## Why Propensity Scores Should Not Be Used For Matching

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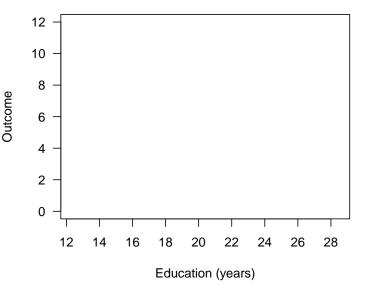
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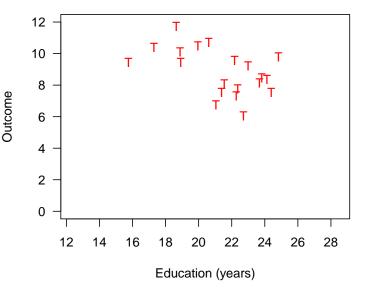
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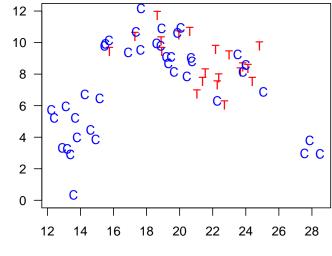
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  - The mathematical theorems about propensity scores: Correct, but inadequate





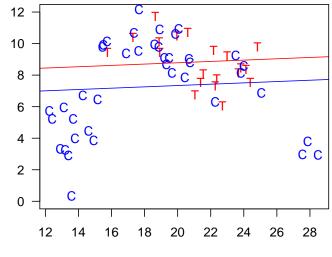
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Outcome

Education (years)

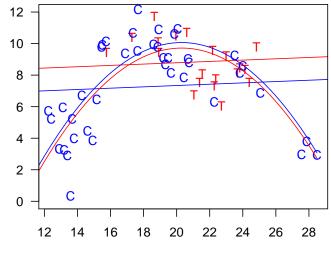
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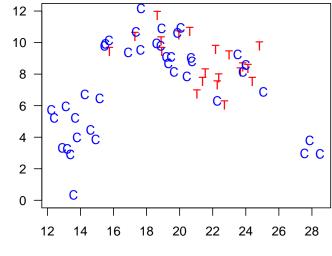
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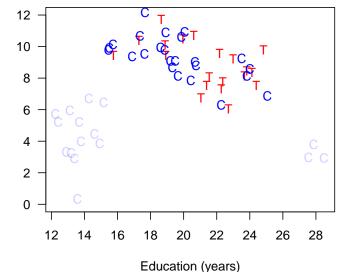
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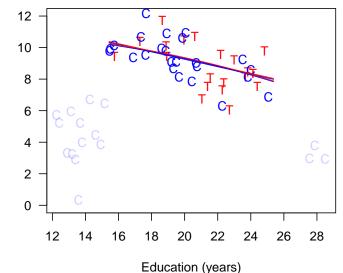
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- "Teaching psychology is mostly a waste of time" (Kahneman 2011)

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A central project of statistics: Automating away human discretion

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- Big convenience: Follow preprocessing with whatever statistical method you'd have used without matching
- Pruning nonmatches makes control vars matter less: reduces imbalance, model dependence, researcher discretion, & bias

Matching: Finding Hidden Randomized Experiments Types of Experiments

**Types of Experiments** 

Complete Randomization Matching: Finding Hidden Randomized Experiments Types of Experiments

> Complete Fully Randomization Blocked

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Goal of Each Matching Method (in Observational Data)

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- Other matching methods dominate PSM

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- PSM: complete randomization
- Other methods: *fully blocked*
- Other matching methods dominate PSM (wait, it gets worse)

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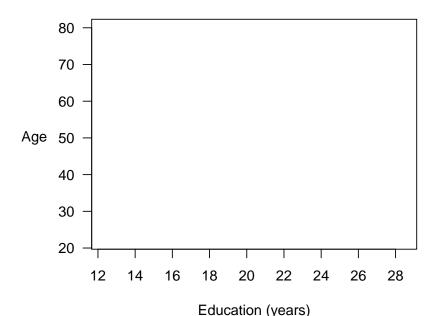
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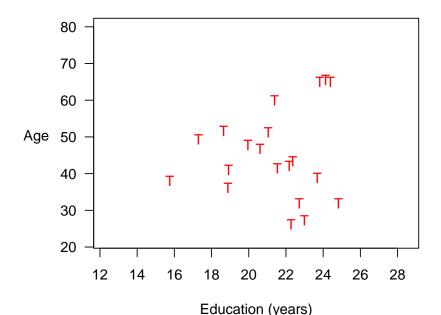
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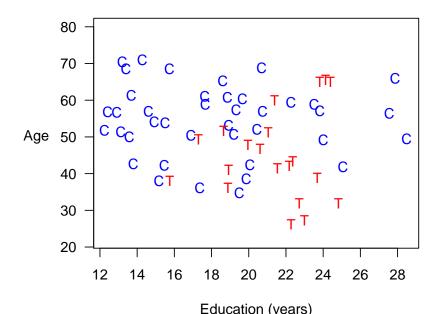
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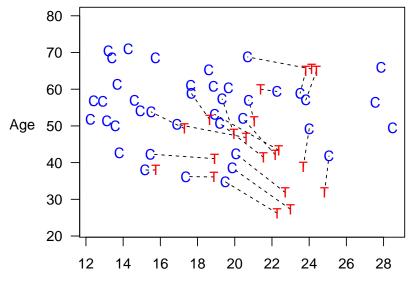
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  - (Many adjustments available to this basic method)
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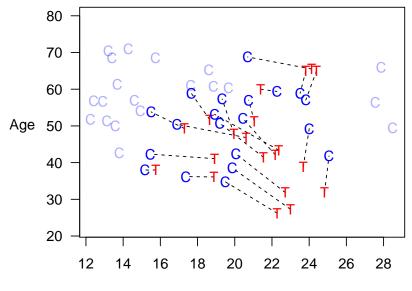




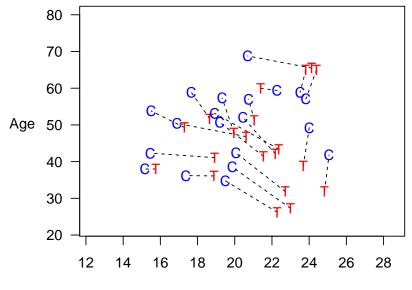




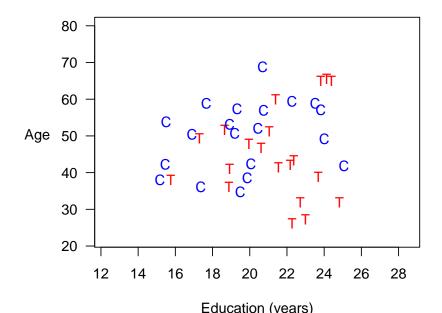
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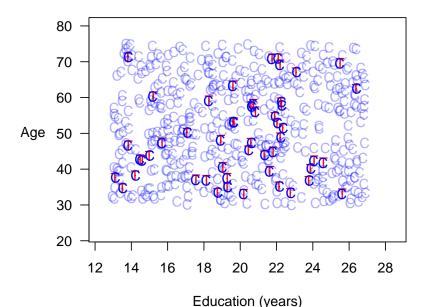
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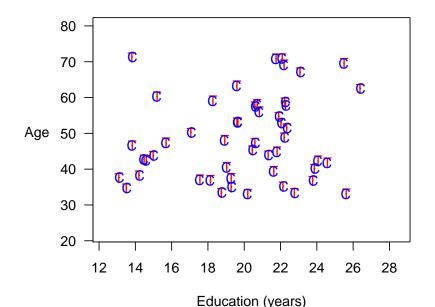
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# Best Case: Mahalanobis Distance Matching

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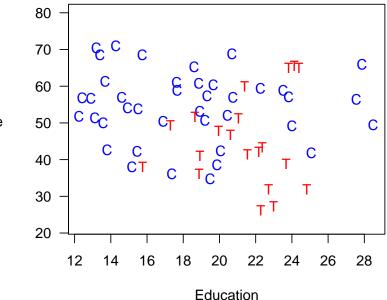
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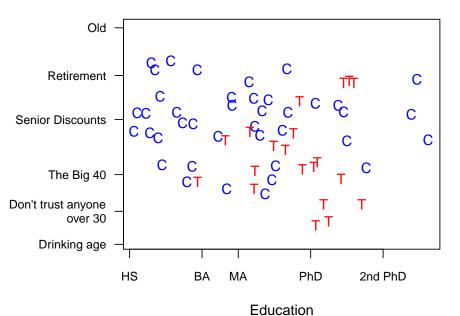
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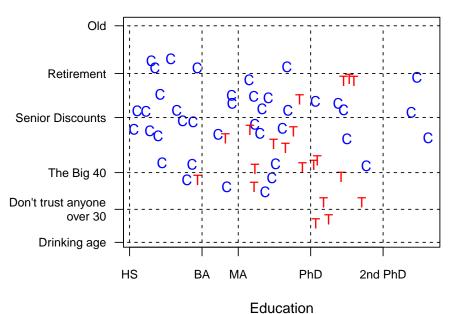
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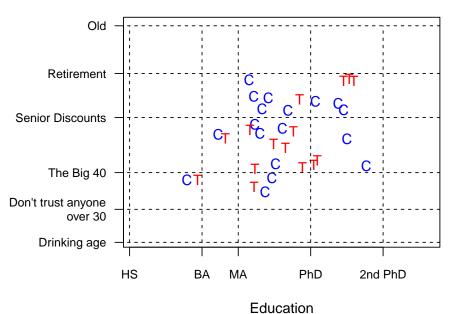
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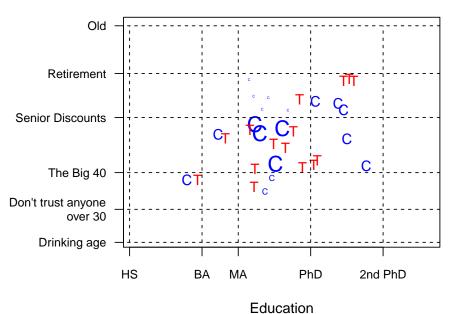


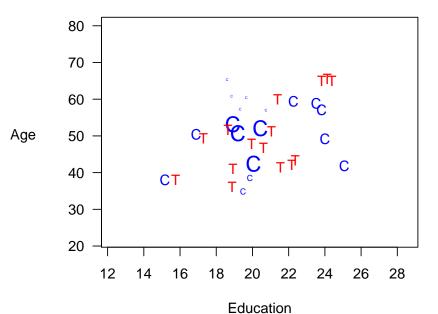
Age

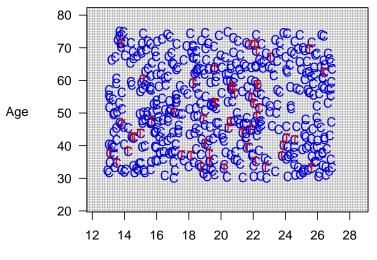




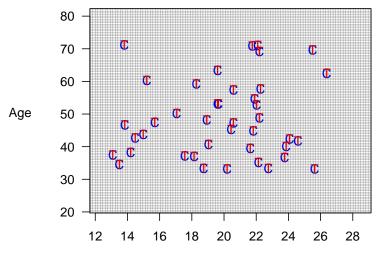




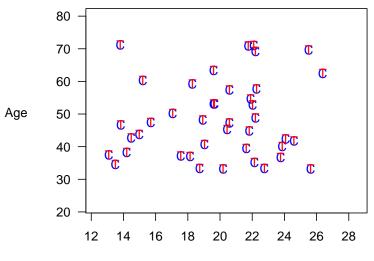




Education



Education



Education

(Approximates Completely Randomized Experiment)

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1. Preprocess (Matching)

(Approximates Completely Randomized Experiment)

### 1. Preprocess (Matching)

• Reduce k elements of X to scalar  

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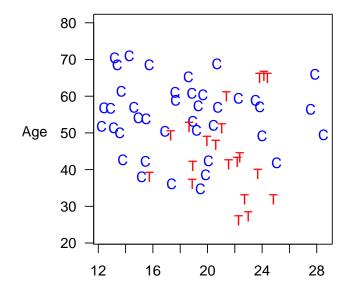
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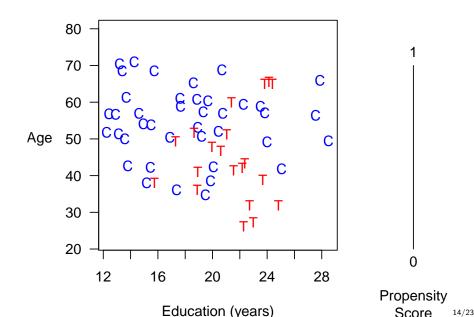
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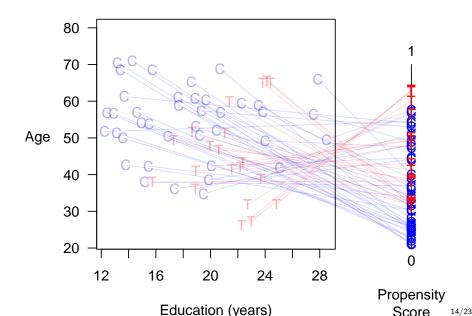
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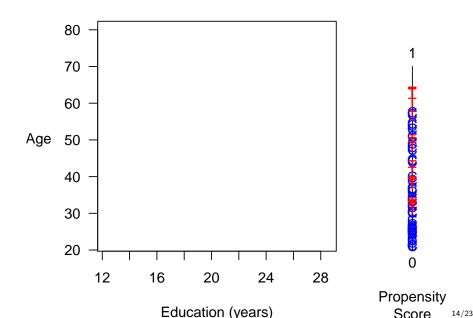
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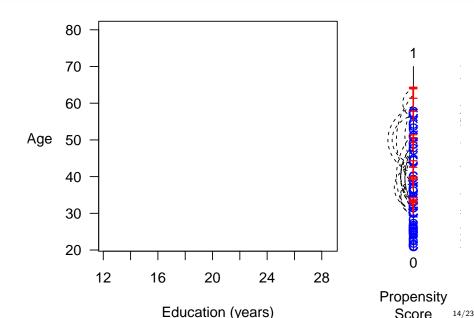


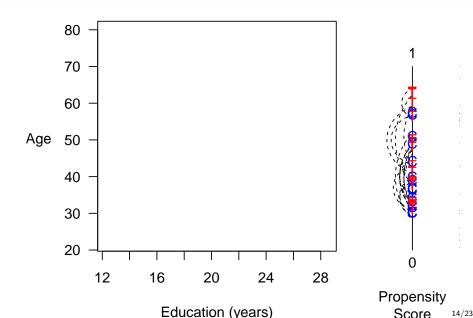
Education (years)

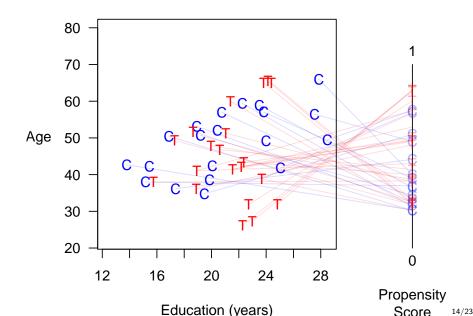


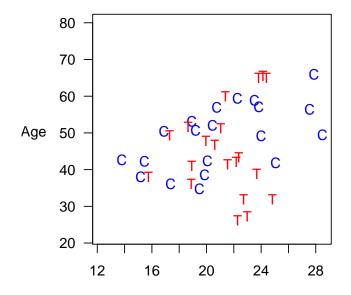




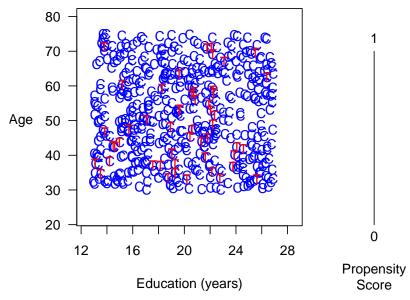


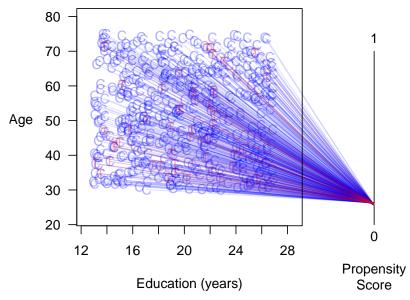


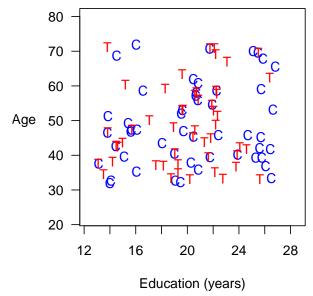




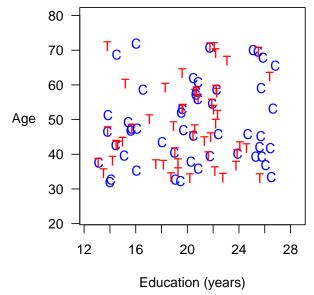
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## Best Case: Propensity Score Matching is Suboptimal



Deleting data only helps if you're careful!

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- Result is completely general (see math in the paper)

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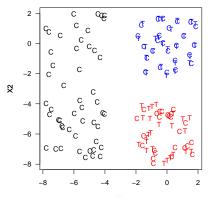
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  - Doesn't PSM solve the curse of dimensionality problem? Nope. The PSM Paradox gets worse with more covariates
  - What if I match on a few important covariates and then use PSM? The low standards will be raised some, but the PSM Paradox will kick in earlier

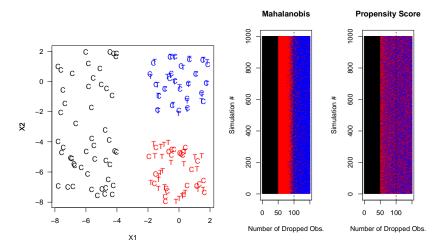
### PSM is Blind Where Other Methods Can See

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X1

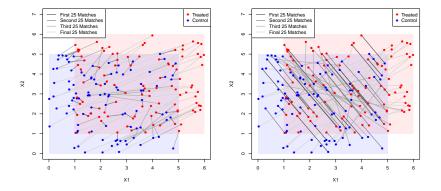
#### PSM is Blind Where Other Methods Can See



#### What Does PSM Match?

#### MDM Matches

#### **PSM Matches**

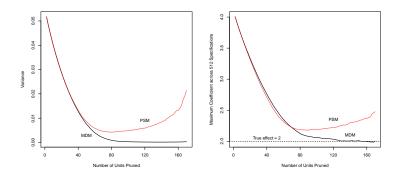


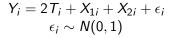
Controls:  $X_1, X_2 \sim \text{Uniform}(0,5)$ Treateds:  $X_1, X_2 \sim \text{Uniform}(1,6)$ 

#### PSM Increases Model Dependence & Bias

Model Dependence

Bias





#### The Propensity Score Paradox in Real Data

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Finkel et al. (JOP, 2012) Nielsen et al. (AJPS, 2011) 30 10-25 8 20 Imbalance Imbalance 6 15 Random Raw 4-Raw PSM 10 1/4 SD caliper △1/4 SD caliper 2 CEM CEM 5 MDN MDN 0 0 500 1000 1500 2000 2500 3000 500 1000 1500 2000 2500 0 Number of units pruned Number of units pruned

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Similar pattern for > 20 other real data sets we checked

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- Low Standards: sometimes helps, never optimizes
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#### For more information, papers, & software



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