone receive with **14** years of education (an Associates Degree)?

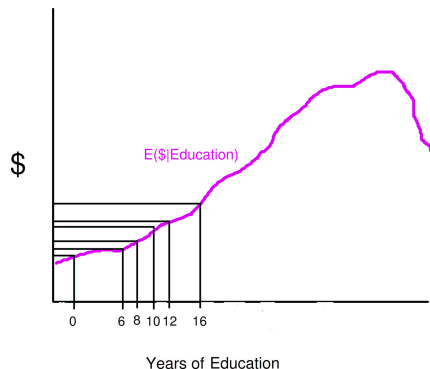
# Counterfactual Inferences with Interpolation



- How much salary would someone receive with **14** years of education (an Associates Degree)?
- Model free estimates impossible.

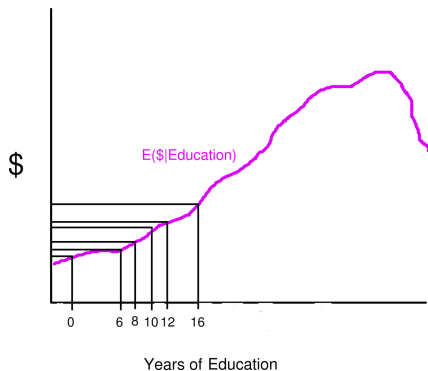


# Counterfactual Inferences with Interpolation

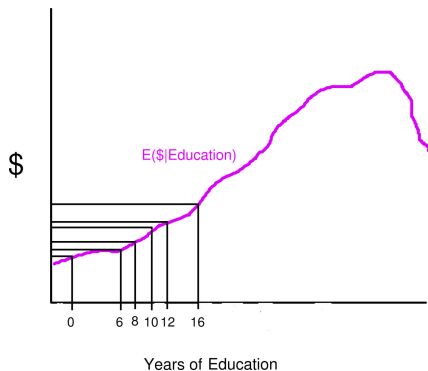


- How much salary would someone receive with **14** years of education (an Associates Degree)?
- Model free estimates impossible.
- $\hat{Y} = X\hat{\beta} = 14 \times \$1,000 = \$14,000$

# Counterfactual Inference with Extrapolation

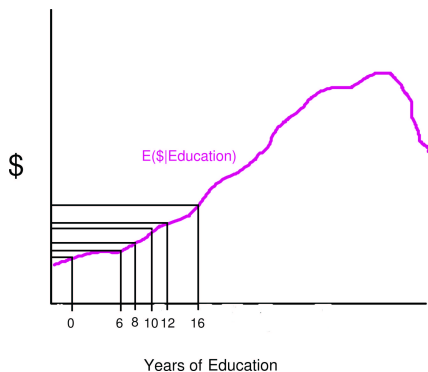


# Counterfactual Inference with Extrapolation



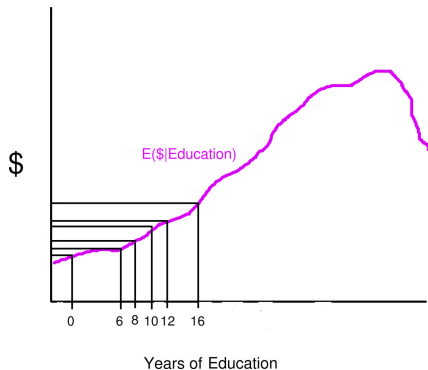
- How much salary would someone receive with 24 years of education (a Ph.D.)?

# Counterfactual Inference with Extrapolation

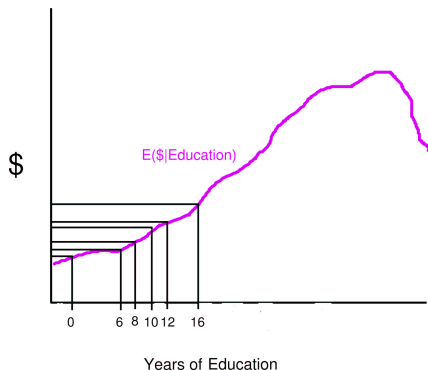


- How much salary would someone receive with 24 years of education (a Ph.D.)?
- $\hat{Y} = X\hat{\beta} = 24 \times \$1,000 = \$24,000$

# Another Counterfactual Inference with Extrapolation

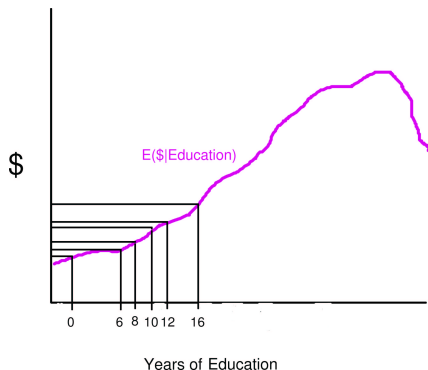


# Another Counterfactual Inference with Extrapolation



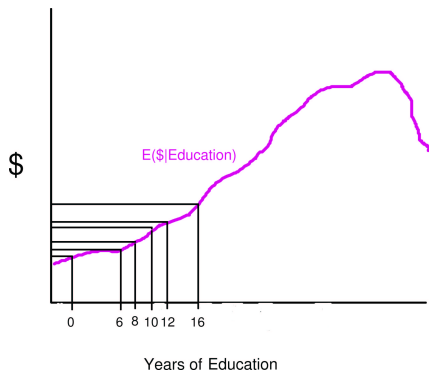
- How much salary would someone receive with **53** years of education?

# Another Counterfactual Inference with Extrapolation



- How much salary would someone receive with **53** years of education?
- $\hat{Y} = X\hat{\beta} = 53 \times \$1,000 = \$53,000$

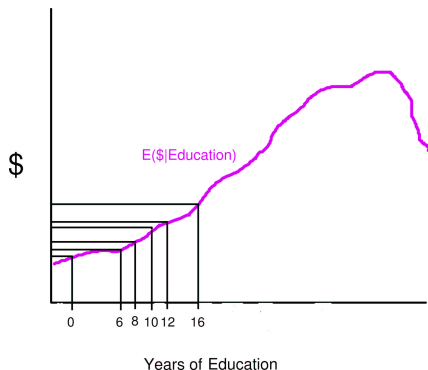
# Another Counterfactual Inference with Extrapolation



- How much salary would someone receive with **53** years of education?
- $\hat{Y} = X\hat{\beta} = 53 \times \$1,000 = \$53,000$
- Recall: the regression passed every test and met every assumption; identical calculations worked for the other questions.



# Another Counterfactual Inference with Extrapolation



- How much salary would someone receive with **53** years of education?
- $\hat{Y} = X\hat{\beta} = 53 \times \$1,000 = \$53,000$
- Recall: the regression passed every test and met every assumption; identical calculations worked for the other questions.
- What's changed? How would we recognize it when the example is less extreme or multidimensional?

# Model Dependence with **One** Explanatory Variable

# Model Dependence with **One** Explanatory Variable

- Suppose  $Y$  is starting salary;  $X$  is education in 10 categories.

# Model Dependence with **One** Explanatory Variable

- Suppose  $Y$  is starting salary;  $X$  is education in 10 categories.
- To estimate  $E(Y|X)$ : we need 10 parameters,  $E(Y|X = x_j)$ ,  $j = 1, \dots, 10$ .

# Model Dependence with **One** Explanatory Variable

- Suppose  $Y$  is starting salary;  $X$  is education in 10 categories.
- To estimate  $E(Y|X)$ : we need 10 parameters,  $E(Y|X = x_j)$ ,  $j = 1, \dots, 10$ .
- **Model-free** method: average 50 observations on  $Y$  for each value of  $X$

# Model Dependence with **One** Explanatory Variable

- Suppose  $Y$  is starting salary;  $X$  is education in 10 categories.
- To estimate  $E(Y|X)$ : we need 10 parameters,  $E(Y|X = x_j)$ ,  $j = 1, \dots, 10$ .
- **Model-free** method: average 50 observations on  $Y$  for each value of  $X$
- **Model-based** method: regress  $Y$  on  $X$ , summarizing 10 parameters with 2 (intercept and slope).

# Model Dependence with **One** Explanatory Variable

- Suppose  $Y$  is starting salary;  $X$  is education in 10 categories.
- To estimate  $E(Y|X)$ : we need 10 parameters,  $E(Y|X = x_j)$ ,  $j = 1, \dots, 10$ .
- **Model-free** method: average 50 observations on  $Y$  for each value of  $X$
- **Model-based** method: regress  $Y$  on  $X$ , summarizing 10 parameters with 2 (intercept and slope).
- The **difference** between the 10 we need and the 2 we estimate with regression is **pure assumption**.

# Model Dependence with **One** Explanatory Variable

- Suppose  $Y$  is starting salary;  $X$  is education in 10 categories.
- To estimate  $E(Y|X)$ : we need 10 parameters,  $E(Y|X = x_j)$ ,  $j = 1, \dots, 10$ .
- **Model-free** method: average 50 observations on  $Y$  for each value of  $X$
- **Model-based** method: regress  $Y$  on  $X$ , summarizing 10 parameters with 2 (intercept and slope).
- The **difference** between the 10 we need and the 2 we estimate with regression is **pure assumption**.
- If  $X$  were continuous, we would be reducing  $\infty$  to 2, also by assumption.



# Model Dependence with **Two** Explanatory Variables

# Model Dependence with **Two** Explanatory Variables

Variables:  $X$  (education) and  $Z$ , parent's income, both with 10 categories

# Model Dependence with **Two** Explanatory Variables

Variables:  $X$  (education) and  $Z$ , parent's income, both with 10 categories

- How many parameters do we now need to estimate?

# Model Dependence with **Two** Explanatory Variables

Variables:  $X$  (education) and  $Z$ , parent's income, both with 10 categories

- How many parameters do we now need to estimate? 20?

# Model Dependence with **Two** Explanatory Variables

Variables:  $X$  (education) and  $Z$ , parent's income, both with 10 categories

- How many parameters do we now need to estimate? 20? Nope.

# Model Dependence with **Two** Explanatory Variables

Variables:  $X$  (education) and  $Z$ , parent's income, both with 10 categories

- How many parameters do we now need to estimate? 20? Nope. Its  $10 \times 10 = 100$ .

# Model Dependence with **Two** Explanatory Variables

Variables:  $X$  (education) and  $Z$ , parent's income, both with 10 categories

- How many parameters do we now need to estimate? 20? Nope. Its  $10 \times 10 = 100$ . This is the curse of dimensionality: the number of parameters goes up geometrically, not additively.

# Model Dependence with **Two** Explanatory Variables

Variables:  $X$  (education) and  $Z$ , parent's income, both with 10 categories

- How many parameters do we now need to estimate? 20? Nope. Its  $10 \times 10 = 100$ . This is the curse of dimensionality: the number of parameters goes up geometrically, not additively.
- If we run a regression, we are summarizing 100 parameters with 3 (an intercept and two slopes).



# Model Dependence with **Two** Explanatory Variables

Variables:  $X$  (education) and  $Z$ , parent's income, both with 10 categories

- How many parameters do we now need to estimate? 20? Nope. Its  $10 \times 10 = 100$ . This is the curse of dimensionality: the number of parameters goes up geometrically, not additively.
- If we run a regression, we are summarizing 100 parameters with 3 (an intercept and two slopes).
- But what about including an interaction? Right, so now we're summarizing 100 parameters with 4.

# Model Dependence with **Two** Explanatory Variables

Variables:  $X$  (education) and  $Z$ , parent's income, both with 10 categories

- How many parameters do we now need to estimate? 20? Nope. Its  $10 \times 10 = 100$ . This is the curse of dimensionality: the number of parameters goes up geometrically, not additively.
- If we run a regression, we are summarizing 100 parameters with 3 (an intercept and two slopes).
- But what about including an interaction? Right, so now we're summarizing 100 parameters with 4.
- The difference is still one enormous assumption based on convenience, and neither evidence nor theory.

# Model Dependence with **Many** Explanatory Variables

# Model Dependence with **Many** Explanatory Variables

- Suppose: 15 explanatory variables, with 10 categories each.

# Model Dependence with **Many** Explanatory Variables

- Suppose: 15 explanatory variables, with 10 categories each.
  - need to estimate  $10^{15}$  (a quadrillion) parameters with how many observations?

# Model Dependence with **Many** Explanatory Variables

- Suppose: 15 explanatory variables, with 10 categories each.
  - need to estimate  $10^{15}$  (a quadrillion) parameters with how many observations?
  - Regression reduces this to 16 parameters, by assumption.

# Model Dependence with **Many** Explanatory Variables

- Suppose: 15 explanatory variables, with 10 categories each.
  - need to estimate  $10^{15}$  (a quadrillion) parameters with how many observations?
  - Regression reduces this to 16 parameters, by assumption.
- Suppose: 80 explanatory variables.

# Model Dependence with **Many** Explanatory Variables

- Suppose: 15 explanatory variables, with 10 categories each.
  - need to estimate  $10^{15}$  (a quadrillion) parameters with how many observations?
  - Regression reduces this to 16 parameters, by assumption.
- Suppose: 80 explanatory variables.
  - $10^{80}$  is more than the number of atoms in the universe.



# Model Dependence with **Many** Explanatory Variables

- Suppose: 15 explanatory variables, with 10 categories each.
  - need to estimate  $10^{15}$  (a quadrillion) parameters with how many observations?
  - Regression reduces this to 16 parameters, by assumption.
- Suppose: 80 explanatory variables.
  - $10^{80}$  is more than the number of atoms in the universe.
  - Yet, with a few simple assumptions, we can still run a regression and estimate only 81 parameters.

# Model Dependence with **Many** Explanatory Variables

- Suppose: 15 explanatory variables, with 10 categories each.
  - need to estimate  $10^{15}$  (a quadrillion) parameters with how many observations?
  - Regression reduces this to 16 parameters, by assumption.
- Suppose: 80 explanatory variables.
  - $10^{80}$  is more than the number of atoms in the universe.
  - Yet, with a few simple assumptions, we can still run a regression and estimate only 81 parameters.
- The curse of dimensionality introduces huge assumptions, often recognized.

# We Ask: How Factual is your Counterfactual?

# We Ask: How Factual is your Counterfactual?

- Readers have the right to know: is your counterfactual close enough to data so that statistical methods provide *empirical* answers?

# We Ask: How Factual is your Counterfactual?

- Readers have the right to know: is your counterfactual close enough to data so that statistical methods provide *empirical* answers?
- If not, the same calculations will be based on indefensible model assumptions. With the curse of dimensionality, its too easy to fall into this trap.

# We Ask: How Factual is your Counterfactual?

- Readers have the right to know: is your counterfactual close enough to data so that statistical methods provide *empirical* answers?
- If not, the same calculations will be based on indefensible model assumptions. With the curse of dimensionality, its too easy to fall into this trap.
- A good existing approach: *Sensitivity testing*, but this requires the user to specify a class of models and then to estimate them all and check how much inferences change

# We Ask: How Factual is your Counterfactual?

- Readers have the right to know: is your counterfactual close enough to data so that statistical methods provide *empirical* answers?
- If not, the same calculations will be based on indefensible model assumptions. With the curse of dimensionality, its too easy to fall into this trap.
- A good existing approach: *Sensitivity testing*, but this requires the user to specify a class of models and then to estimate them all and check how much inferences change
- Our alternative approach:

# We Ask: How Factual is your Counterfactual?

- Readers have the right to know: is your counterfactual close enough to data so that statistical methods provide *empirical* answers?
- If not, the same calculations will be based on indefensible model assumptions. With the curse of dimensionality, its too easy to fall into this trap.
- A good existing approach: *Sensitivity testing*, but this requires the user to specify a class of models and then to estimate them all and check how much inferences change
- Our alternative approach:
  - Specify your explanatory variables,  $X$ .



# We Ask: How Factual is your Counterfactual?

- Readers have the right to know: is your counterfactual close enough to data so that statistical methods provide *empirical* answers?
- If not, the same calculations will be based on indefensible model assumptions. With the curse of dimensionality, its too easy to fall into this trap.
- A good existing approach: *Sensitivity testing*, but this requires the user to specify a class of models and then to estimate them all and check how much inferences change
- Our alternative approach:
  - Specify your explanatory variables,  $X$ .
  - Assume  $E(Y|X)$  is (minimally) smooth in  $X$

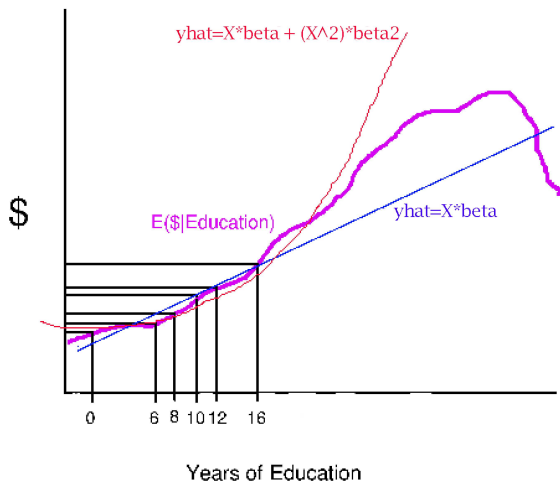
# We Ask: How Factual is your Counterfactual?

- Readers have the right to know: is your counterfactual close enough to data so that statistical methods provide *empirical* answers?
- If not, the same calculations will be based on indefensible model assumptions. With the curse of dimensionality, its too easy to fall into this trap.
- A good existing approach: *Sensitivity testing*, but this requires the user to specify a class of models and then to estimate them all and check how much inferences change
- Our alternative approach:
  - Specify your explanatory variables,  $X$ .
  - Assume  $E(Y|X)$  is (minimally) smooth in  $X$
  - No need to specify models (or a class of models), estimators, or dependent variables.

# We Ask: How Factual is your Counterfactual?

- Readers have the right to know: is your counterfactual close enough to data so that statistical methods provide *empirical* answers?
- If not, the same calculations will be based on indefensible model assumptions. With the curse of dimensionality, its too easy to fall into this trap.
- A good existing approach: *Sensitivity testing*, but this requires the user to specify a class of models and then to estimate them all and check how much inferences change
- Our alternative approach:
  - Specify your explanatory variables,  $X$ .
  - Assume  $E(Y|X)$  is (minimally) smooth in  $X$
  - No need to specify models (or a class of models), estimators, or dependent variables.
  - Results of one run apply to the class of all models, all estimators, and all dependent variables.

# Interpolation vs Extrapolation in one Dimension



# Interpolation or Extrapolation in One and Two Dimensions

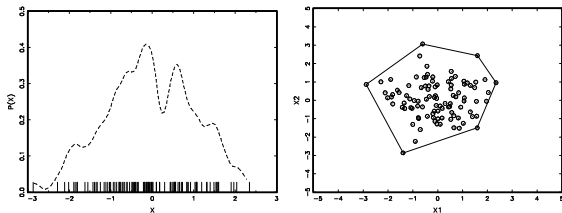


Figure: The Convex Hull

# Interpolation or Extrapolation in One and Two Dimensions

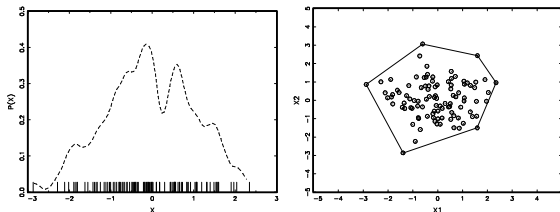


Figure: The Convex Hull

- **Interpolation:** Inside the convex hull

# Interpolation or Extrapolation in One and Two Dimensions

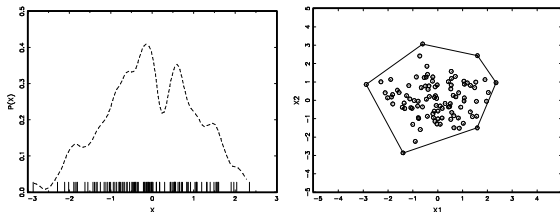


Figure: The Convex Hull

- **Interpolation:** Inside the convex hull
- **Extrapolation:** Outside the convex hull

# Interpolation or Extrapolation in One and Two Dimensions

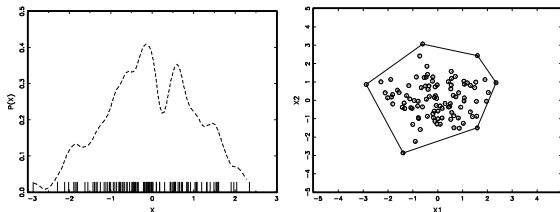


Figure: The Convex Hull

- **Interpolation:** Inside the convex hull
- **Extrapolation:** Outside the convex hull
- Works mathematically for any number of  $X$  variables



# Interpolation or Extrapolation in One and Two Dimensions

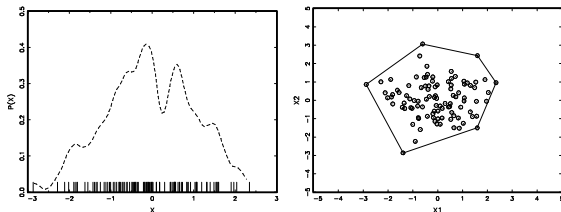


Figure: The Convex Hull

- **Interpolation:** Inside the convex hull
- **Extrapolation:** Outside the convex hull
- Works mathematically for any number of  $X$  variables
- We show how to determine whether a point is in the hull without calculating the hull, so its fast; see <http://GKing.harvard.edu/whatif>

# Replication: Doyle and Sambanis, APSR 2000

# Replication: Doyle and Sambanis, APSR 2000

- Data: 124 Post-World War II civil wars

# Replication: Doyle and Sambanis, APSR 2000

- Data: 124 Post-World War II civil wars
- Dependent variable: peacebuilding success

# Replication: Doyle and Sambanis, APSR 2000

- Data: 124 Post-World War II civil wars
- Dependent variable: peacebuilding success
- Treatment variable: multilateral UN peacekeeping intervention (0/1)

# Replication: Doyle and Sambanis, APSR 2000

- Data: 124 Post-World War II civil wars
- Dependent variable: peacebuilding success
- Treatment variable: multilateral UN peacekeeping intervention (0/1)
- Control variables: war type, severity, and duration; development status; etc...

# Replication: Doyle and Sambanis, APSR 2000

- Data: 124 Post-World War II civil wars
- Dependent variable: peacebuilding success
- Treatment variable: multilateral UN peacekeeping intervention (0/1)
- Control variables: war type, severity, and duration; development status; etc...
- Counterfactuals: UN intervention switched (0/1 to 1/0) for each observation

# Replication: Doyle and Sambanis, APSR 2000

- Data: 124 Post-World War II civil wars
- Dependent variable: peacebuilding success
- Treatment variable: multilateral UN peacekeeping intervention (0/1)
- Control variables: war type, severity, and duration; development status; etc...
- Counterfactuals: UN intervention switched (0/1 to 1/0) for each observation
- Percent of counterfactuals in the convex hull:



- Data: 124 Post-World War II civil wars
- Dependent variable: peacebuilding success
- Treatment variable: multilateral UN peacekeeping intervention (0/1)
- Control variables: war type, severity, and duration; development status; etc...
- Counterfactuals: UN intervention switched (0/1 to 1/0) for each observation
- Percent of counterfactuals in the convex hull: 0%

# Replication: Doyle and Sambanis, APSR 2000

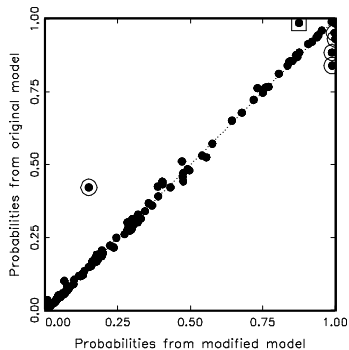
- Data: 124 Post-World War II civil wars
- Dependent variable: peacebuilding success
- Treatment variable: multilateral UN peacekeeping intervention (0/1)
- Control variables: war type, severity, and duration; development status; etc...
- Counterfactuals: UN intervention switched (0/1 to 1/0) for each observation
- **Percent of counterfactuals in the convex hull: 0%**
- Thus, without estimating any models, we know inferences will be model dependent; for illustration, let's find an example. . . .

# Doyle and Sambanis, Logit Model

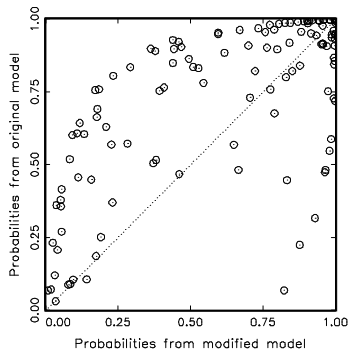
Variables	Original Model			Modified Model		
	Coeff	SE	P-val	Coeff	SE	P-val
Wartype	-1.742	.609	.004	-1.666	.606	.006
Logdead	-.445	.126	.000	-.437	.125	.000
Wardur	.006	.006	.258	.006	.006	.342
Factnum	-1.259	.703	.073	-1.045	.899	.245
Factnum2	.062	.065	.346	.032	.104	.756
Trnsfcap	.004	.002	.010	.004	.002	.017
Develop	.001	.000	.065	.001	.000	.068
Exp	-6.016	3.071	.050	-6.215	3.065	.043
Decade	-.299	.169	.077	-0.284	.169	.093
Treaty	2.124	.821	.010	2.126	.802	.008
UNOP4	3.135	1.091	.004	.262	1.392	.851
Wardur*UNOP4	—	—	—	.037	.011	.001
Constant	8.609	2.157	0.000	7.978	2.350	.000
N		122			122	
Log-likelihood		-45.649			-44.902	
Pseudo $R^2$		.423			.433	

# Doyle and Sambanis: Model Dependence

In Sample Fit



Counterfactual Prediction



# Biases in Causal Inference: A New Decomposition

$$d = \text{mean}(Y|D = 1) - \text{mean}(Y|D = 0)$$

# Biases in Causal Inference: A New Decomposition

$$d = \text{mean}(Y|D = 1) - \text{mean}(Y|D = 0)$$

# Biases in Causal Inference: A New Decomposition

$$d = \text{mean}(Y|D = 1) - \text{mean}(Y|D = 0)$$

$$\text{bias} \equiv E(d) - \theta$$

# Biases in Causal Inference: A New Decomposition

$$d = \text{mean}(Y|D = 1) - \text{mean}(Y|D = 0)$$

$$\text{bias} \equiv E(d) - \theta = \Delta_o + \Delta_p + \Delta_i + \Delta_e$$



# Biases in Causal Inference: A New Decomposition

$$d = \text{mean}(Y|D = 1) - \text{mean}(Y|D = 0)$$

$$\text{bias} \equiv E(d) - \theta = \Delta_o + \Delta_p + \Delta_i + \Delta_e$$

- $\Delta_o$  Omitted variable bias

# Biases in Causal Inference: A New Decomposition

$$d = \text{mean}(Y|D = 1) - \text{mean}(Y|D = 0)$$

$$\text{bias} \equiv E(d) - \theta = \Delta_o + \Delta_p + \Delta_i + \Delta_e$$

- $\Delta_o$  Omitted variable bias
- $\Delta_p$  Post-treatment bias

# Biases in Causal Inference: A New Decomposition

$$d = \text{mean}(Y|D = 1) - \text{mean}(Y|D = 0)$$

$$\text{bias} \equiv E(d) - \theta = \Delta_o + \Delta_p + \Delta_i + \Delta_e$$

- $\Delta_o$  Omitted variable bias
- $\Delta_p$  Post-treatment bias
- $\Delta_i$  Interpolation bias

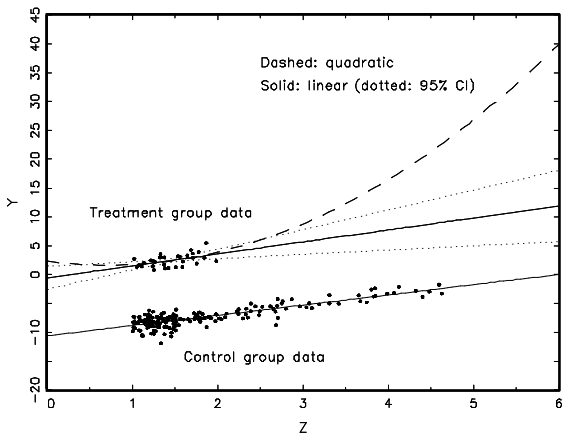
# Biases in Causal Inference: A New Decomposition

$$d = \text{mean}(Y|D = 1) - \text{mean}(Y|D = 0)$$

$$\text{bias} \equiv E(d) - \theta = \Delta_o + \Delta_p + \Delta_i + \Delta_e$$

- $\Delta_o$  Omitted variable bias
- $\Delta_p$  Post-treatment bias
- $\Delta_i$  Interpolation bias
- $\Delta_e$  Extrapolation bias

# Interpolation vs Extrapolation Bias



# Causal Effect of Multidimensional UN Peacekeeping Operations

