Why Propensity Scores Should Not Be Used For Matching

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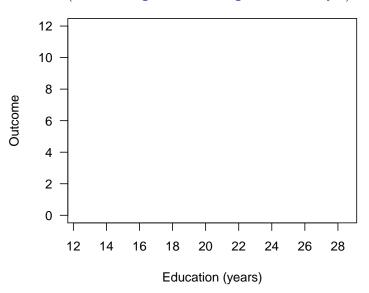
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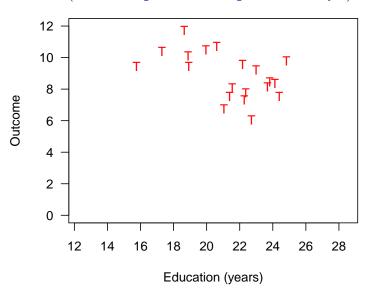
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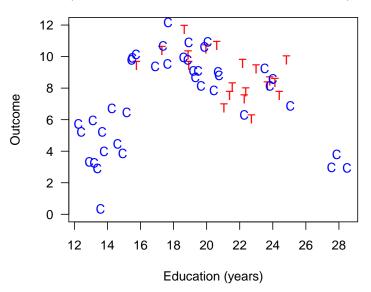
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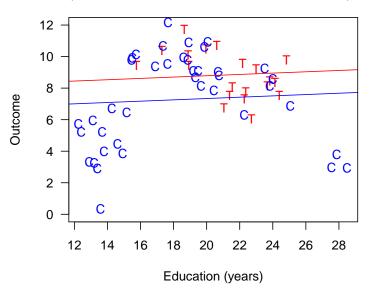
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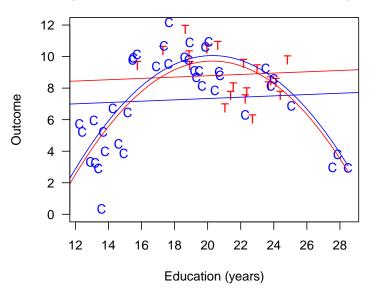
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 - The mathematical theorems about propensity scores

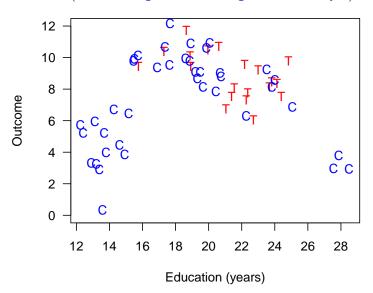


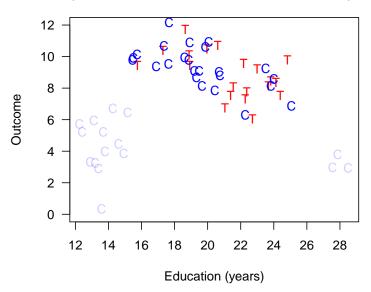


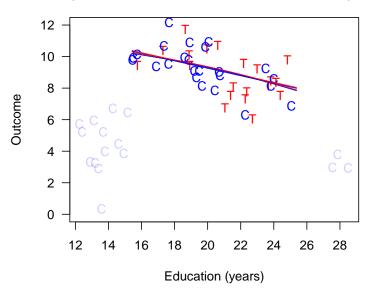












Without Matching:

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Imbalance

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Imbalance → Model Dependence

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- "Teaching psychology is mostly a waste of time" (Kahneman 2011)

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A central project of statistics: Automating away human discretion

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- Pruning nonmatches makes control vars matter less: reduces imbalance, model dependence, researcher discretion, & bias

Types of Experiments

Complete Randomization

Matching: Finding Hidden Randomized Experiments Types of Experiments

Complete Fully Randomization Blocked

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Covariates:	Randomization	Blocked	
Observed			
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- Other matching methods dominate PSM (wait, it gets worse)

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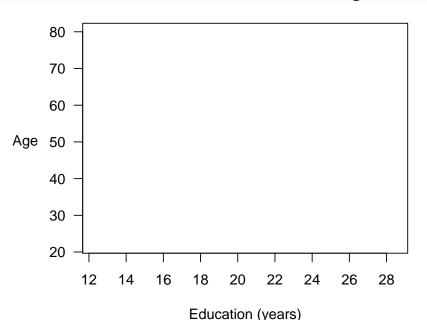
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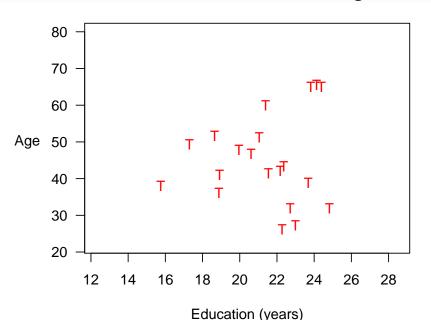
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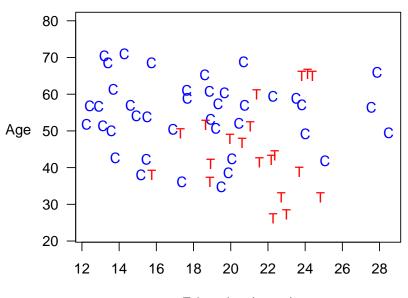
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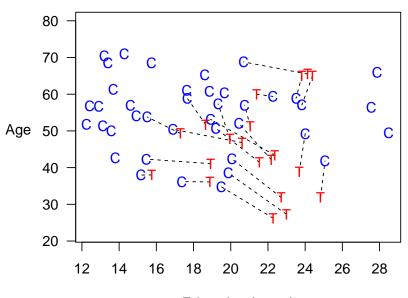
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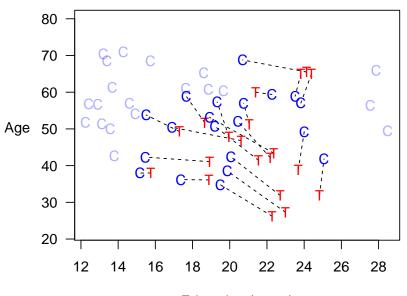
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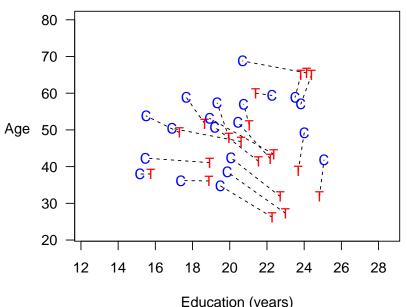


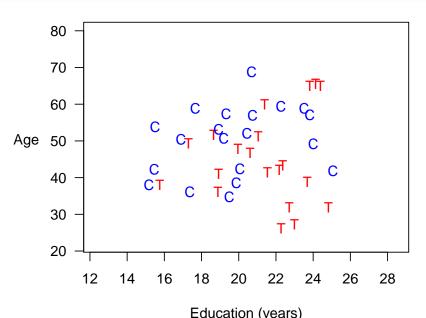






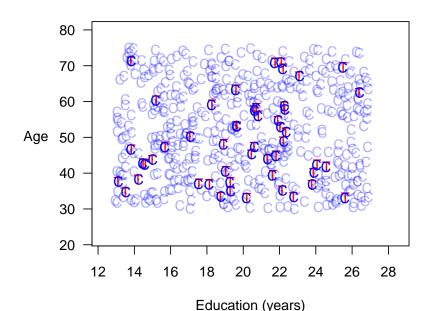




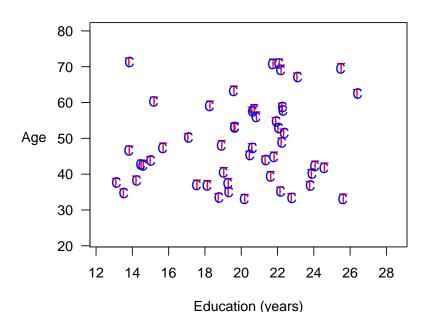


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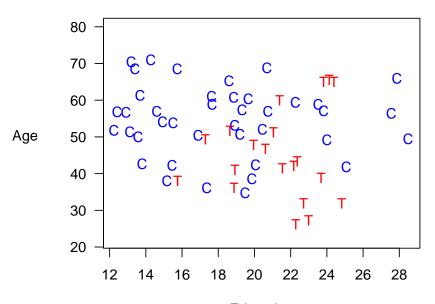
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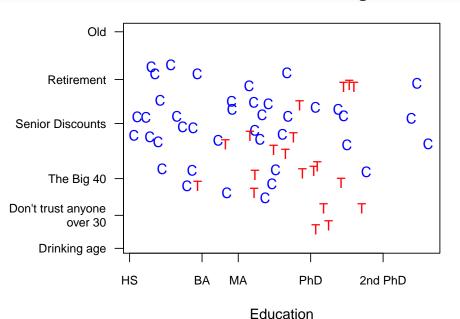
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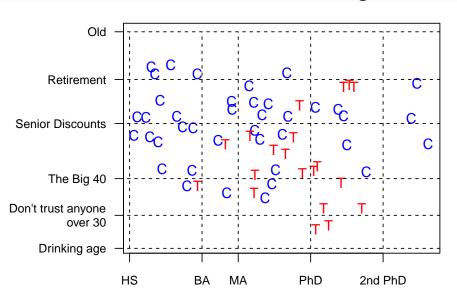
Method 2: Coarsened Exact Matching

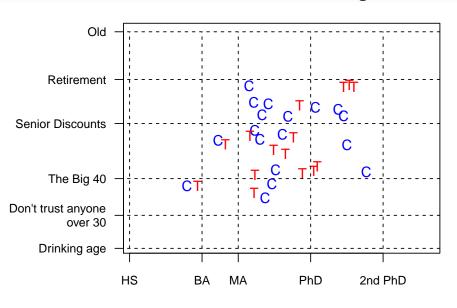
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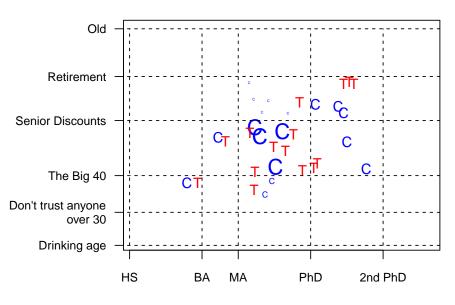
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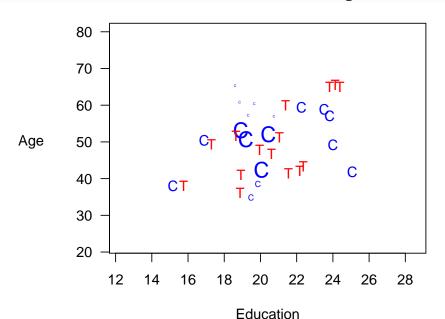


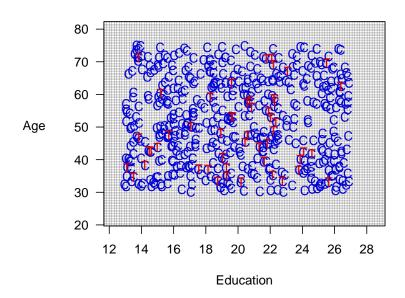


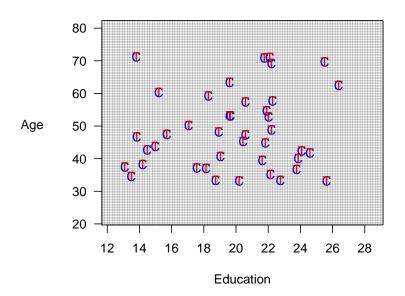


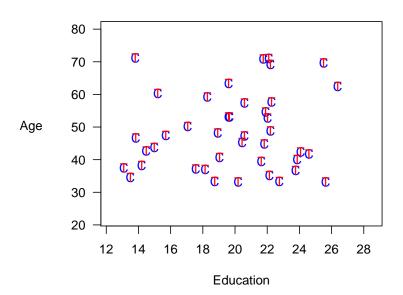












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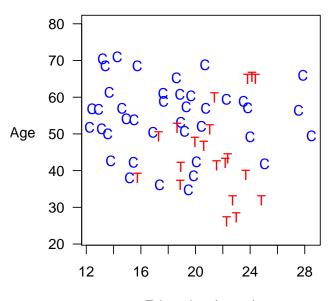
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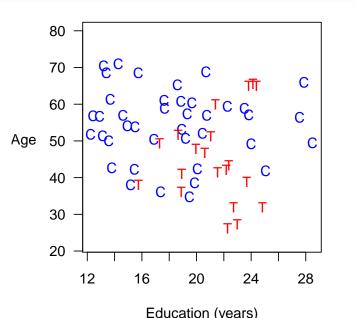
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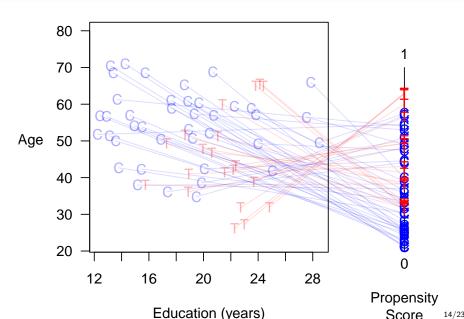
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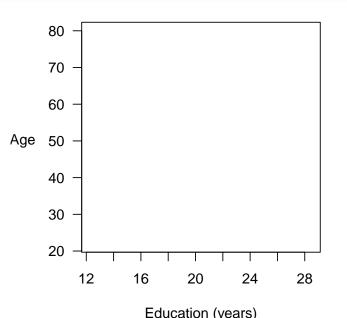
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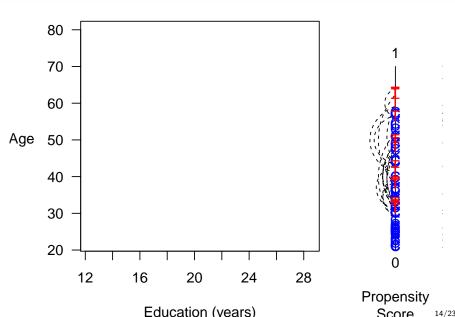


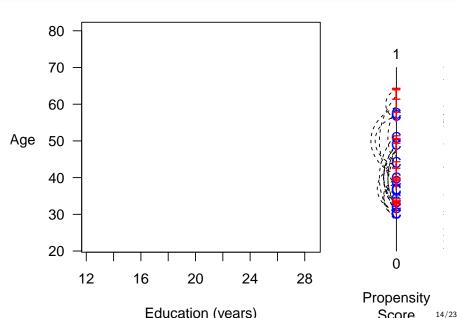
Propensity
Score 14/23

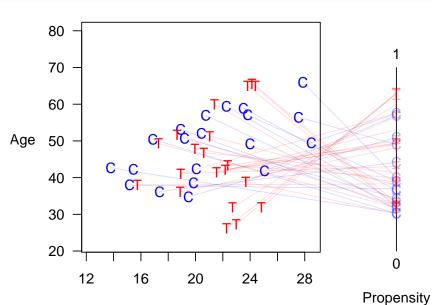








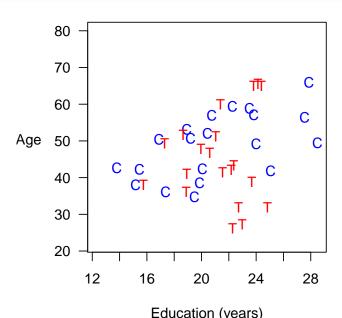


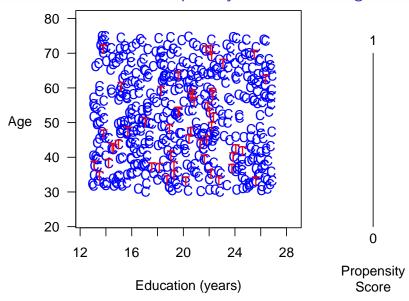


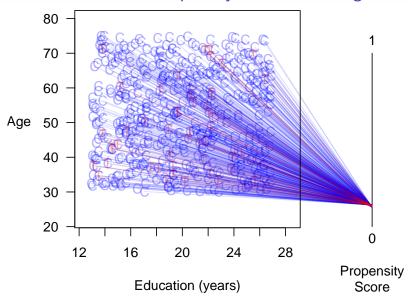
Education (vears)

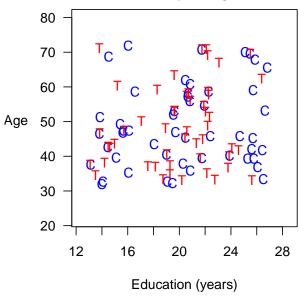
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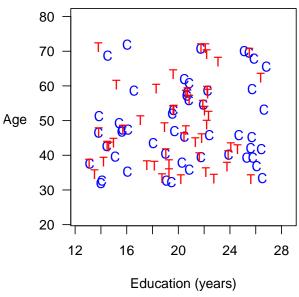








Best Case: Propensity Score Matching is Suboptimal



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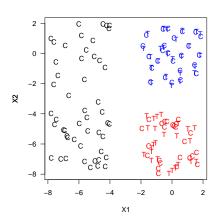
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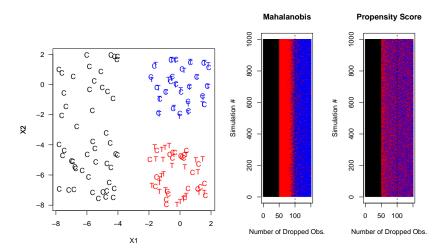
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PSM is Blind Where Other Methods Can See

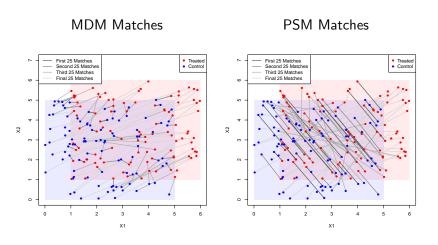
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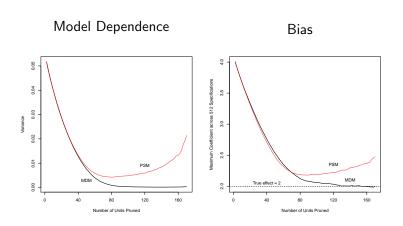


What Does PSM Match?



Controls: $X_1, X_2 \sim \mathsf{Uniform}(0,5)$ Treateds: $X_1, X_2 \sim \mathsf{Uniform}(1,6)$

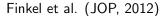
PSM Increases Model Dependence & Bias

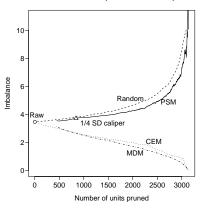


$$Y_i = 2T_i + X_{1i} + X_{2i} + \epsilon_i$$
$$\epsilon_i \sim N(0, 1)$$

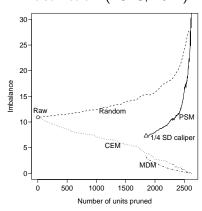
The Propensity Score Paradox in Real Data

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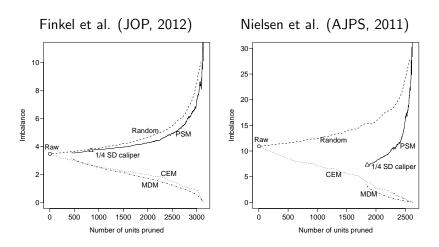




Nielsen et al. (AJPS, 2011)



The Propensity Score Paradox in Real Data



Similar pattern for > 20 other real data sets we checked

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- Matching methods still highly recommended; choose one with higher standards

For more information, papers, & software



 $\begin{array}{c} {\tt GaryKing.org} \\ {\tt www.mit.edu/}{\sim} {\tt rnielsen} \end{array}$