Demographic Forecasting

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Harvard University

Joint work with Federico Girosi (RAND)
with contributions from Kevin Quinn and Gregory Wawro

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- Approach: Formalizing qualitative insights in quantitative models

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- Better ways of creating Bayesian priors
- Produces forecasts and farcasts using considerably more information

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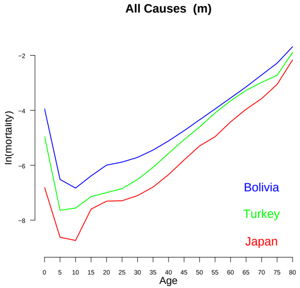

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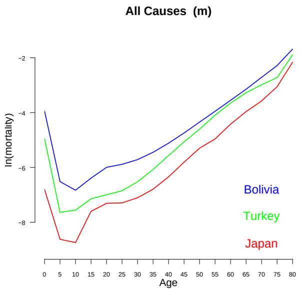

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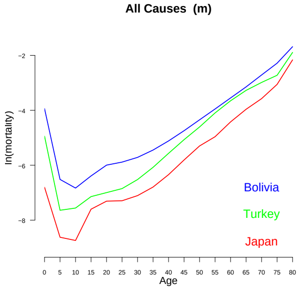

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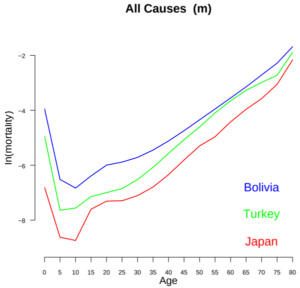

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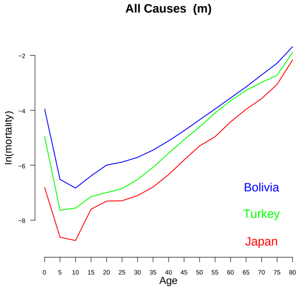

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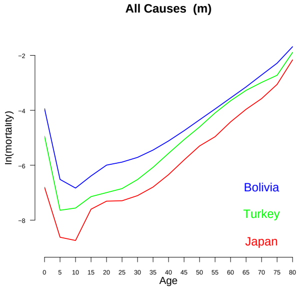

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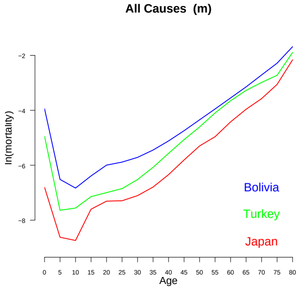

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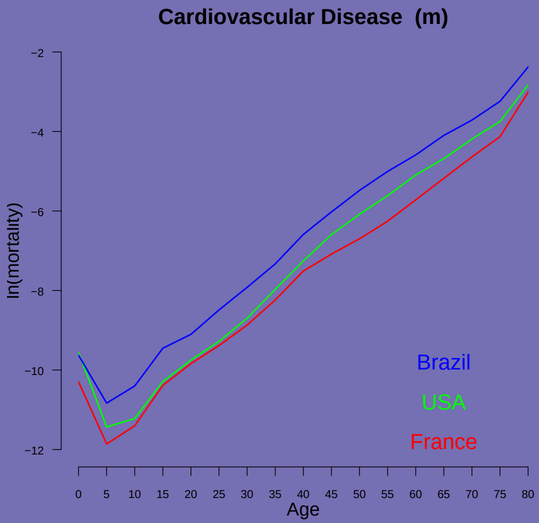

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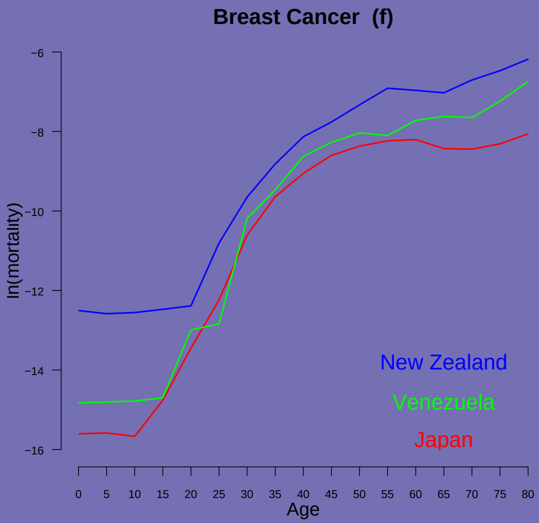

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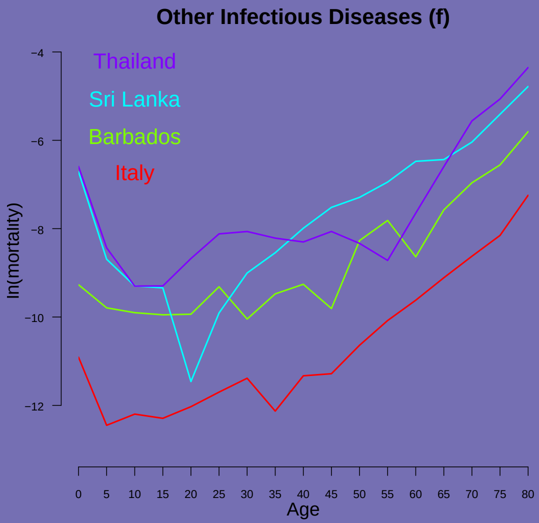

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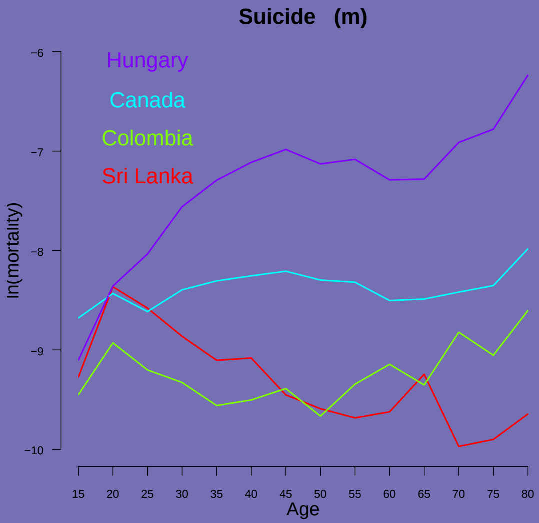

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- Also: Method ignores covariate information; the leading current method (McNown-Rogers) not replicable

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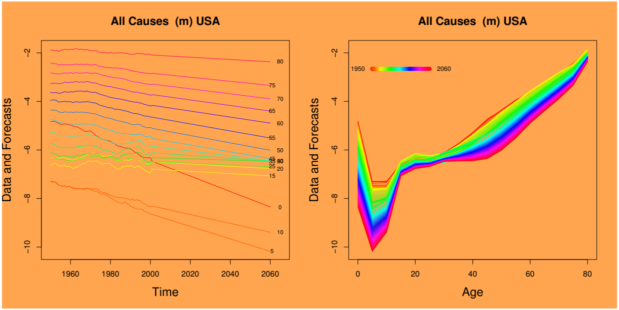

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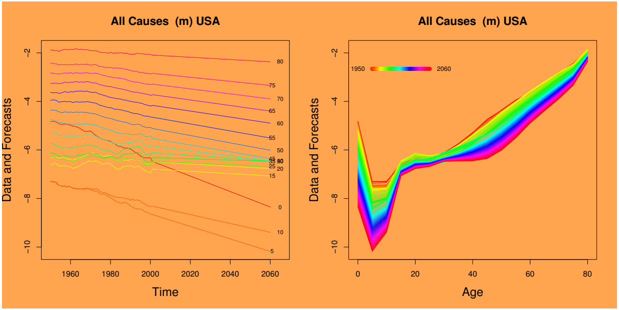

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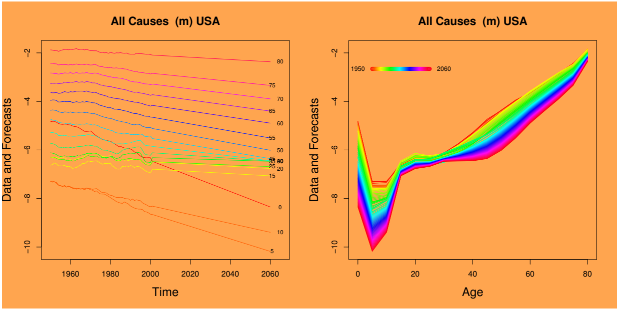

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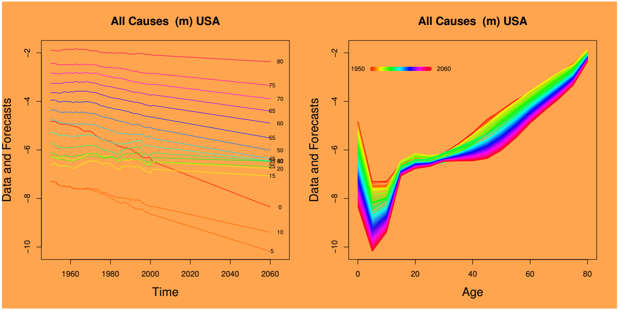

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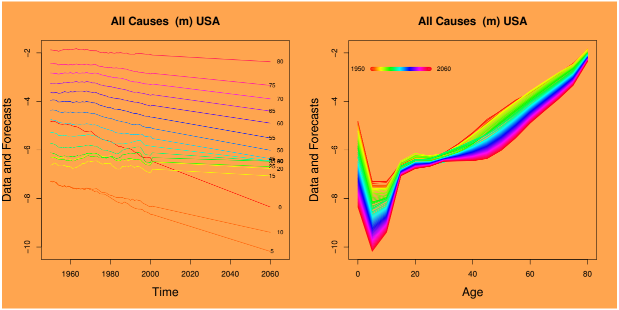

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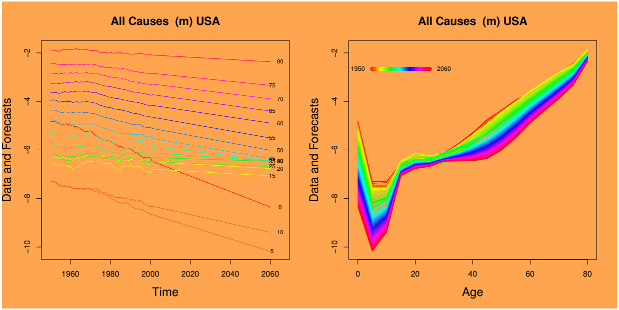

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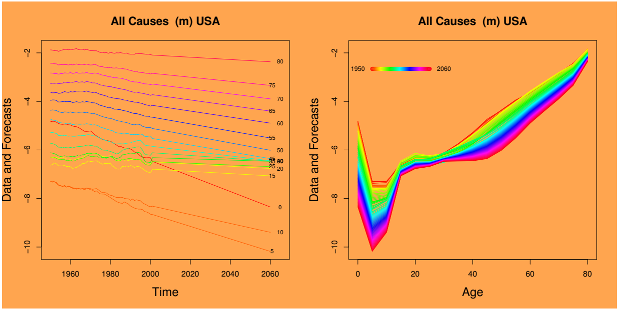

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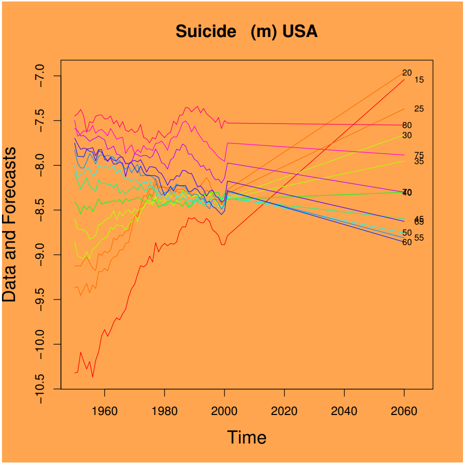

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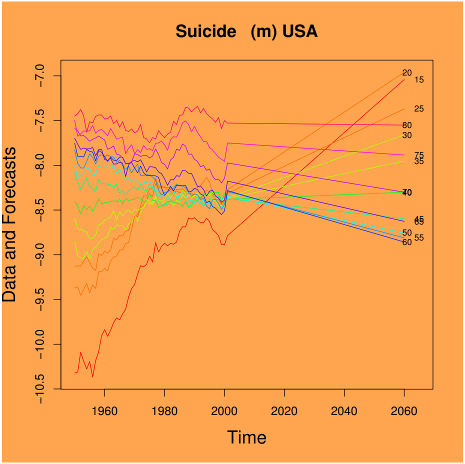

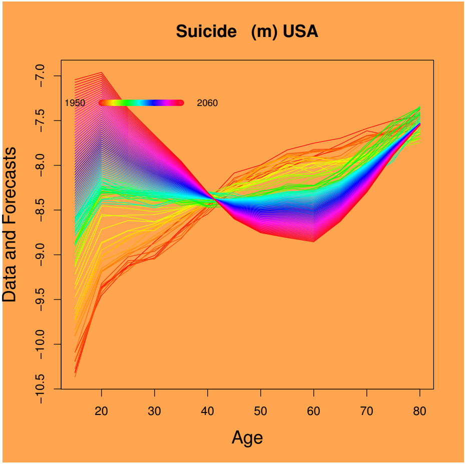

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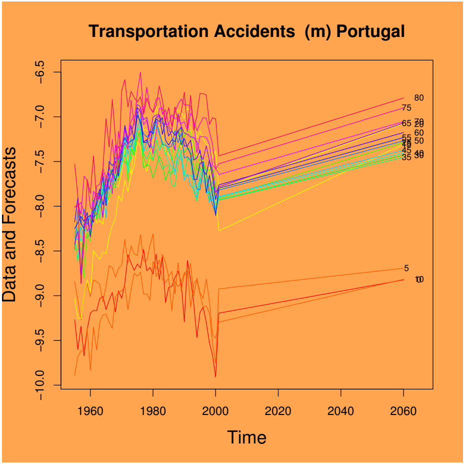

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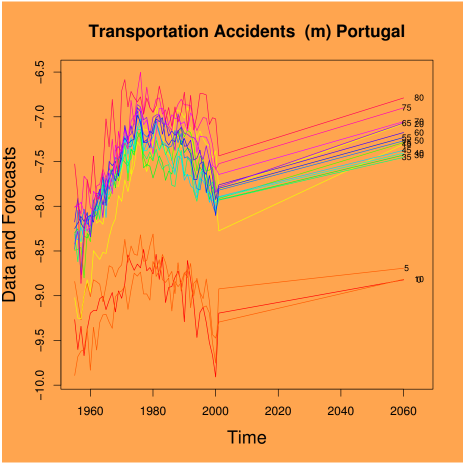

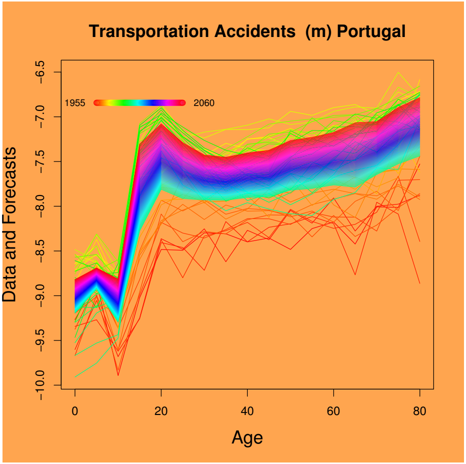

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Partial Pooling via a Bayesian Hierarchical Approach

- Likelihood for equation-by-equation least squares:

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- The easy part: *easy-to-use software* to implement everything we discuss today.

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Natural choice for the prior:

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- Extensive trial-and-error runs, yielded no useful parameter values.

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3. In the subspace, we can invert $\mu = \mathbf{Z}\boldsymbol{\beta}$ as $\boldsymbol{\beta} = (\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\mu$, giving:

$$\mathcal{P}(\boldsymbol{\beta} \mid \theta) \propto \exp\left(-\frac{1}{2}H[\mu,\theta]\right) = \exp\left(-\frac{1}{2}H[\mathbf{Z}\boldsymbol{\beta},\theta]\right)$$

   the same prior on μ, expressed as a function of $\boldsymbol{\beta}$ (with constant Jacobian).

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- We start with one-dimensional $P(\mu_{cat})$, and treat it as the multidimensional $P(\boldsymbol{\beta}_{ca})$, constant in all directions but $Z_{cat}\boldsymbol{\beta}_{ca}$.

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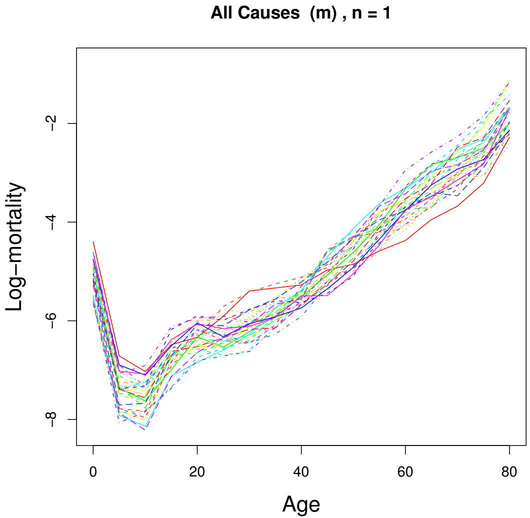
- Different age groups can have different covariates: the matrices $\mathbf{C}_{aa'} \equiv \frac{1}{T}\mathbf{Z}'_a\mathbf{Z}_{a'}$ are rectangular $(d_a \times d_{a'})$.

Samples From Age Prior



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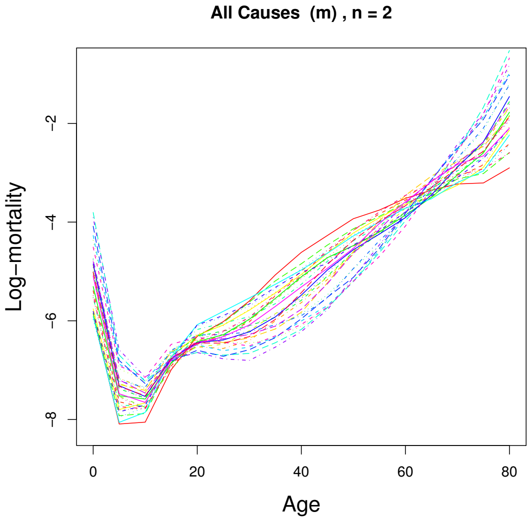

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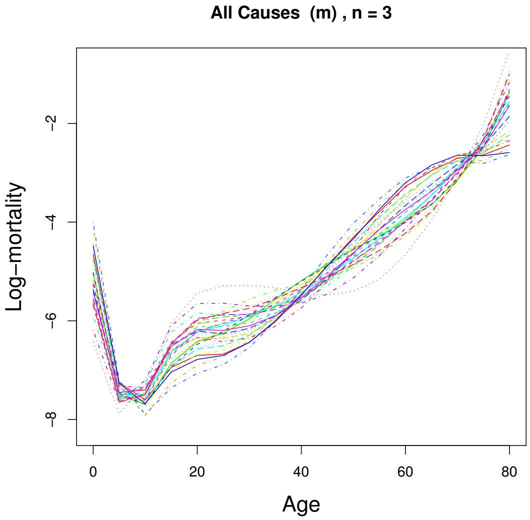

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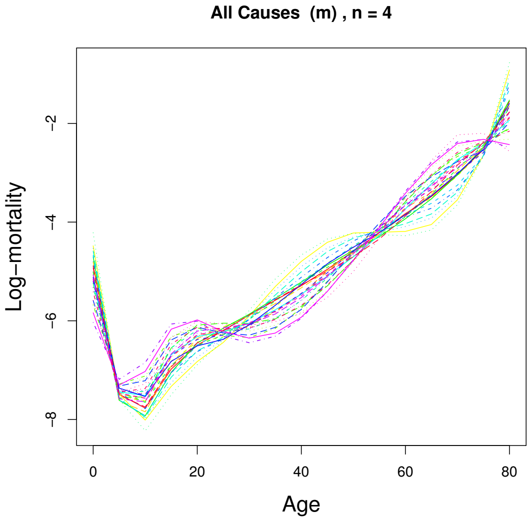

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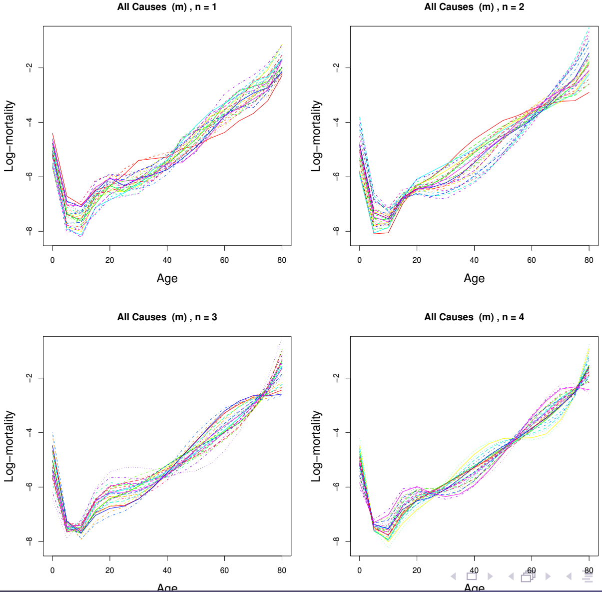

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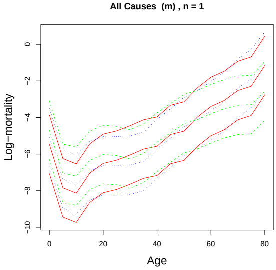

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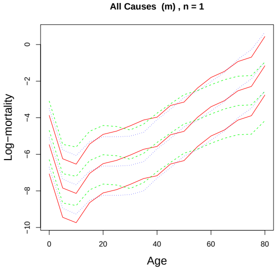
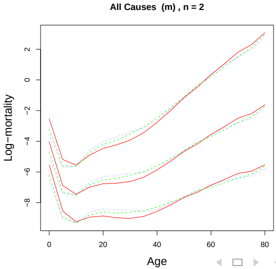


Level and slope indifference



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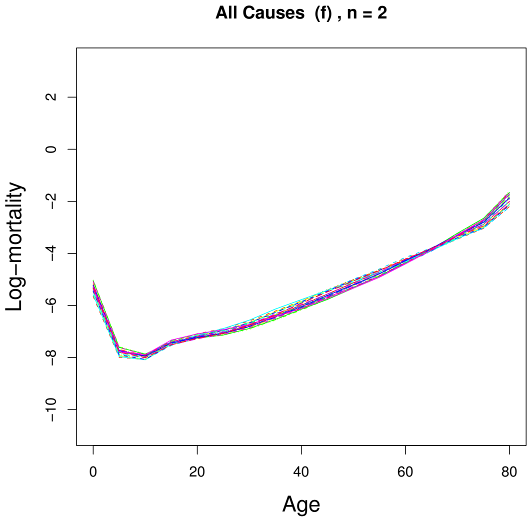
Smoothness Parameter

- The prior:

$$\mathcal{P}(\beta \mid \theta) \propto \exp\left(-\theta \sum_{aa'} W^{n}_{aa'} \beta'_{a} \mathbf{C}_{aa'} \beta_{a'}\right)$$

- We figured out what n is
- but what is the smoothness parameter, θ?
- θ controls the prior standard deviation

Samples from Age Prior



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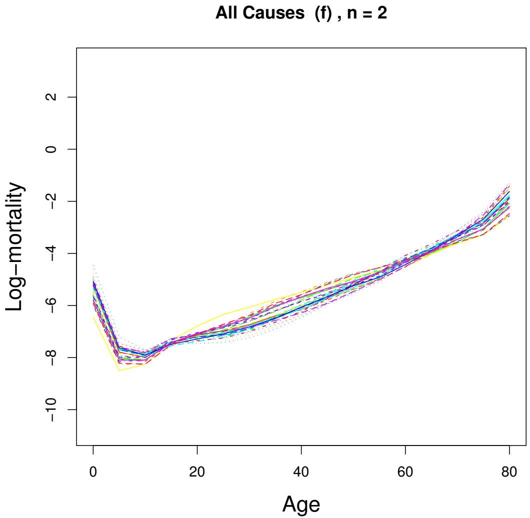

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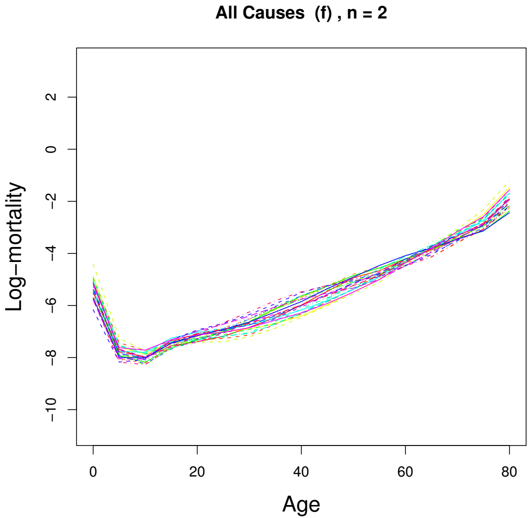

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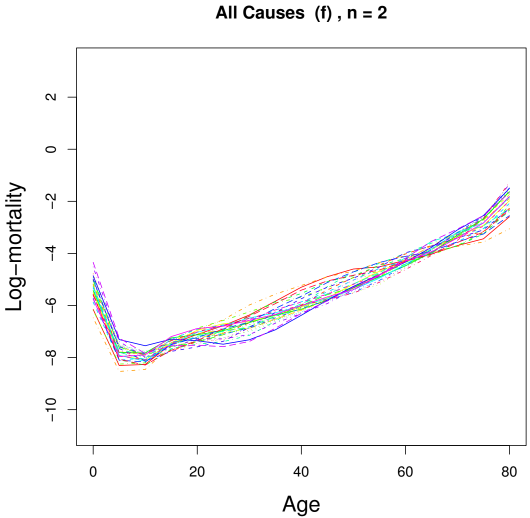

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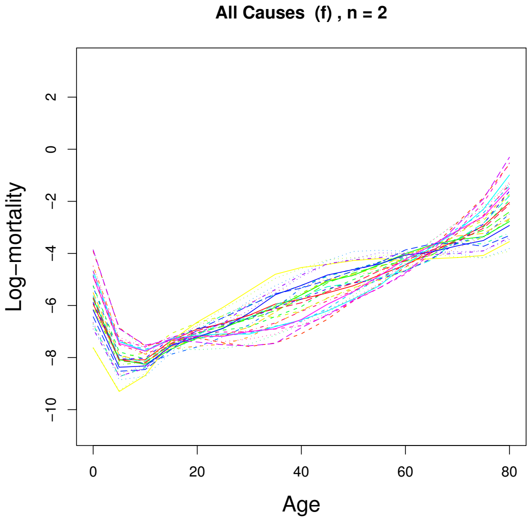

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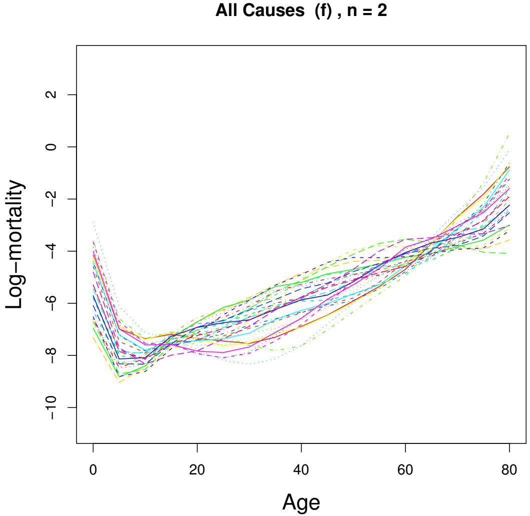

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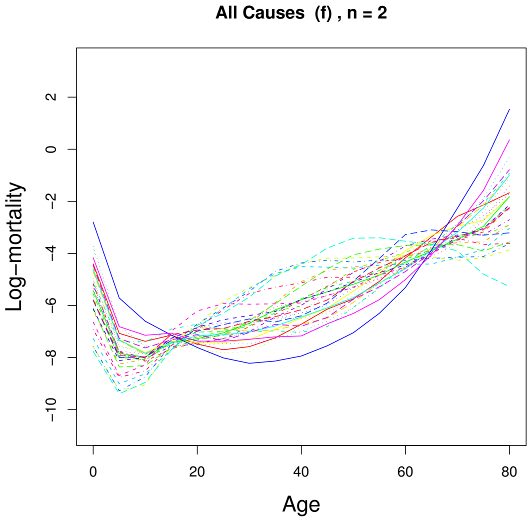

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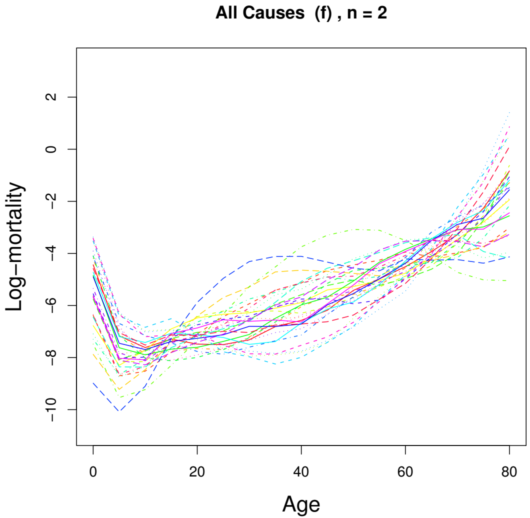

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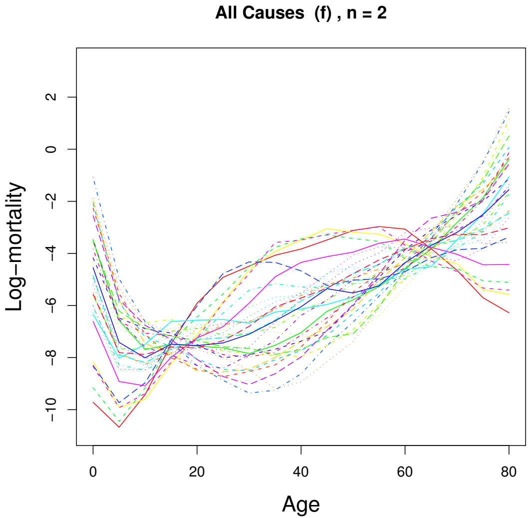

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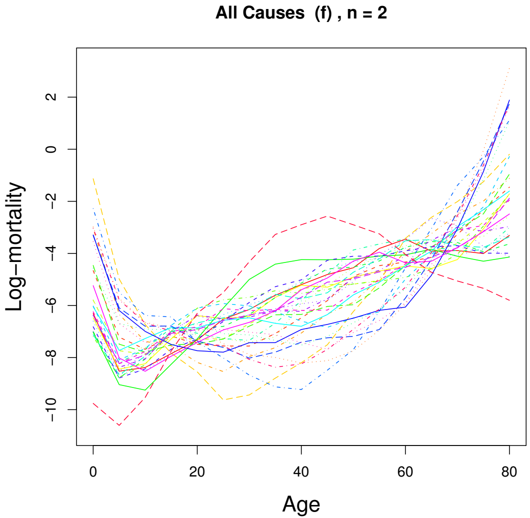

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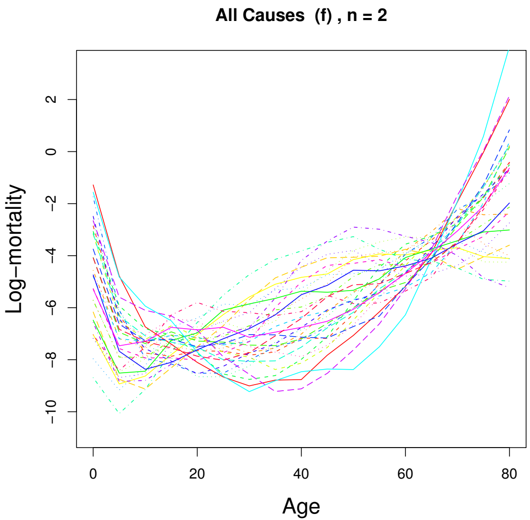

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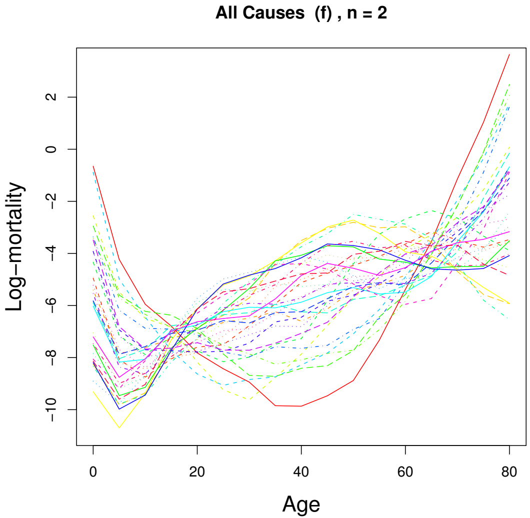

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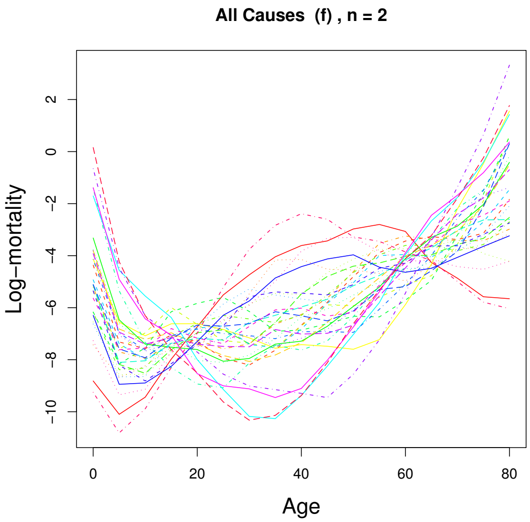

Generalizations

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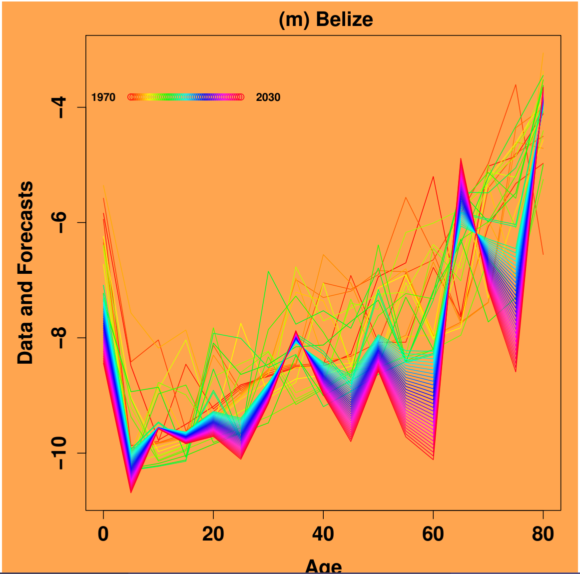
  - Smoothing trends over age groups as they vary across countries, etc.
- The mathematical form for *all* these (separately or together) turns out to be the same:

$$\mathcal{P}(\beta \mid \theta) \propto \exp\left(-\frac{\theta}{2}\sum_{ij} W_{ij}\beta_i' \mathbf{C}_{ij}\beta_j\right), \qquad \mathbf{C}_{aa'} \equiv \frac{1}{T}\mathbf{Z}_a\mathbf{Z}_{a'}$$

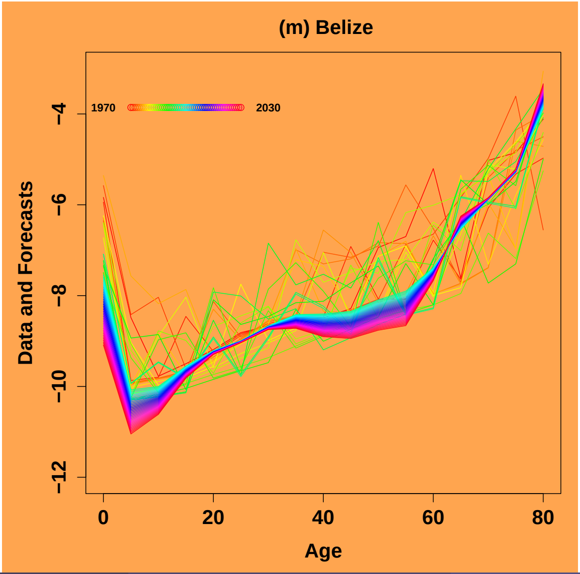
Mortality from Respiratory Infections, Males

Least Squares



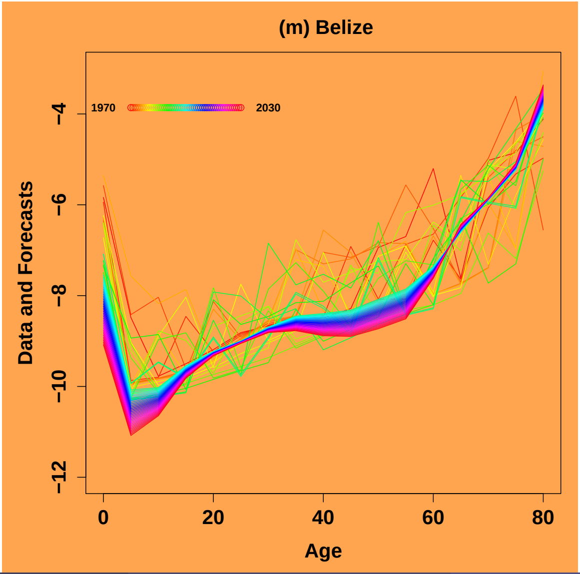
Mortality from Respiratory Infections, males, $\sigma = 2.00$

Smoothing over Age Groups



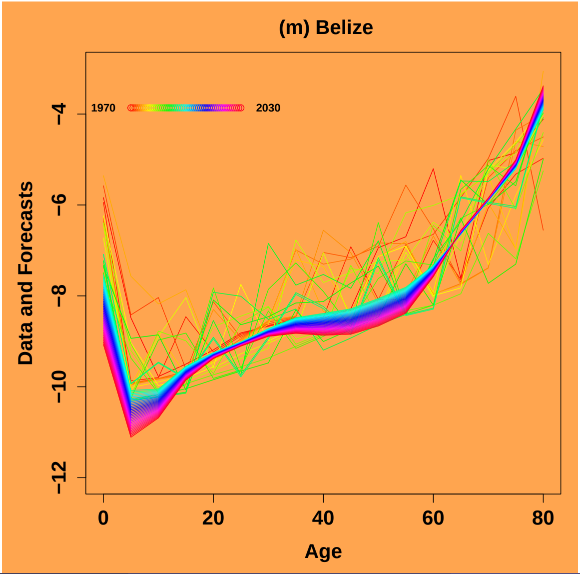
Mortality from Respiratory Infections, males, $\sigma = 1.51$

Smoothing over Age Groups



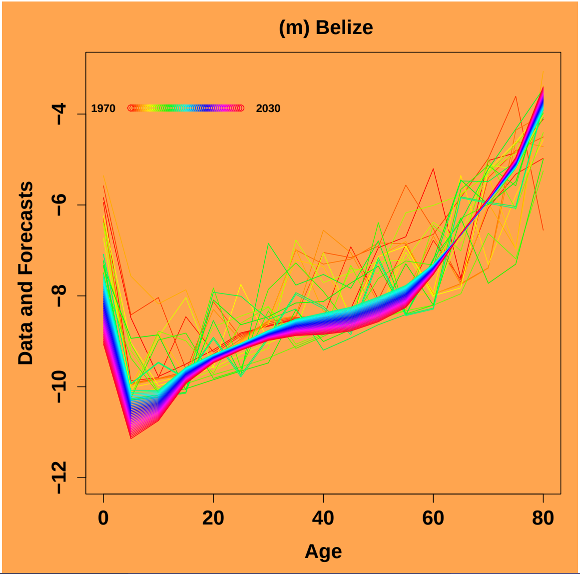
Mortality from Respiratory Infections, males, $\sigma = 1.15$

Smoothing over Age Groups



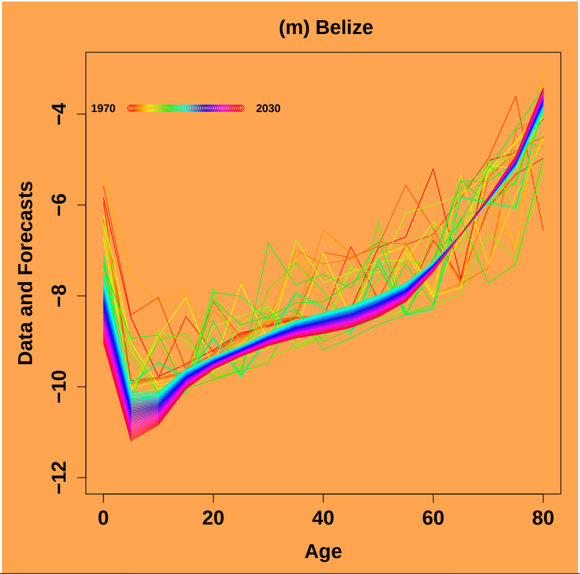
Mortality from Respiratory Infections, males, $\sigma = 0.87$

Smoothing over Age Groups



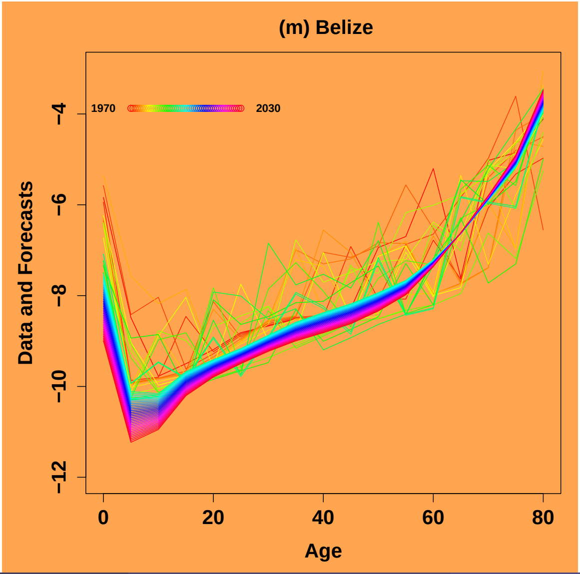
Mortality from Respiratory Infections, males, $\sigma = 0.66$

Smoothing over Age Groups



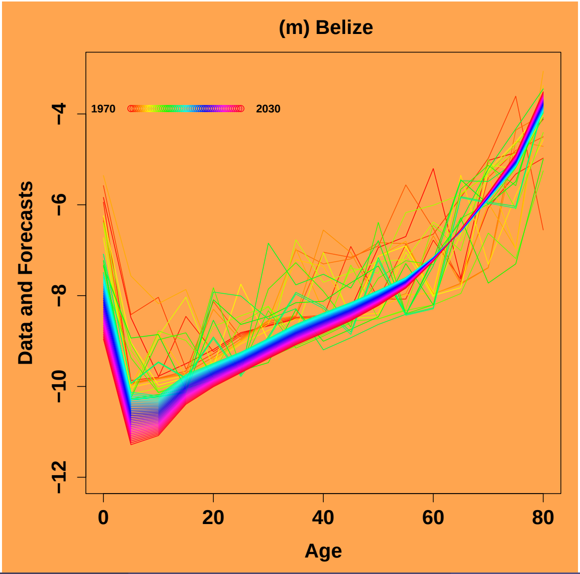
Mortality from Respiratory Infections, males, $\sigma = 0.50$

Smoothing over Age Groups



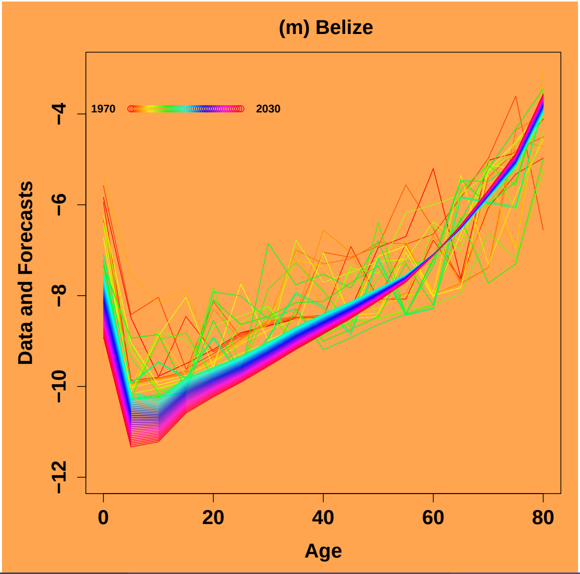
Mortality from Respiratory Infections, males, $\sigma = 0.38$

Smoothing over Age Groups



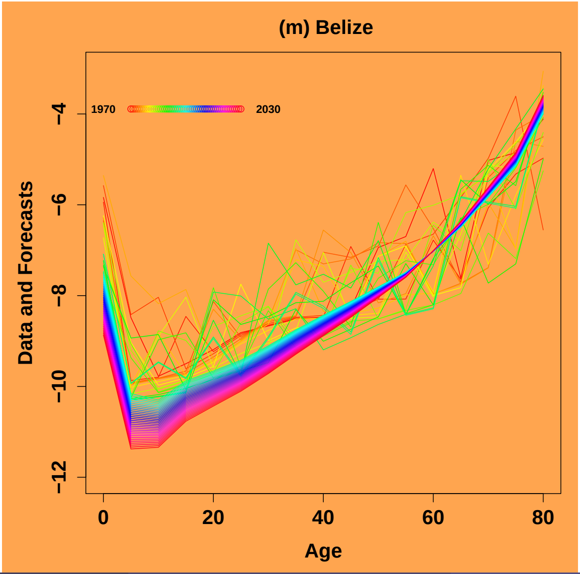
Mortality from Respiratory Infections, males, $\sigma = 0.28$

Smoothing over Age Groups



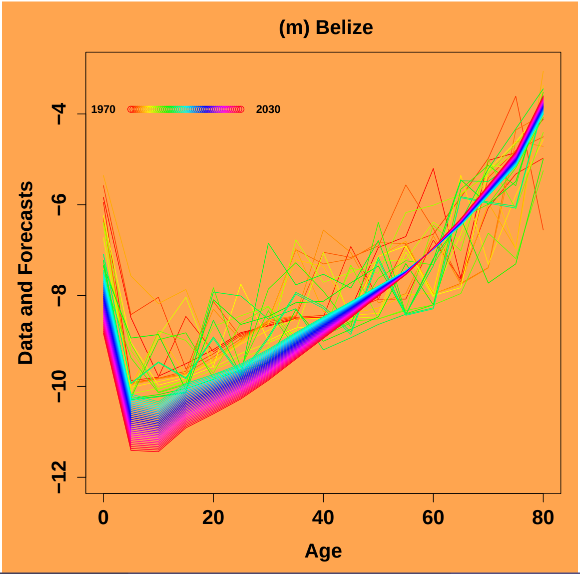
Mortality from Respiratory Infections, males, $\sigma = 0.21$

Smoothing over Age Groups



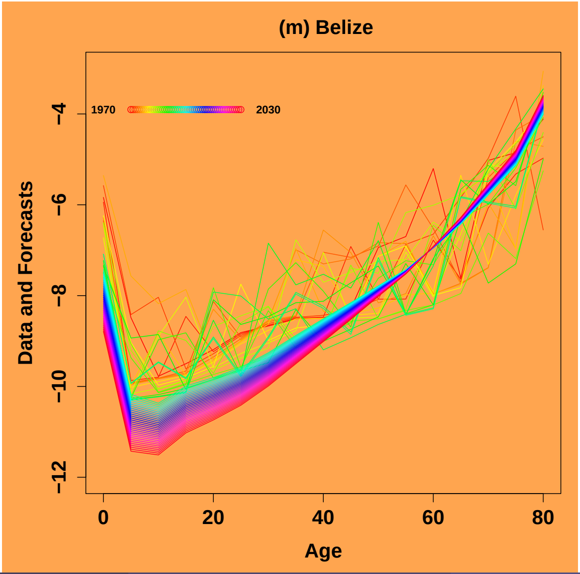
Mortality from Respiratory Infections, males, $\sigma = 0.16$

Smoothing over Age Groups



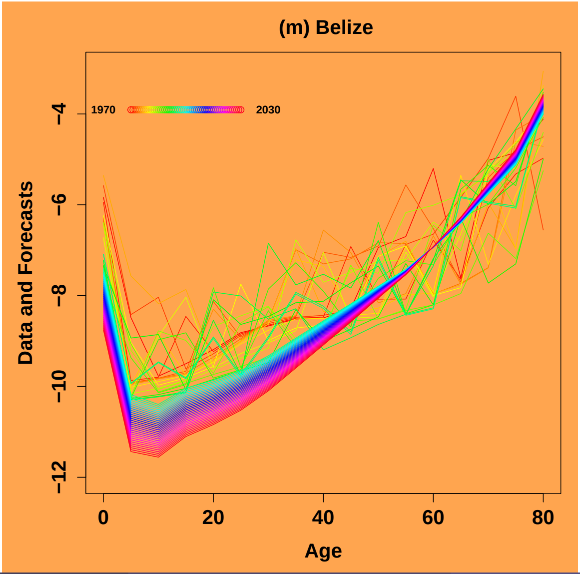
Mortality from Respiratory Infections, males, $\sigma = 0.12$

Smoothing over Age Groups



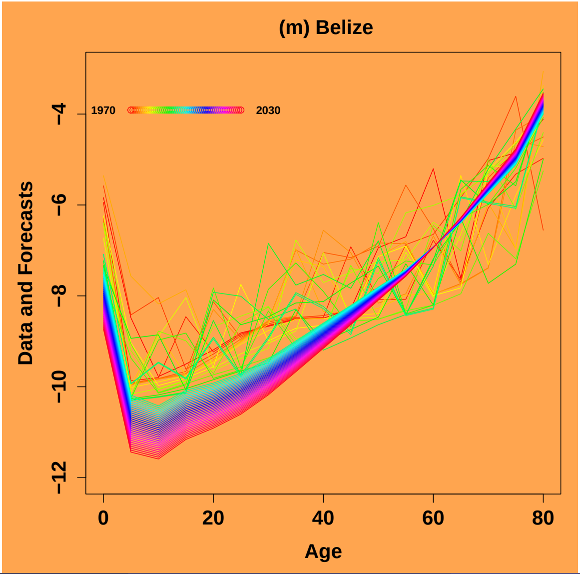
Mortality from Respiratory Infections, males, $\sigma = 0.09$

Smoothing over Age Groups



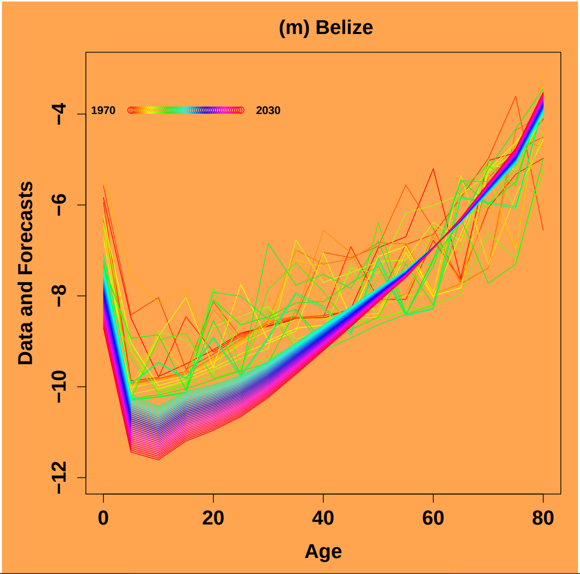
Mortality from Respiratory Infections, males, $\sigma = 0.07$

Smoothing over Age Groups



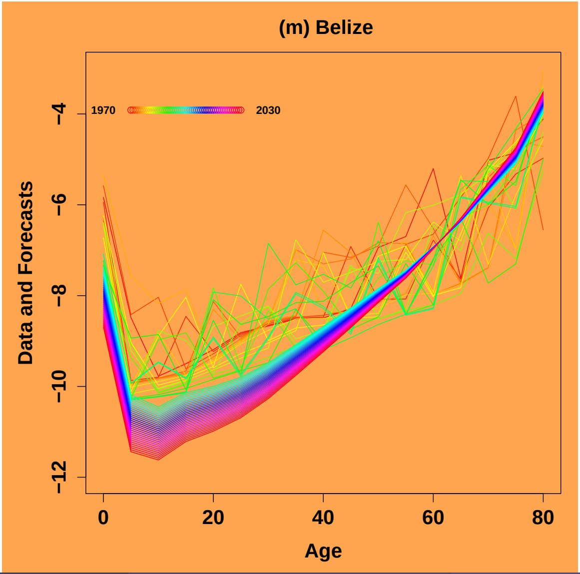
Mortality from Respiratory Infections, males, $\sigma = 0.05$

Smoothing over Age Groups



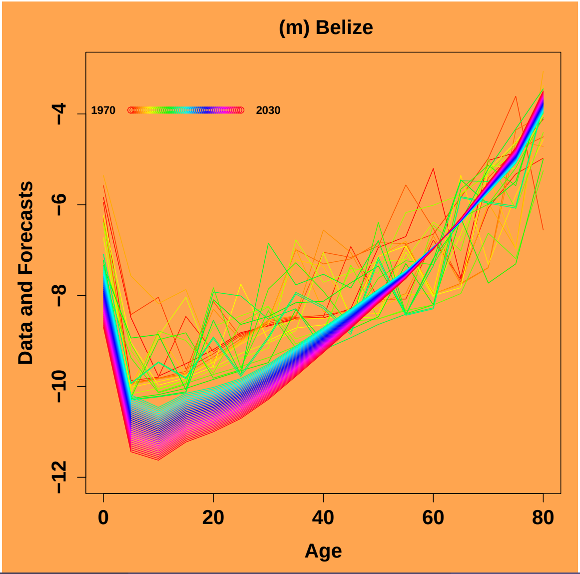
Mortality from Respiratory Infections, males, $\sigma = 0.04$

Smoothing over Age Groups



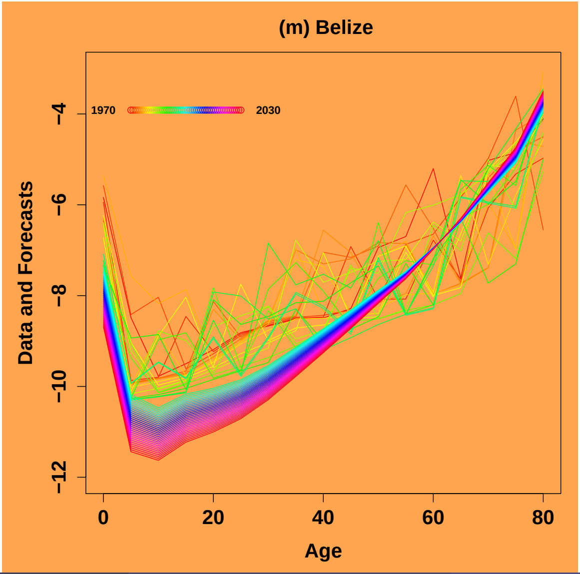
Mortality from Respiratory Infections, males, $\sigma = 0.03$

Smoothing over Age Groups



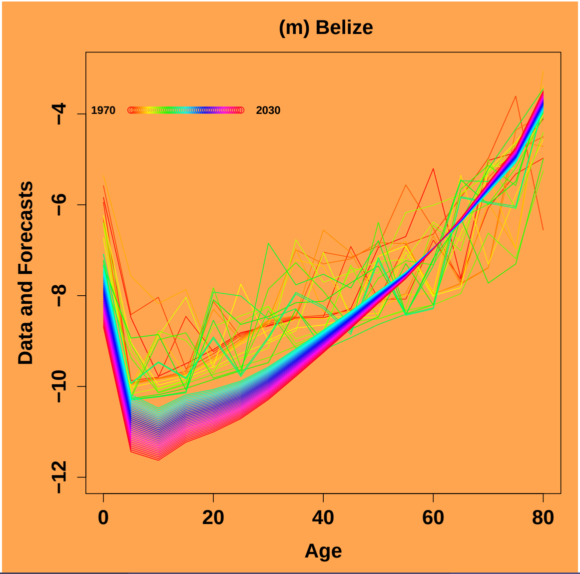
Mortality from Respiratory Infections, males, $\sigma = 0.02$

Smoothing over Age Groups



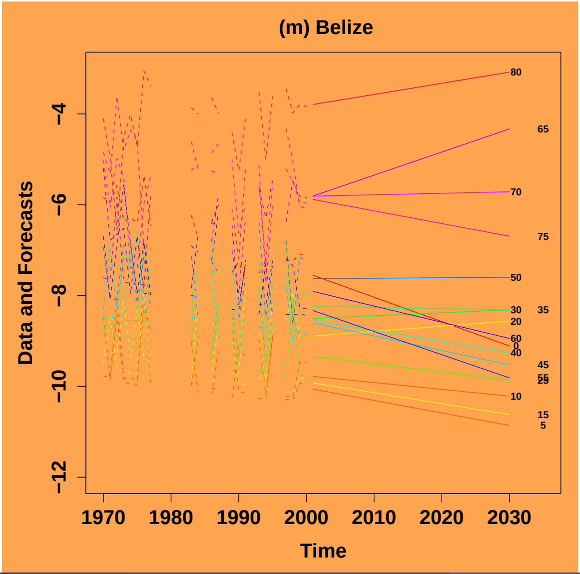
Mortality from Respiratory Infections, males, $\sigma = 0.01$

Smoothing over Age Groups



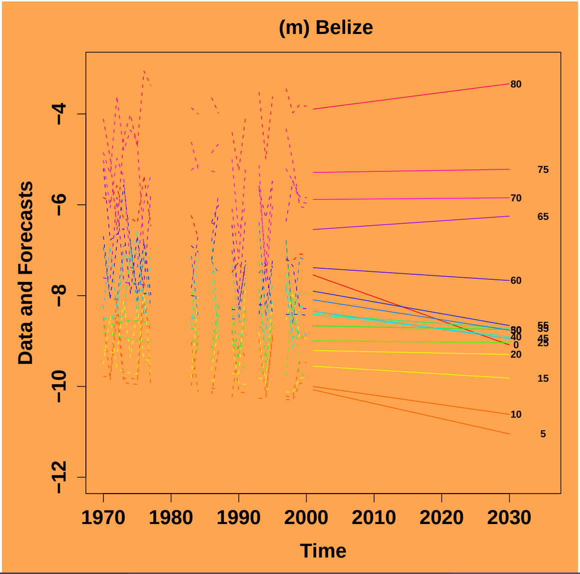
Mortality from Respiratory Infections, males

Least Squares



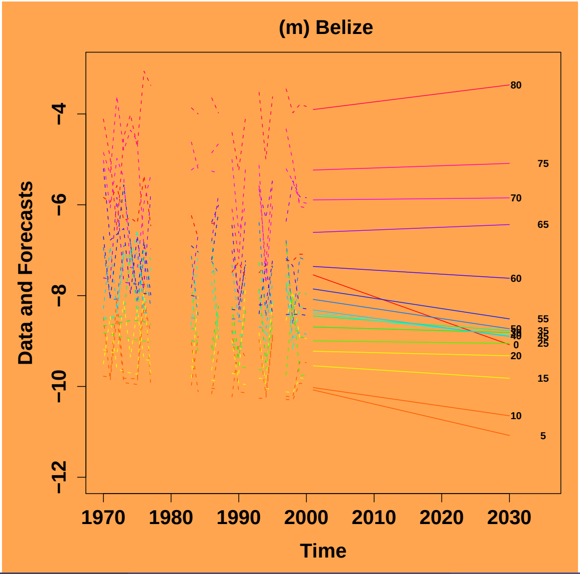
Mortality from Respiratory Infections, males, $\sigma = 2.00$

Smoothing over Age Groups



Mortality from Respiratory Infections, males, $\sigma = 1.51$

Smoothing over Age Groups



Mortality from Respiratory Infections, males, $\sigma = 1.15$

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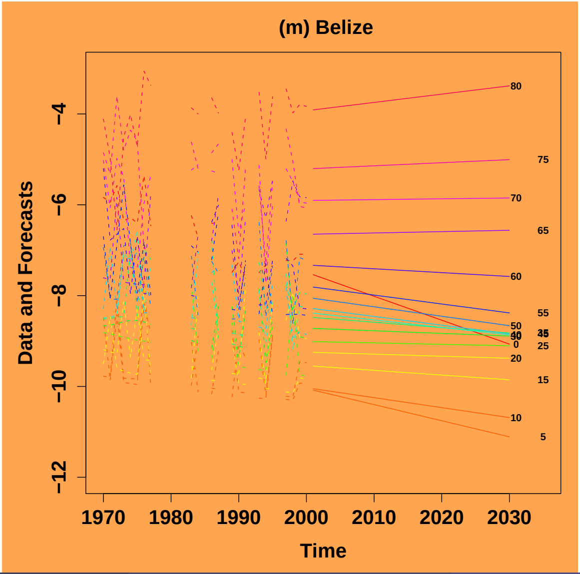
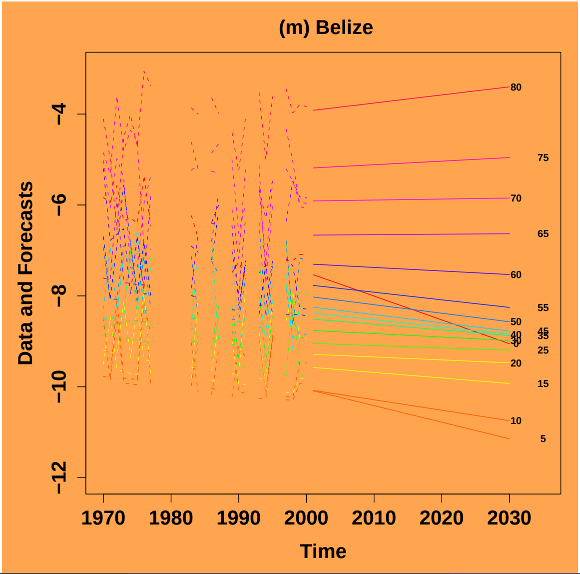


Mortality from Respiratory Infections, males, $\sigma = 0.87$

Smoothing over Age Groups



Mortality from Respiratory Infections, males, $\sigma = 0.66$

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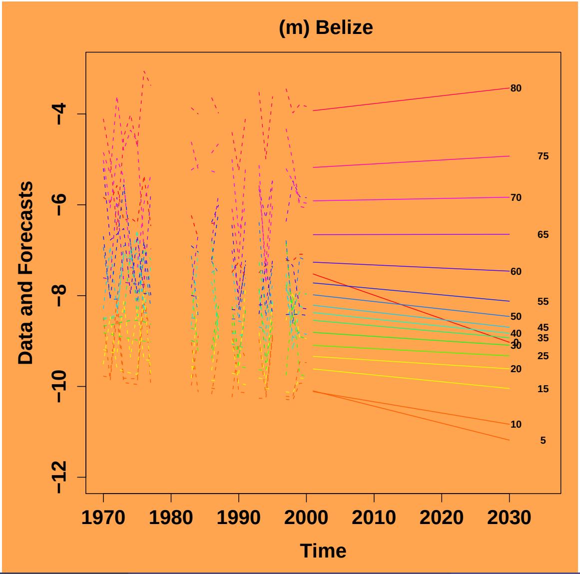

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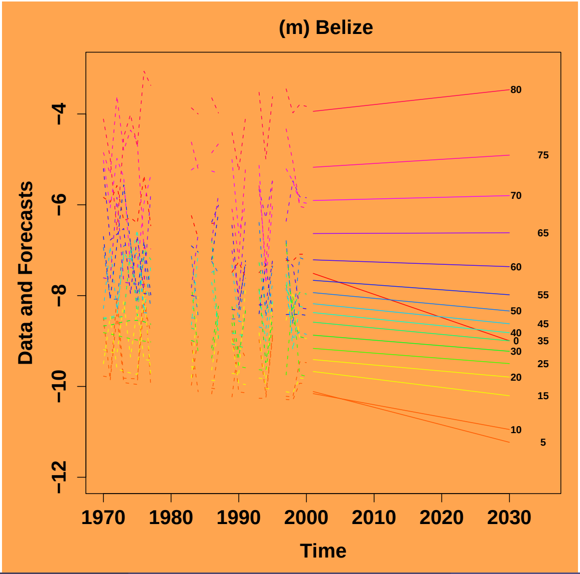

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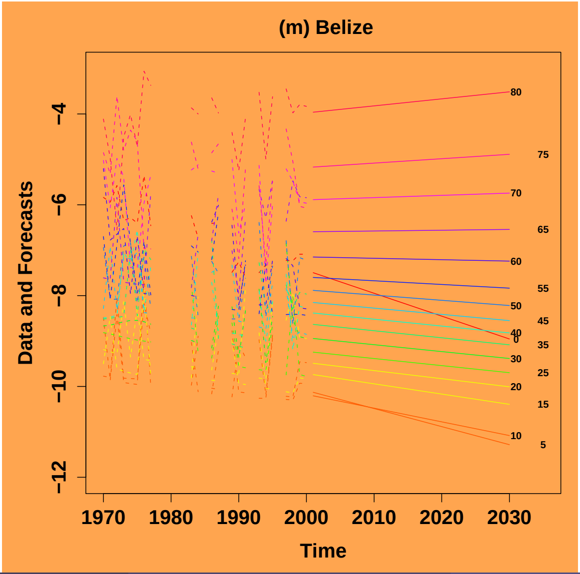

Mortality from Respiratory Infections, males, $\sigma = 0.28$

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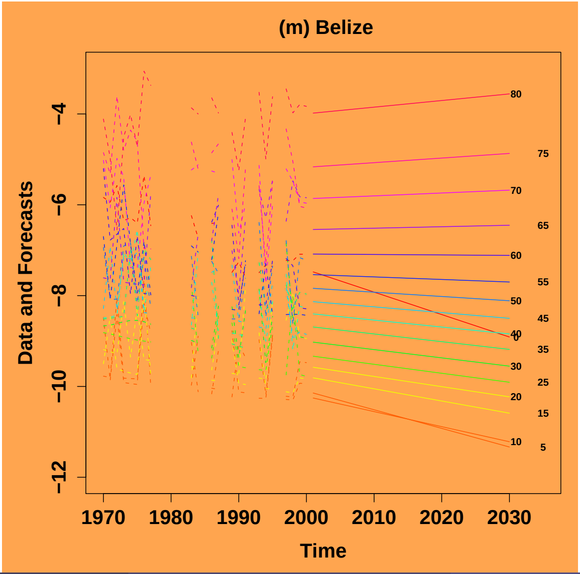

Mortality from Respiratory Infections, males, $\sigma = 0.21$

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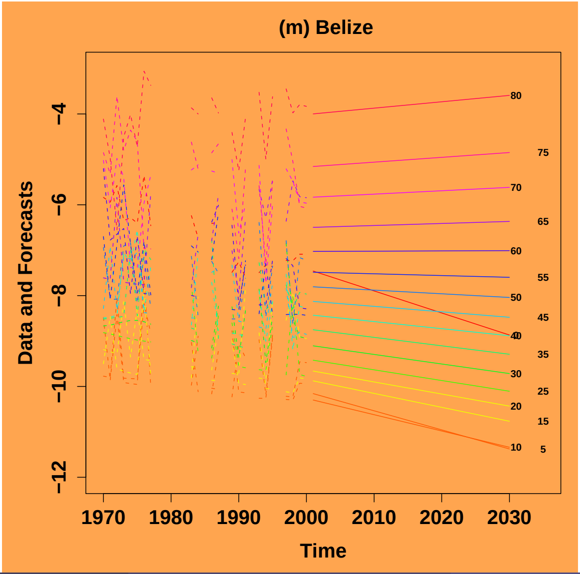

Mortality from Respiratory Infections, males, $\sigma = 0.16$

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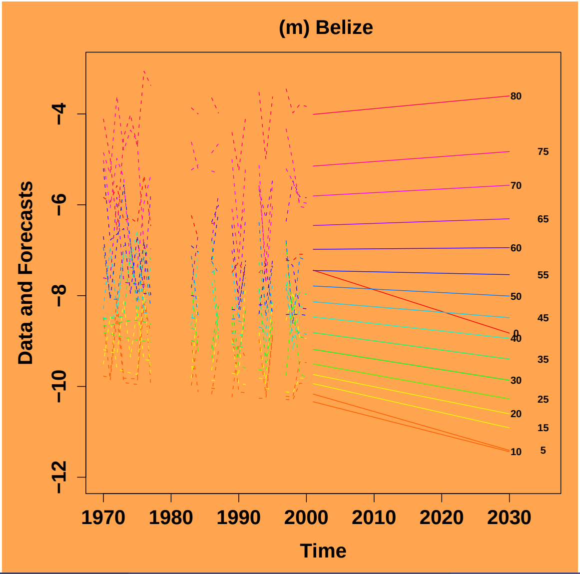
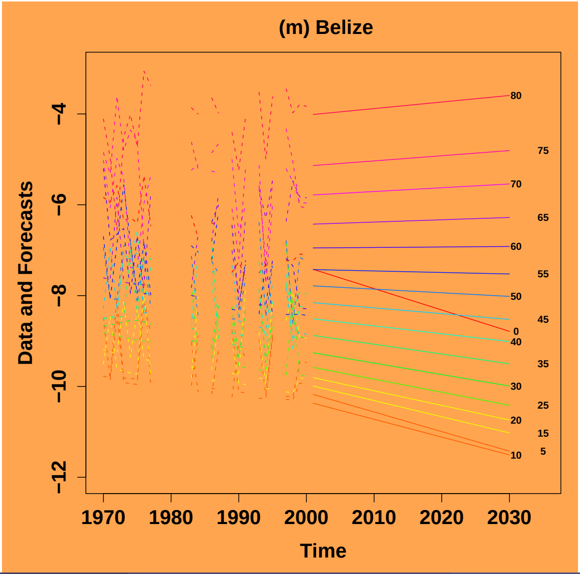


Mortality from Respiratory Infections, males, $\sigma = 0.12$

Smoothing over Age Groups



Mortality from Respiratory Infections, males, $\sigma = 0.09$

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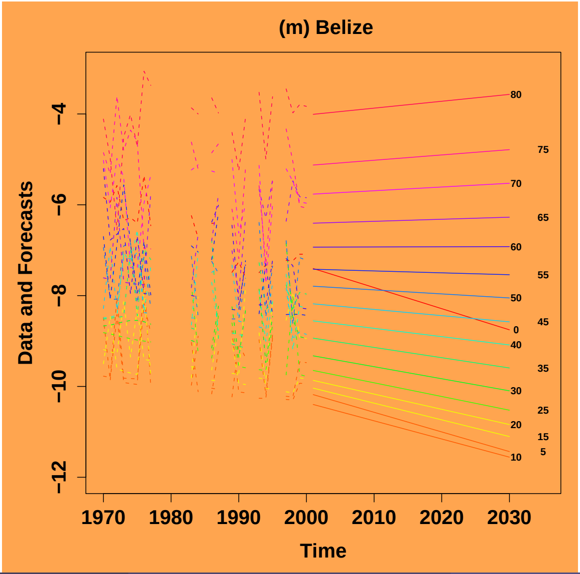
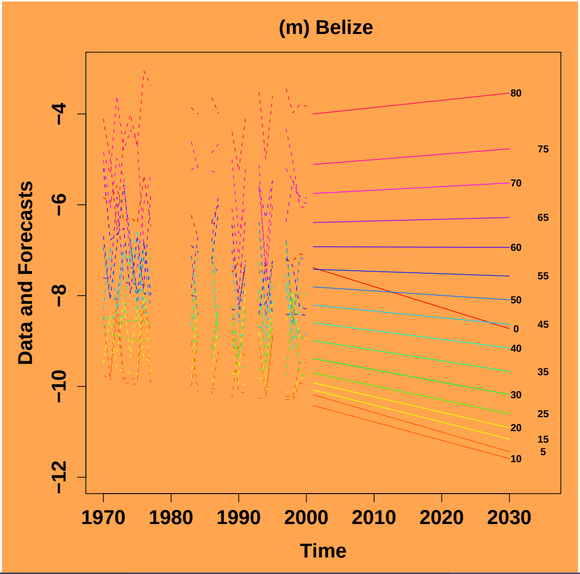


Mortality from Respiratory Infections, males, $\sigma = 0.07$

Smoothing over Age Groups



Mortality from Respiratory Infections, males, $\sigma = 0.05$

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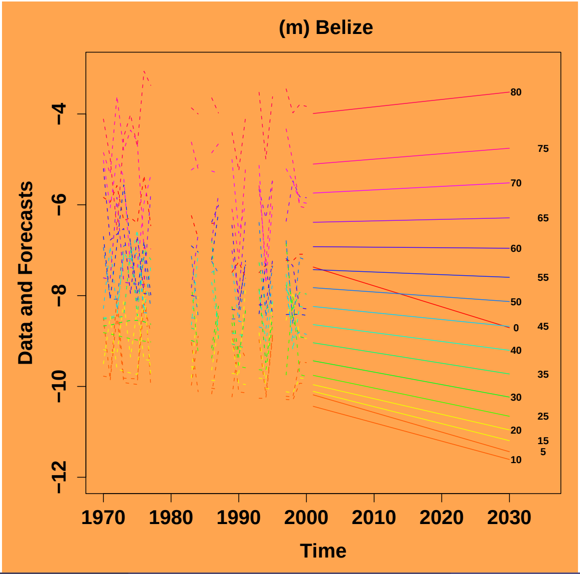

Mortality from Respiratory Infections, males, $\sigma = 0.04$

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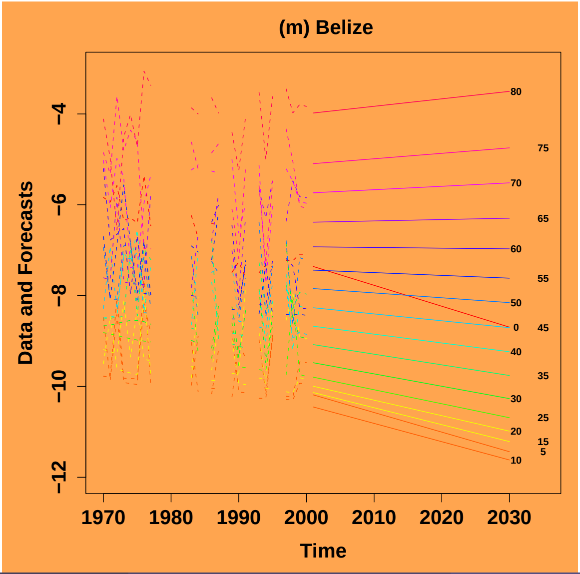
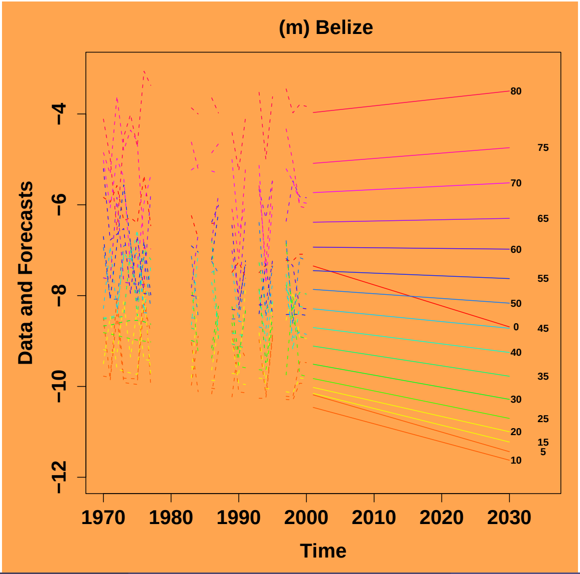


Mortality from Respiratory Infections, males, $\sigma = 0.03$

Smoothing over Age Groups



Mortality from Respiratory Infections, males, $\sigma = 0.02$

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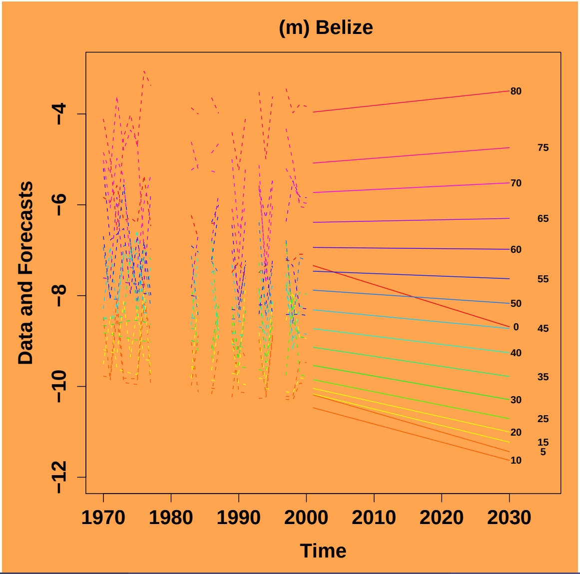
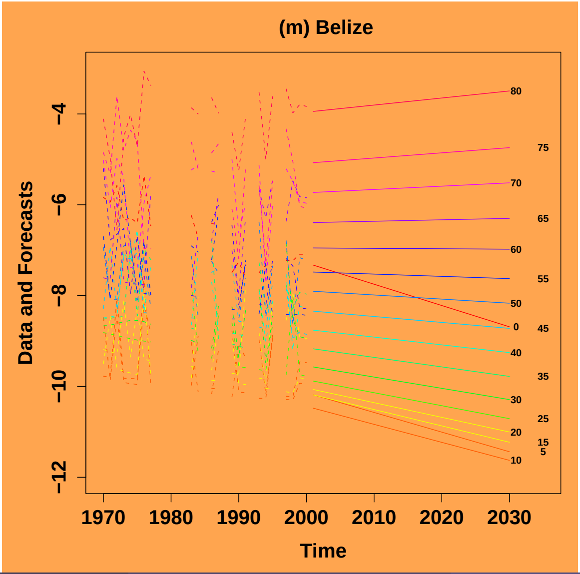


Mortality from Respiratory Infections, males, $\sigma = 0.01$

Smoothing over Age Groups



Smoothing Trends over Age Groups

Smoothing Trends over Age Groups

Log-mortality in Belize males from respiratory infections

Least Squares

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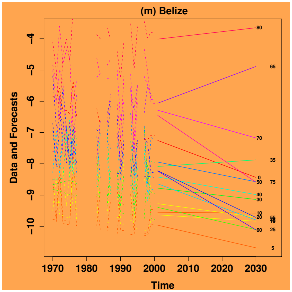

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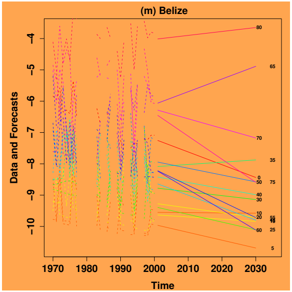

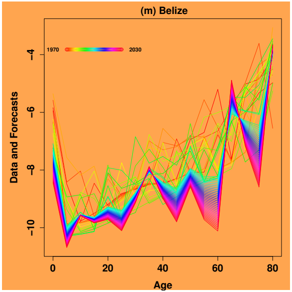

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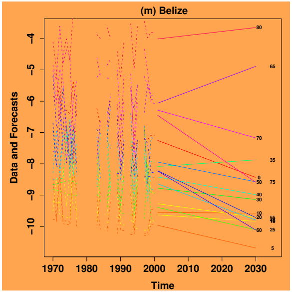

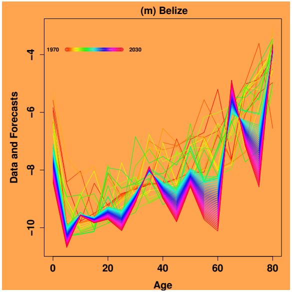

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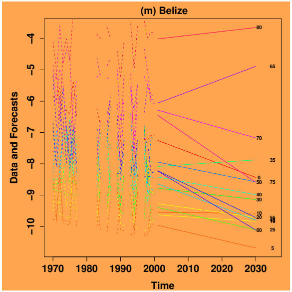
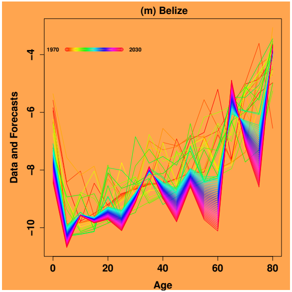
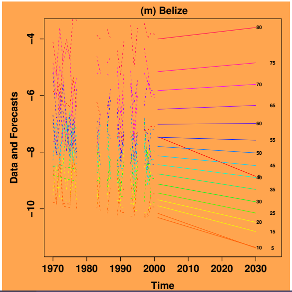




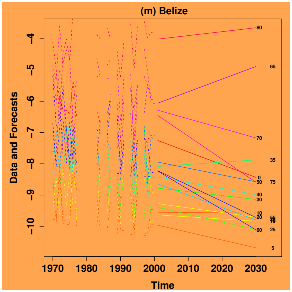
Smoothing
Age Groups

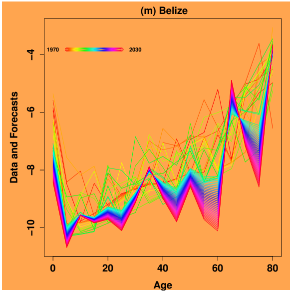


Smoothing Trends over Age Groups

Log-mortality in Belize males from respiratory infections

Least Squares





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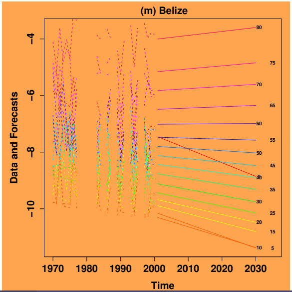

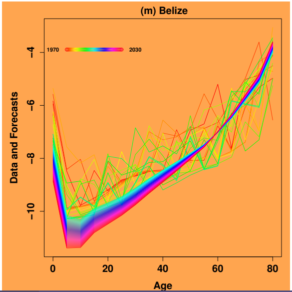

Smoothing Trends over Age Groups and Time

Smoothing Trends over Age Groups and Time

Log-Mortality in Bulgarian males from respiratory infections

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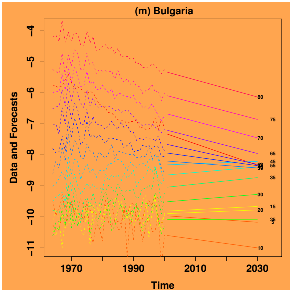

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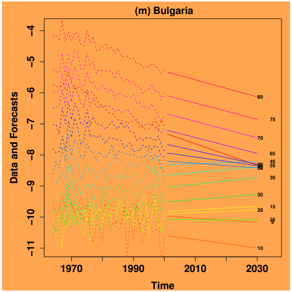

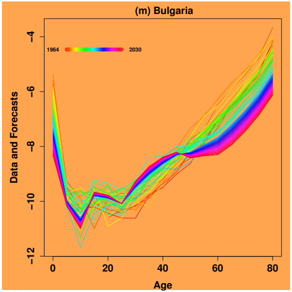

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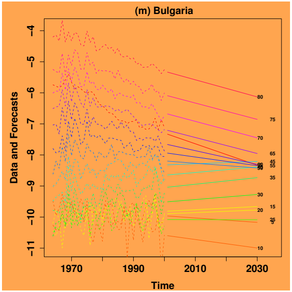

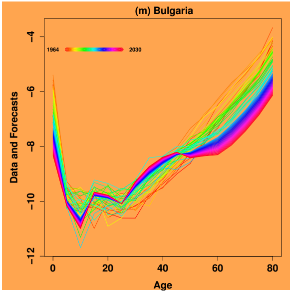

Smoothing
Age and Time

Smoothing Trends over Age Groups and Time

Log-Mortality in Bulgarian males from respiratory infections

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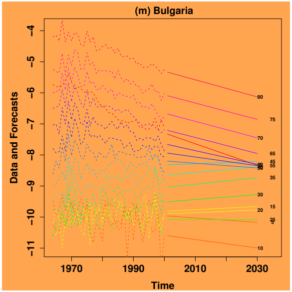

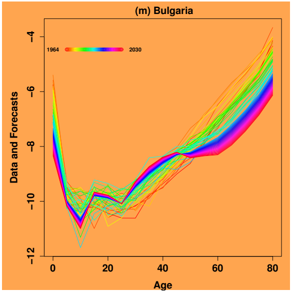

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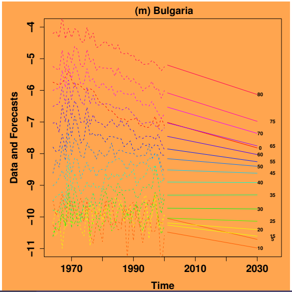

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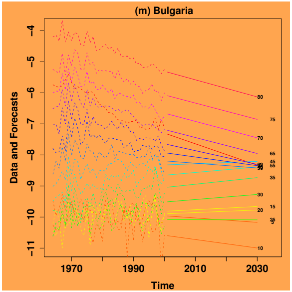

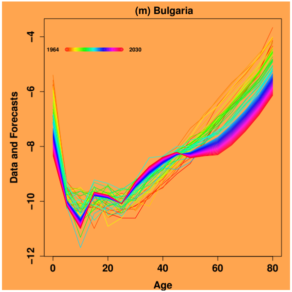

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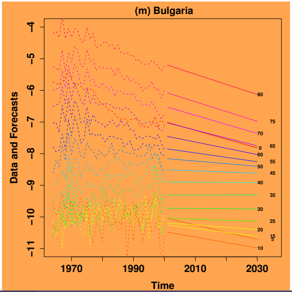

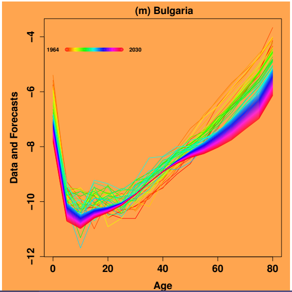

Using Covariates (GDP, tobacco, trend, log trend)

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Lung cancer in Korean Males

Least Squares

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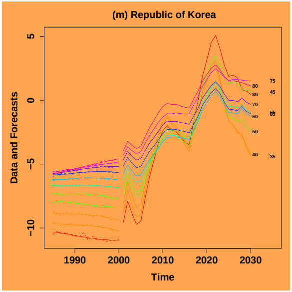

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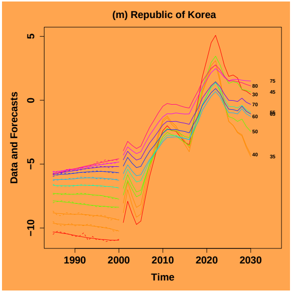

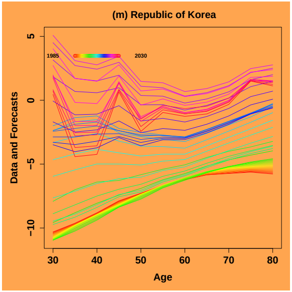

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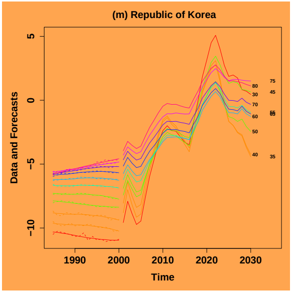

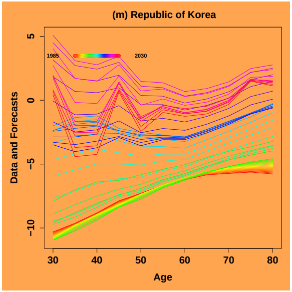

Smooth over age, time, age/time

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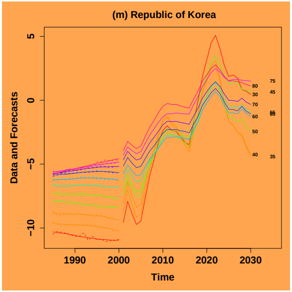

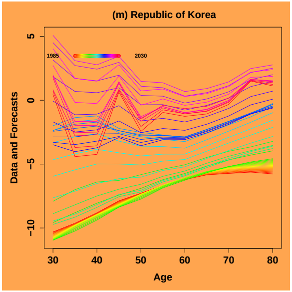

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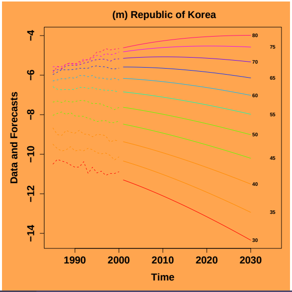

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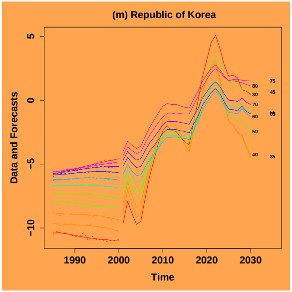

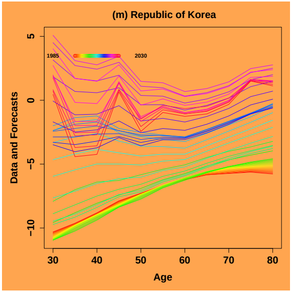

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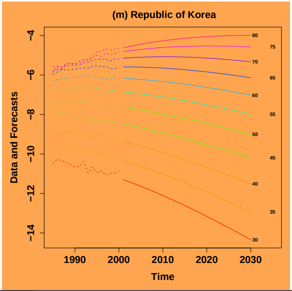

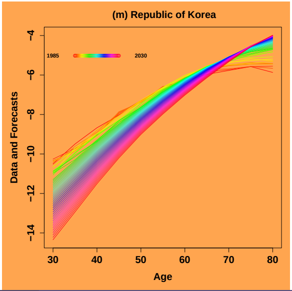

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Lung cancer in Males, Singapore

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Least Squares

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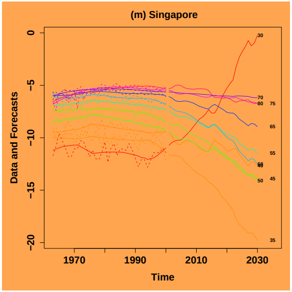

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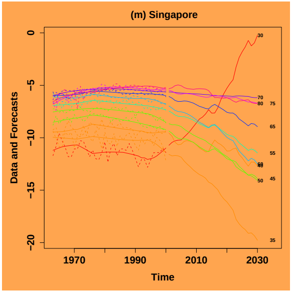

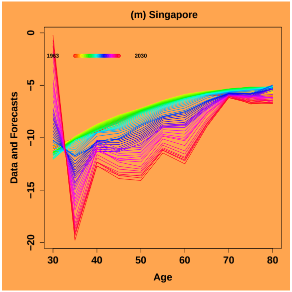

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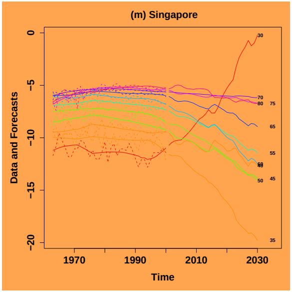

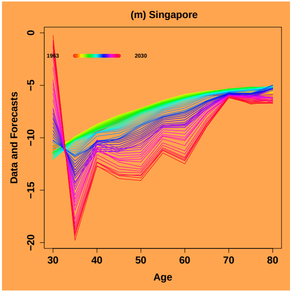

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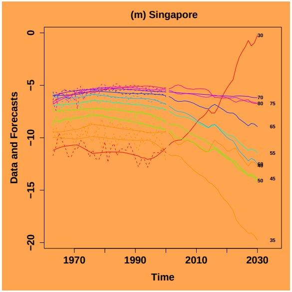

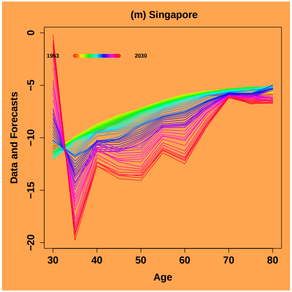

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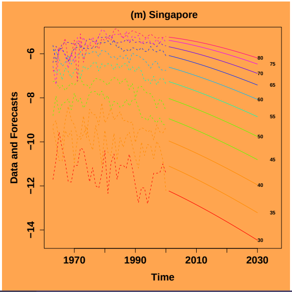

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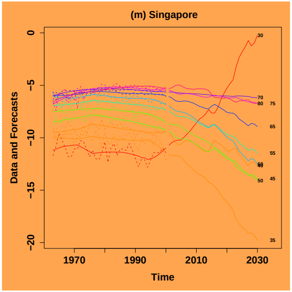

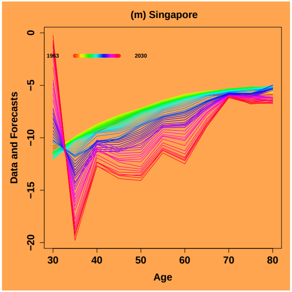

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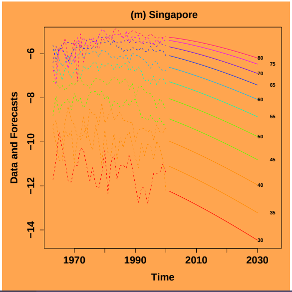
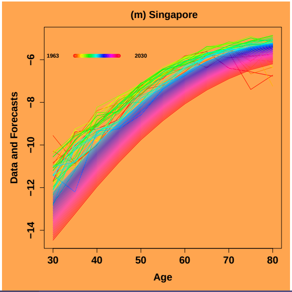
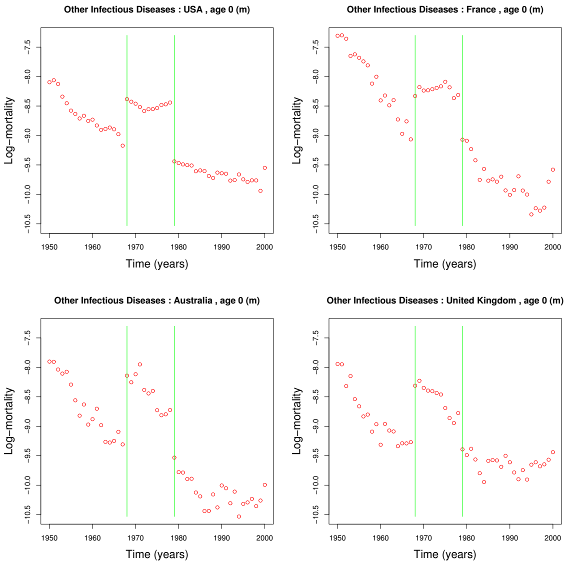
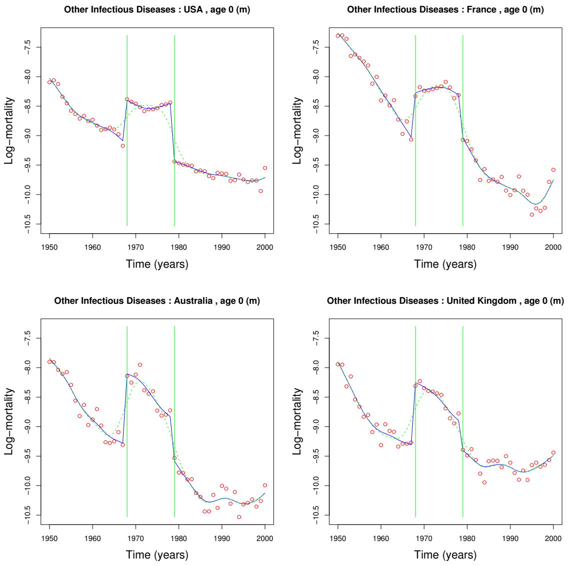




What about ICD Changes?



Fixing ICD Changes



A book manuscript, YourCast software, etc.

http://GKing.Harvard.edu

Preview of Results: Out-of-Sample Evaluation

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Mean Absolute Error in Males (over age and country)

Preview of Results: Out-of-Sample Evaluation

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	% Improvement	
	Over Best Previous	to Best Conceivable
Cardiovascular	22	49
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Transportation	16	31
Respiratory Chronic	13	30
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Stomach Cancer	8	24
All-Cause	12	22
Suicide	7	17
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- Does *considerably* better with more informative covariates

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$$\mathcal{P}(\mu \mid \theta) \propto \exp\left(-\frac{1}{2}\theta \sum_{aa't} (\mu_{at} - \bar{\mu}_a)' W^n_{aa'} (\mu_{a't} - \bar{\mu}_{a'}) \right)$$

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- where W^n is a matrix uniquely determined by n and θ

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where we have defined:

$$\mathbf{C}_{aa'} \equiv \frac{1}{T}\mathbf{Z}'_a\mathbf{Z}_{a'} \qquad \mathbf{Z}_a \text{ is a } T \times d_a \text{ data matrix for age group } a$$

The Prior on the Coefficients $\boldsymbol{\beta}$

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- The prior is normal (and improper);
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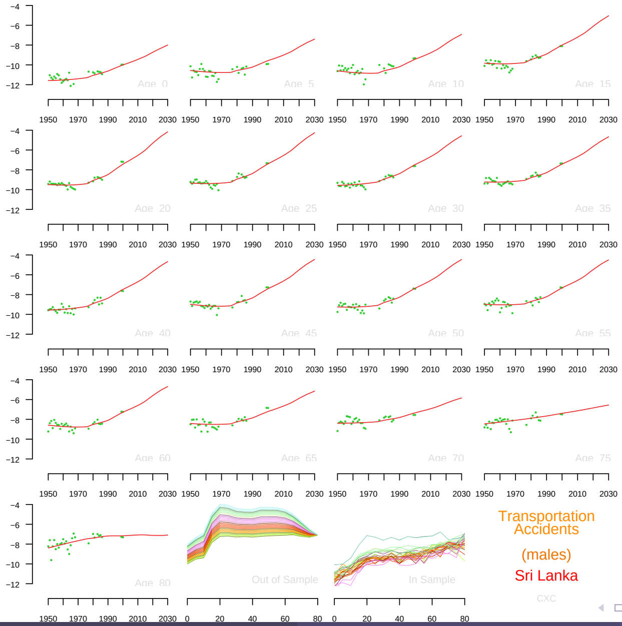
- Different age groups can have different covariates: the matrices $\mathbf{C}_{aa'}$ are rectangular ($d_a \times d_{a'}$).

The Prior on the Coefficients $\boldsymbol{\beta}$

$$\mathcal{P}(\boldsymbol{\beta} \mid \theta) \propto \exp\left(-\theta \sum_{aa'} W^{n}_{aa'} \boldsymbol{\beta}'_{a} \mathbf{C}_{aa'} \boldsymbol{\beta}_{a'}\right)$$

- The prior is normal (and improper);
- The prior is uniquely determined by the choice of n, through the "interaction" matrix W^n.
- Different age groups can have different covariates: the matrices $\mathbf{C}_{aa'}$ are rectangular ($d_a \times d_{a'}$).
- The variance of the prior is inversely proportional to θ, which controls the "strength" of the prior.

Without Country Smoothing



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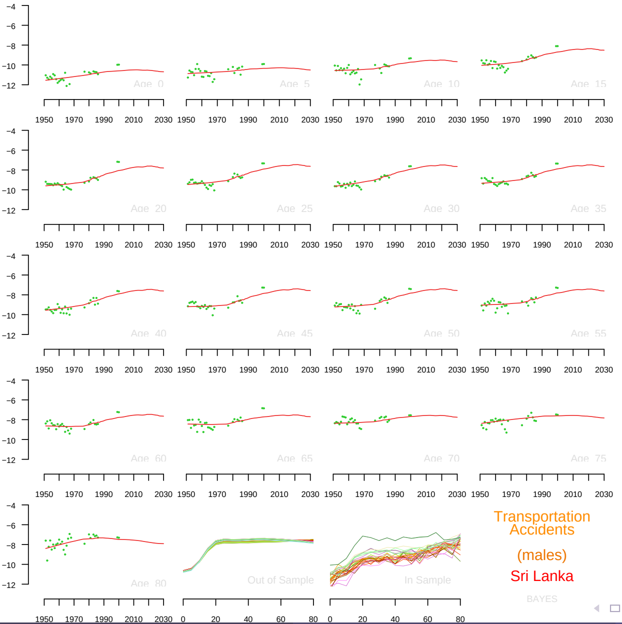

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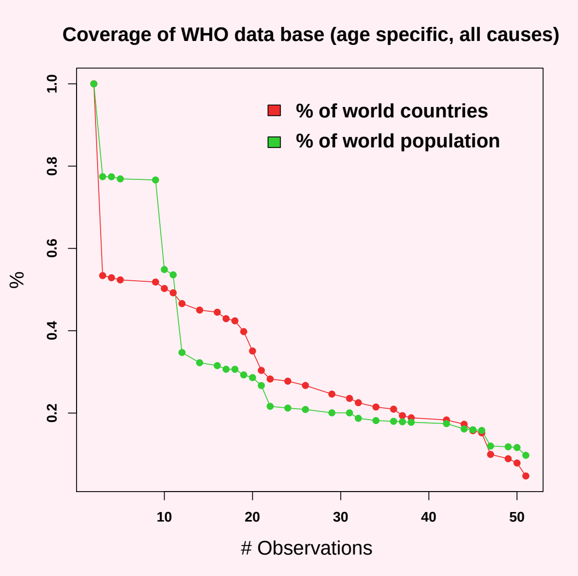
- Covariates with the same name can have different meanings

Many Short Time Series



Preview of Results: Out-of-Sample Evaluation

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Mean Absolute Error in Males (over age and country)

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	Mean Absolute Error			% Improvement	
	Best Previous	Our Method	Best Conceivable	Over Best Previous	to Best Conceivable
Cardiovascular	0.34	0.27	0.19	22	49
Lung Cancer	0.36	0.27	0.17	24	47
Transportation	0.37	0.31	0.18	16	31
Respiratory Chronic	0.45	0.39	0.26	13	30
Other Infectious	0.55	0.48	0.32	12	30
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- Does much better with better covariates