Why Propensity Scores Should Not Be Used For Matching

Gary King¹

Richard Nielsen²

Institute for Quantitative Social Science Harvard University MIT

(Talk at HMS/BWH Division of Pharmacoepidemiology and Pharmacoeconomics, 9/23/2015)

¹GaryKing.org ²www.mit.edu/~rnielsen

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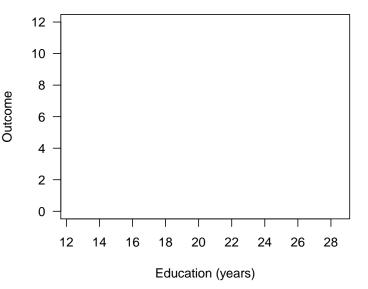
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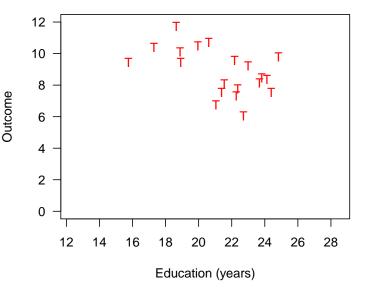
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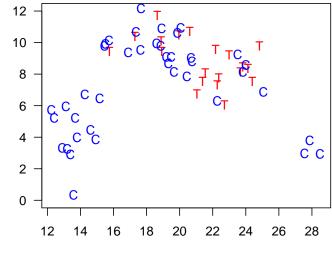
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 - Other uses of propensity scores: E.g., regression adjustment, inverse weighting, stratification, pscores used in other methods
 - The mathematical theorems about propensity scores





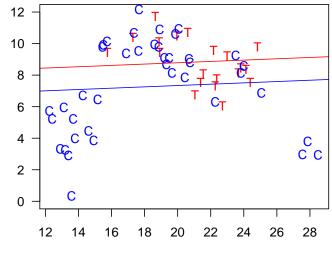
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Outcome

Education (years)

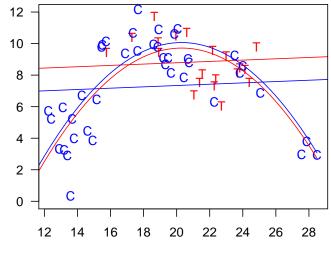
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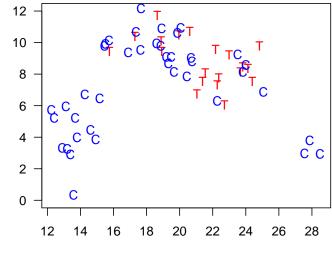
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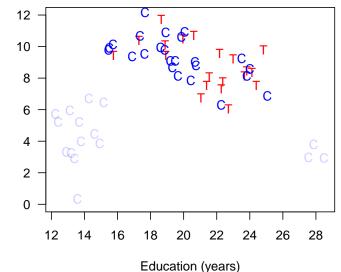
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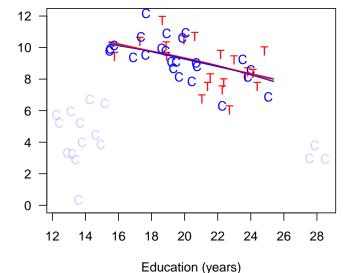
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Imbalance

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- "Teaching psychology is mostly a waste of time" (Kahneman 2011)

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A central project of statistics: Automating away human discretion

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- Big convenience: Follow preprocessing with whatever statistical method you'd have used without matching
- Pruning nonmatches makes control vars matter less: reduces imbalance, model dependence, researcher discretion, & bias

Types of Experiments

Complete Randomization

> Complete Fully Randomization Blocked

Types of Experiments

Balance Covariates: *Observed Unobserved*

Balance Complete Fully Covariates: Randomization Blocked

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Goal of Each Matching Method (in Observational Data)

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- Other matching methods dominate PSM

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- PSM: complete randomization
- Other methods: *fully blocked*
- Other matching methods dominate PSM (wait, it gets worse)

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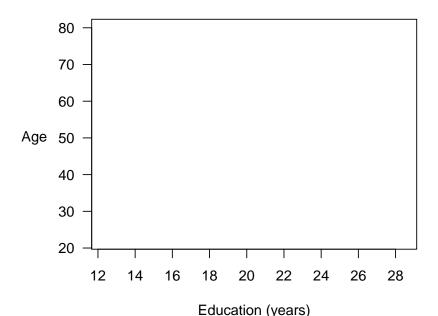
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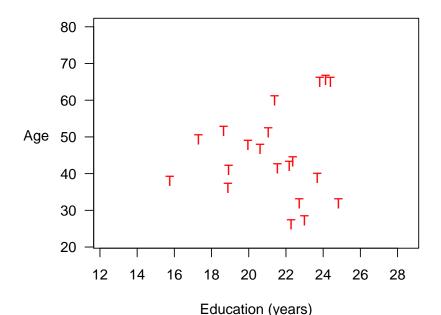
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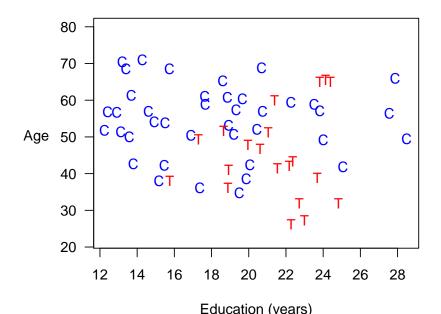
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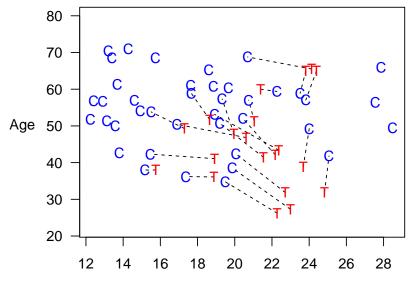
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 - (Many adjustments available to this basic method)
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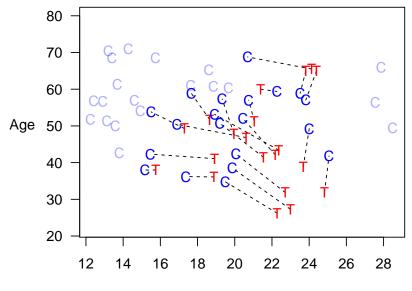




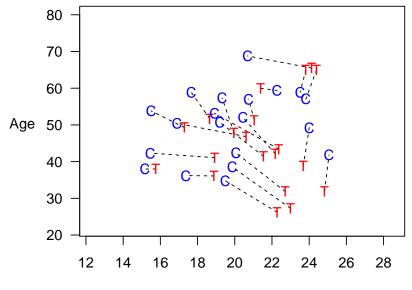




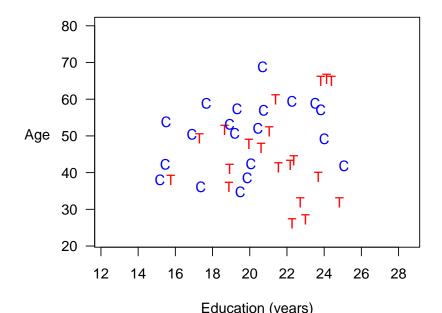
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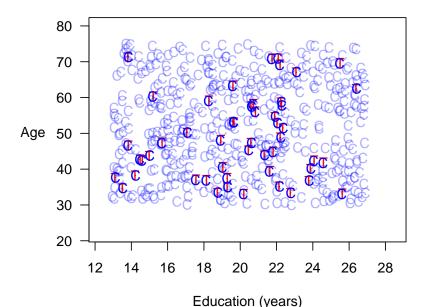
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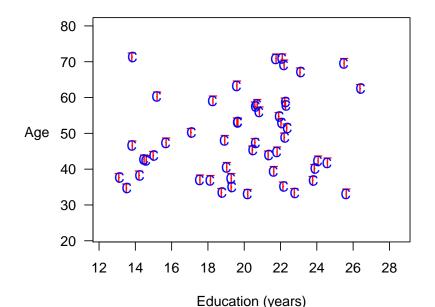
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Best Case: Mahalanobis Distance Matching

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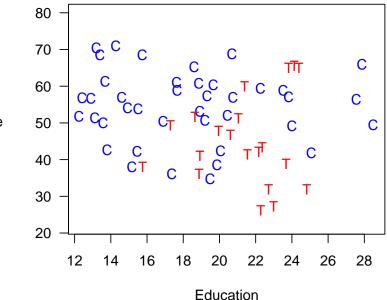
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Method 2: Coarsened Exact Matching

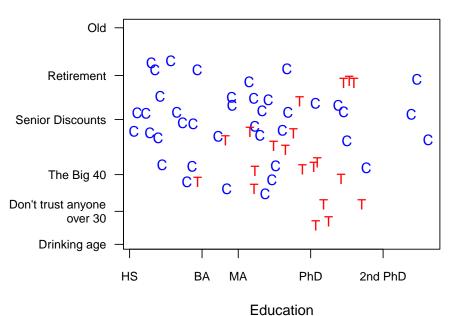
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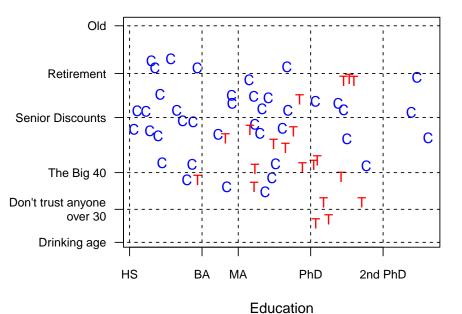
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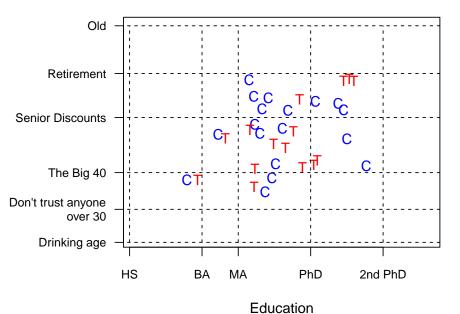
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 - Weight controls in each stratum to equal treateds

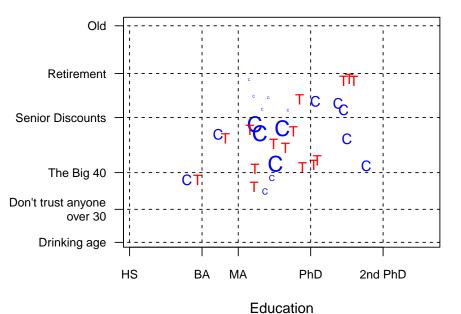


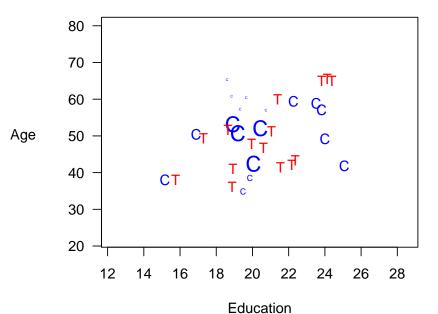
Age

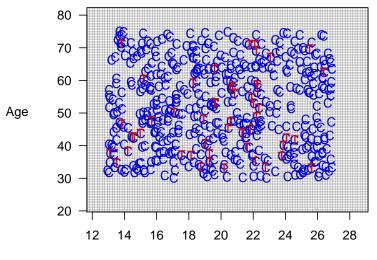




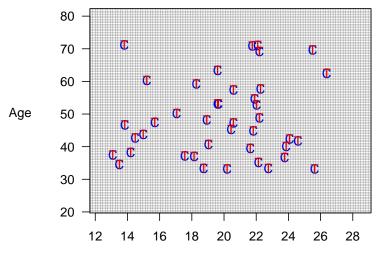




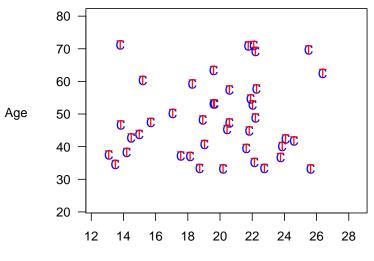




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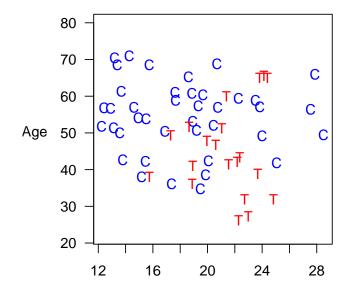
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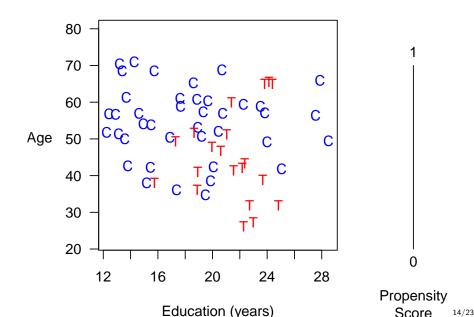
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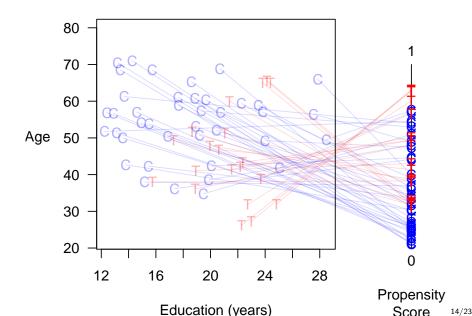
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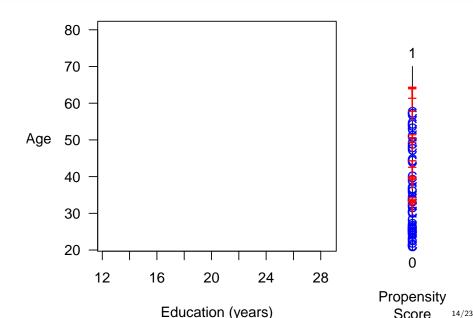
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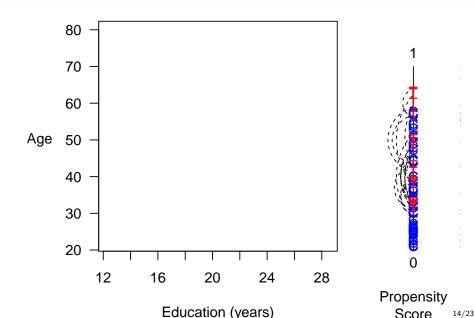


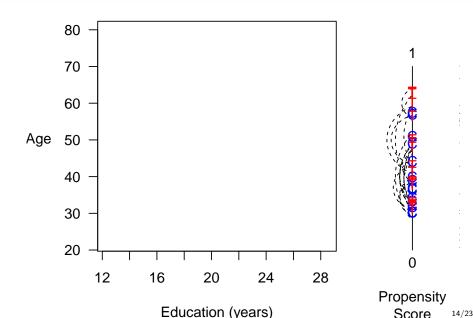
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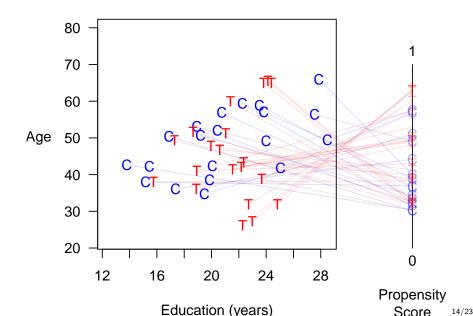


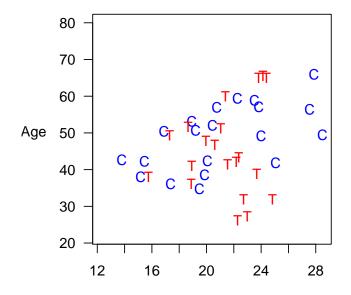




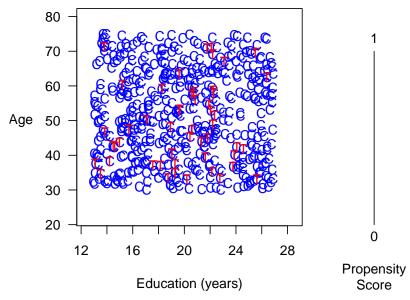


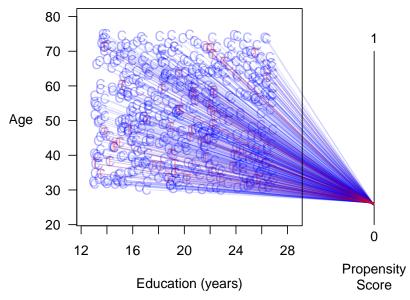


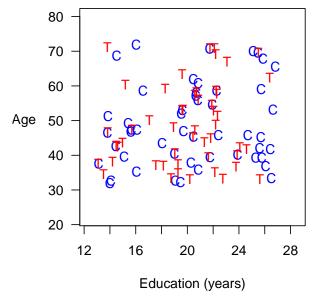




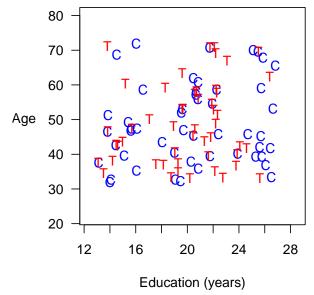
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Best Case: Propensity Score Matching is Suboptimal



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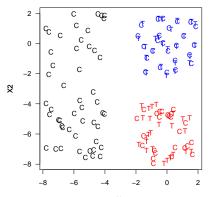
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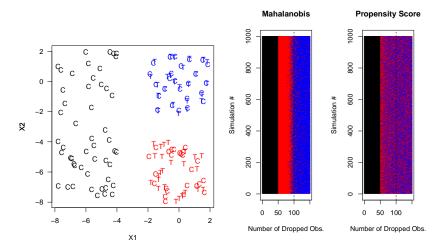
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X1

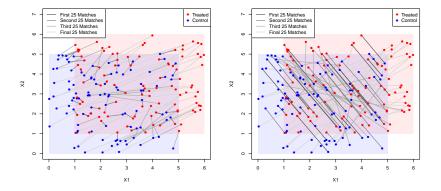
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What Does PSM Match?

MDM Matches

PSM Matches

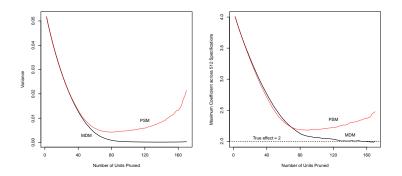


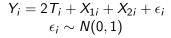
Controls: $X_1, X_2 \sim \text{Uniform}(0,5)$ Treateds: $X_1, X_2 \sim \text{Uniform}(1,6)$

PSM Increases Model Dependence & Bias

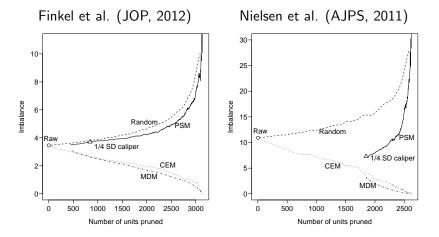
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The Propensity Score Paradox in Real Data



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For more information, papers, & software



GaryKing.org www.mit.edu/~rnielsen