

# Matching Methods for Causal Inference

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Harvard University

(Talk at University of Kansas, 12/2/2011)

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- $\rightsquigarrow$  Lots of insights revealed in the process

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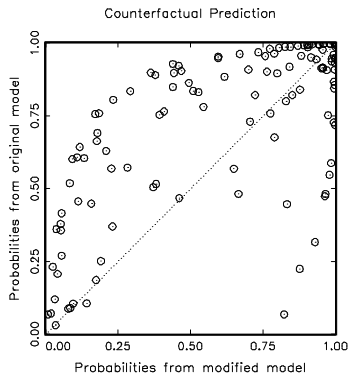
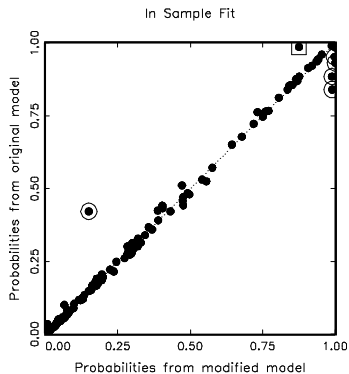
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- **Data analysis:** Logit model
- **The question:** How *model dependent* are the results?

# Two Logit Models, Apparently Similar Results

Variables	Original “Interactive” Model			Modified Model		
	Coeff	SE	P-val	Coeff	SE	P-val
Wartype	-1.742	.609	.004	-1.666	.606	.006
Logdead	-.445	.126	.000	-.437	.125	.000
Wardur	.006	.006	.258	.006	.006	.342
Factnum	-1.259	.703	.073	-1.045	.899	.245
Factnum2	.062	.065	.346	.032	.104	.756
Trnsfcap	.004	.002	.010	.004	.002	.017
Develop	.001	.000	.065	.001	.000	.068
Exp	-6.016	3.071	.050	-6.215	3.065	.043
Decade	-.299	.169	.077	-0.284	.169	.093
Treaty	2.124	.821	.010	2.126	.802	.008
UNOP4	3.135	1.091	.004	.262	1.392	.851
Wardur*UNOP4	—	—	—	.037	.011	.001
Constant	8.609	2.157	0.000	7.978	2.350	.000
N		122			122	
Log-likelihood		-45.649			-44.902	
Pseudo $R^2$		.423			.433	

# Doyle and Sambanis: Model Dependence



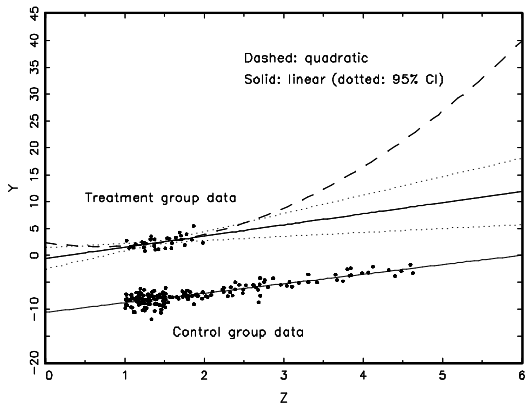
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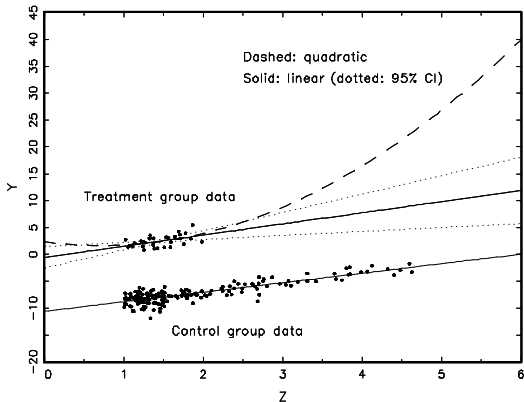
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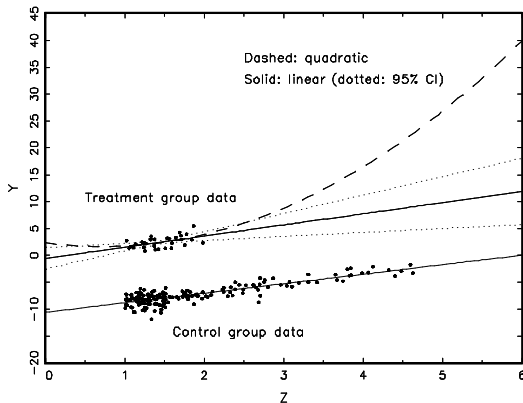


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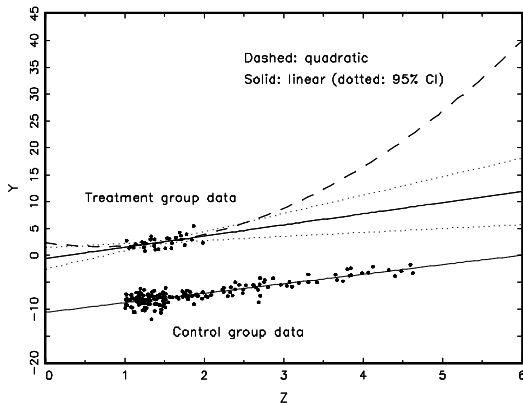


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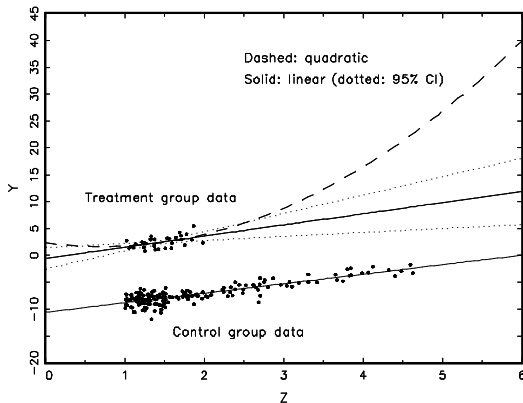


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- Preprocess II: Match (prune bad matches) within interpolation region

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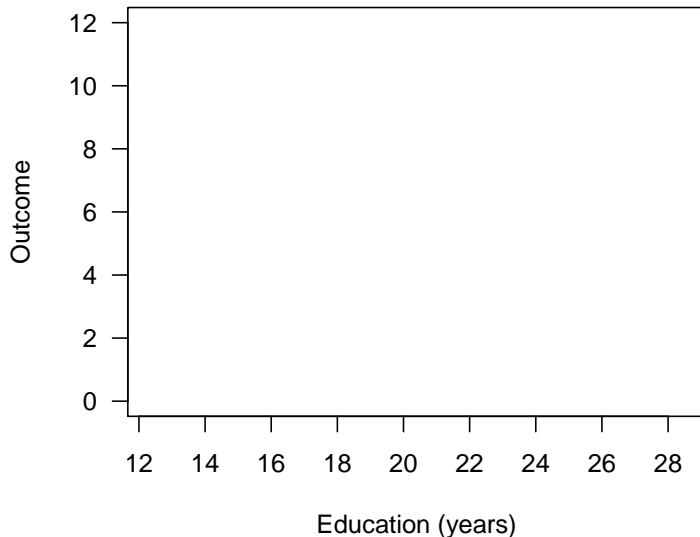


What to do?

- Preprocess I: Eliminate extrapolation region
- Preprocess II: Match (prune bad matches) within interpolation region
- Model remaining imbalance

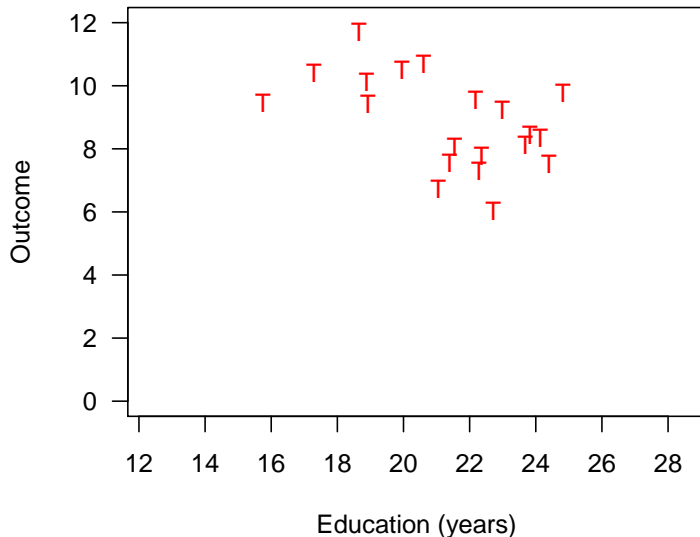
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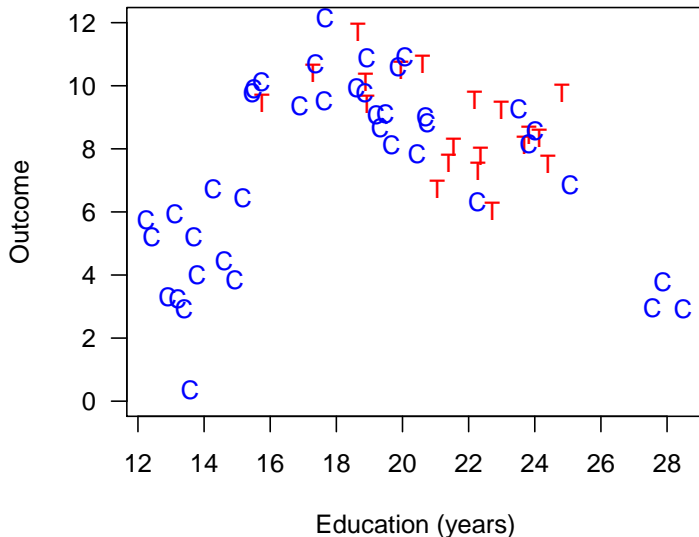
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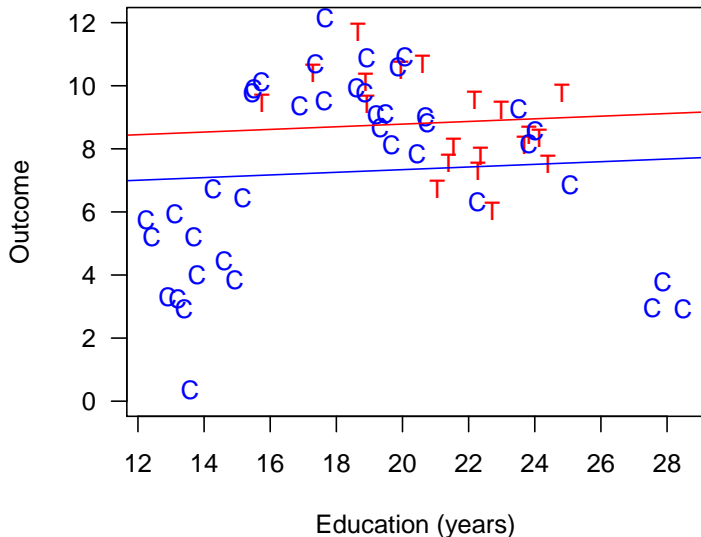
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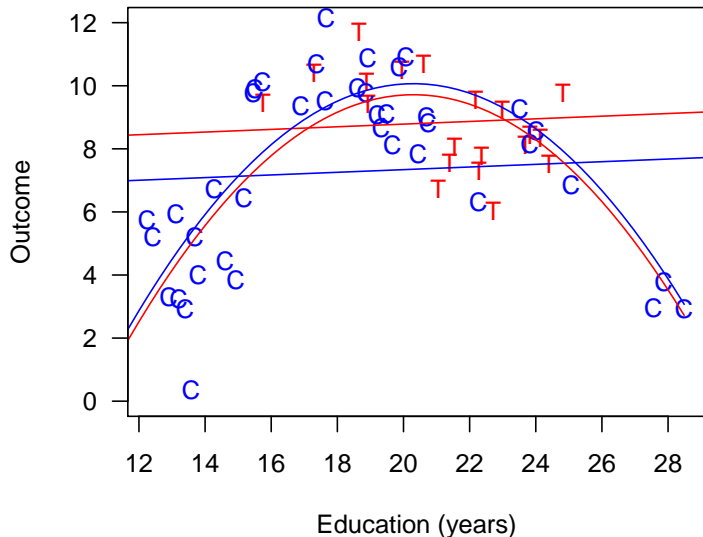
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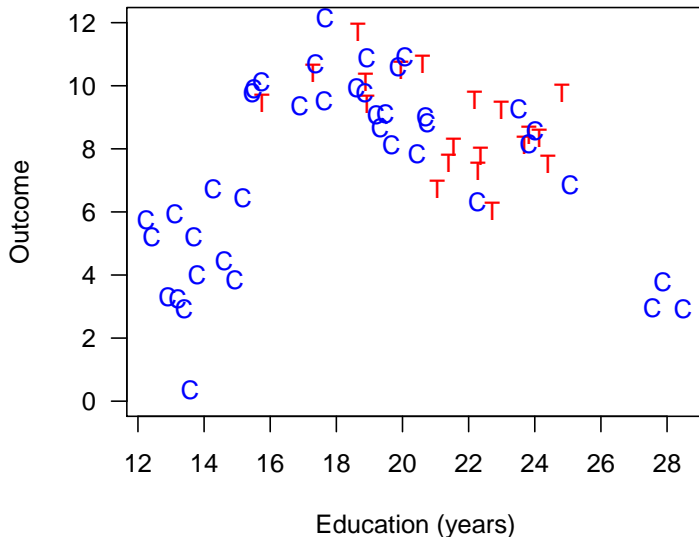
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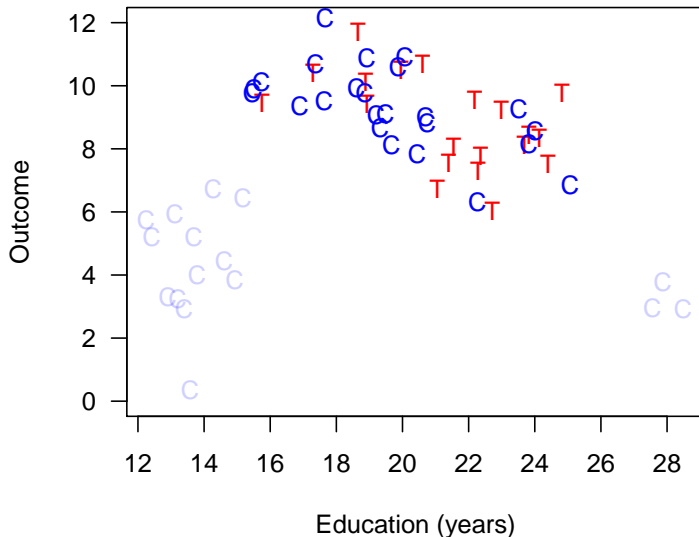
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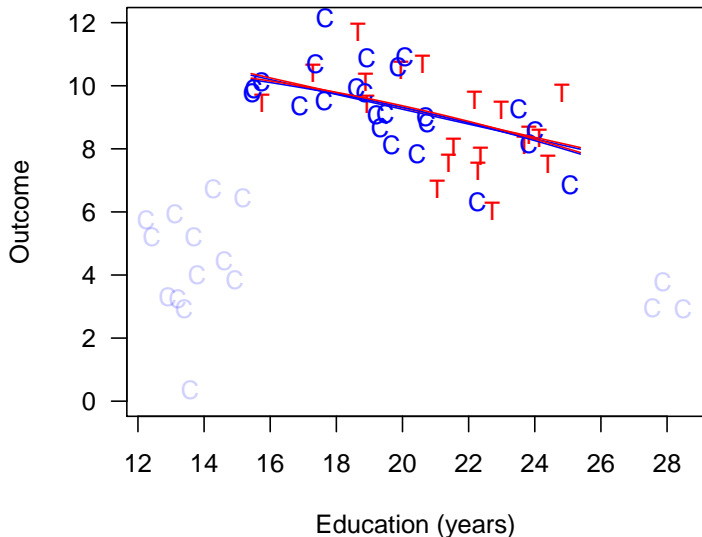
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Matching reduces model dependence, bias, and variance

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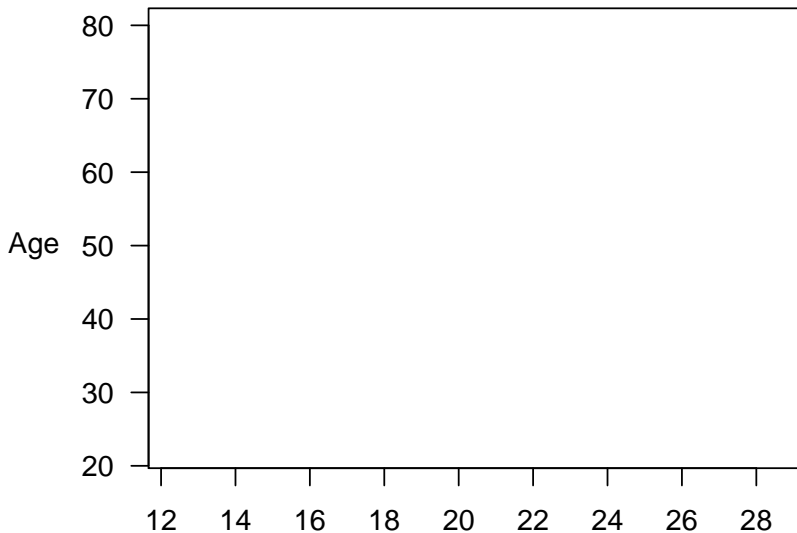
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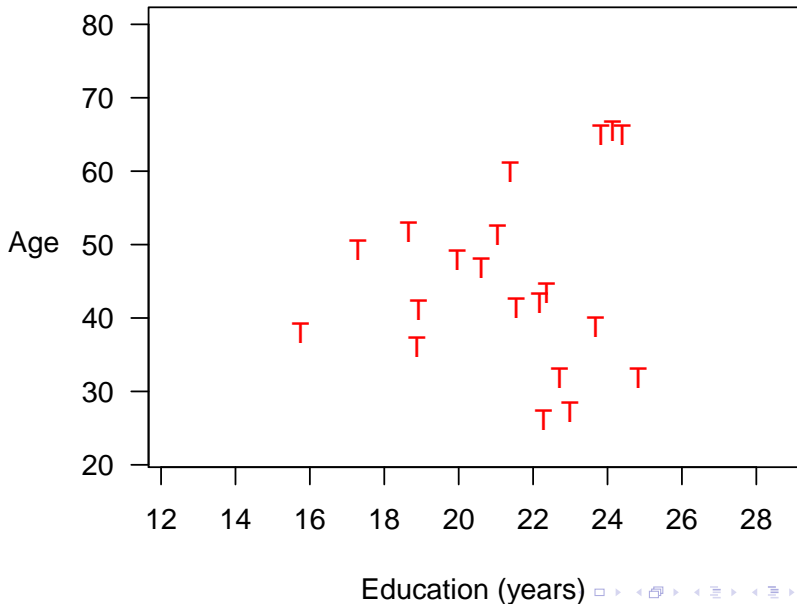
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Education (years)

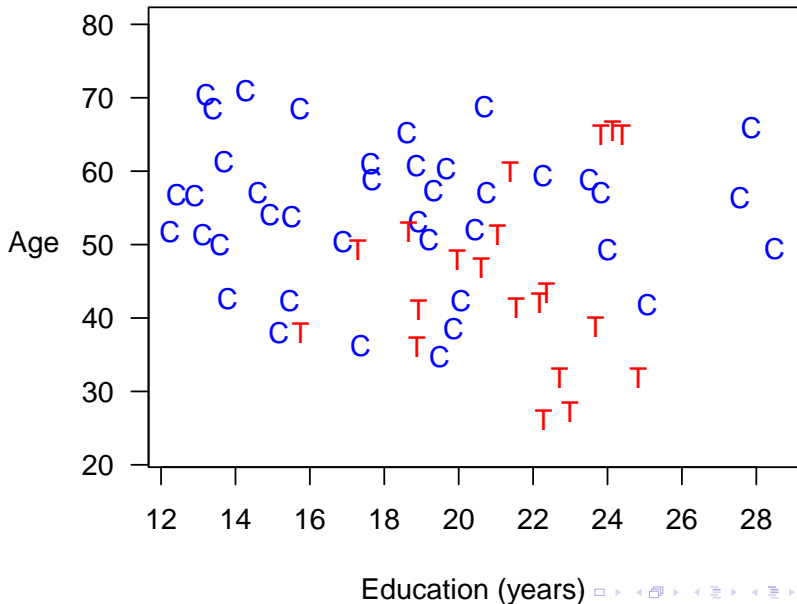


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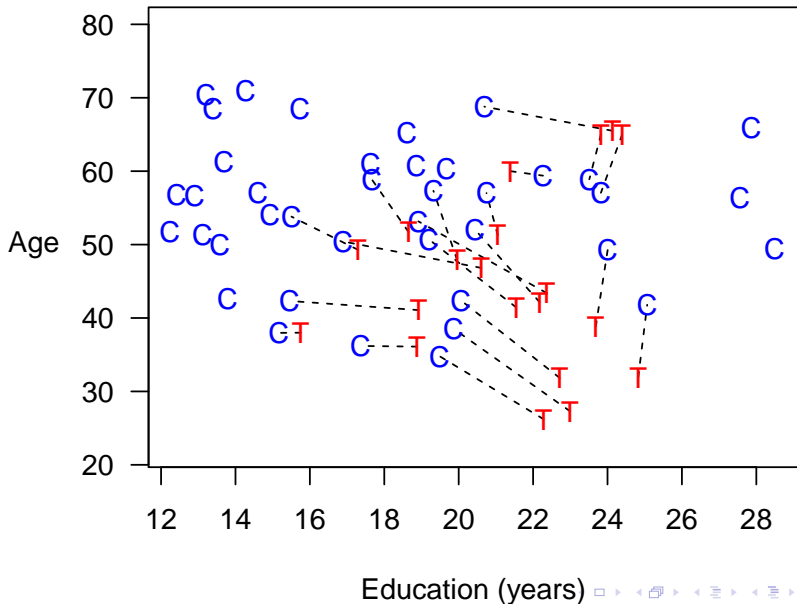




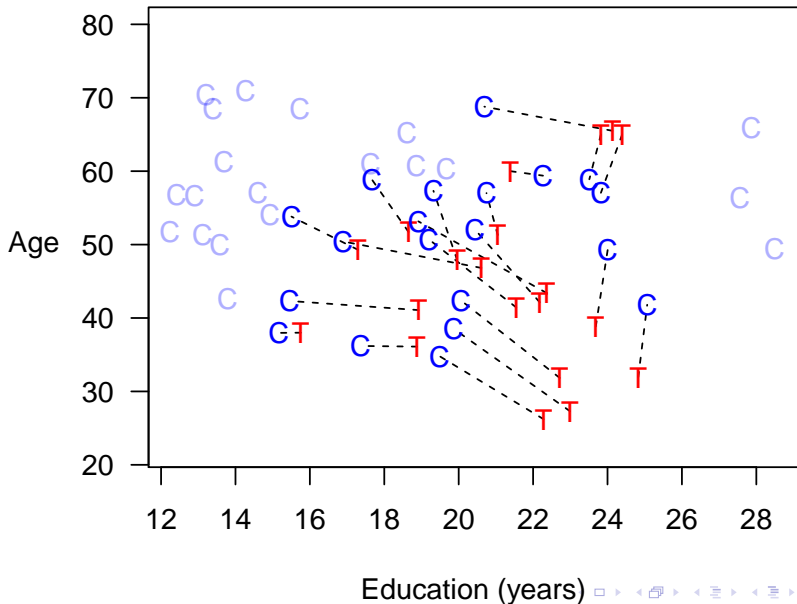
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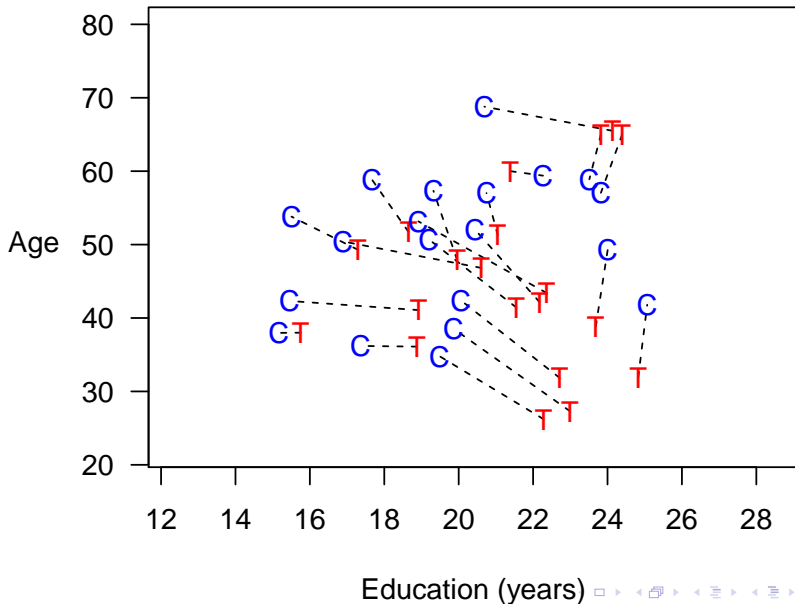
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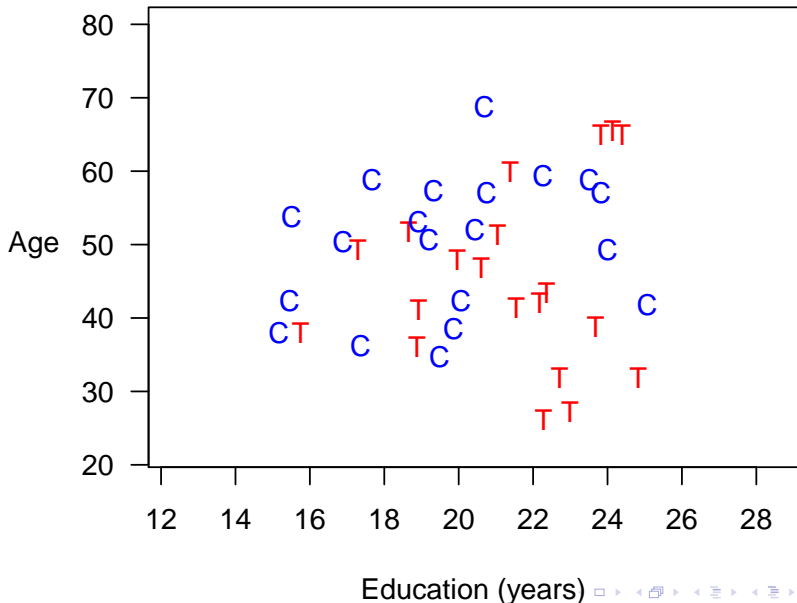
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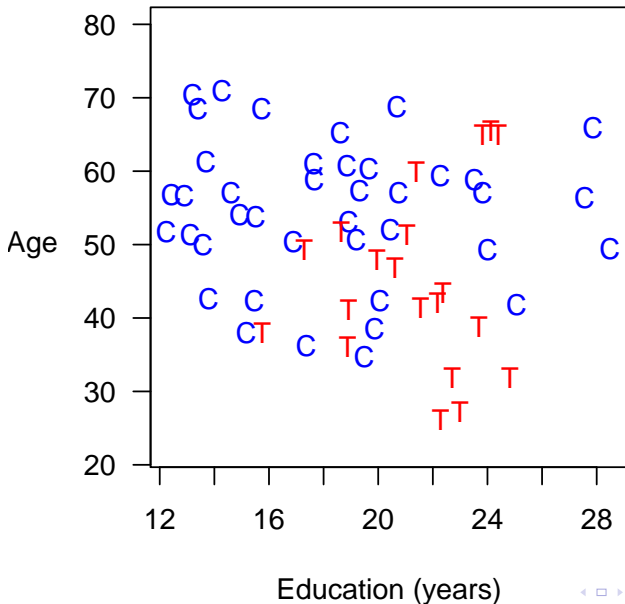
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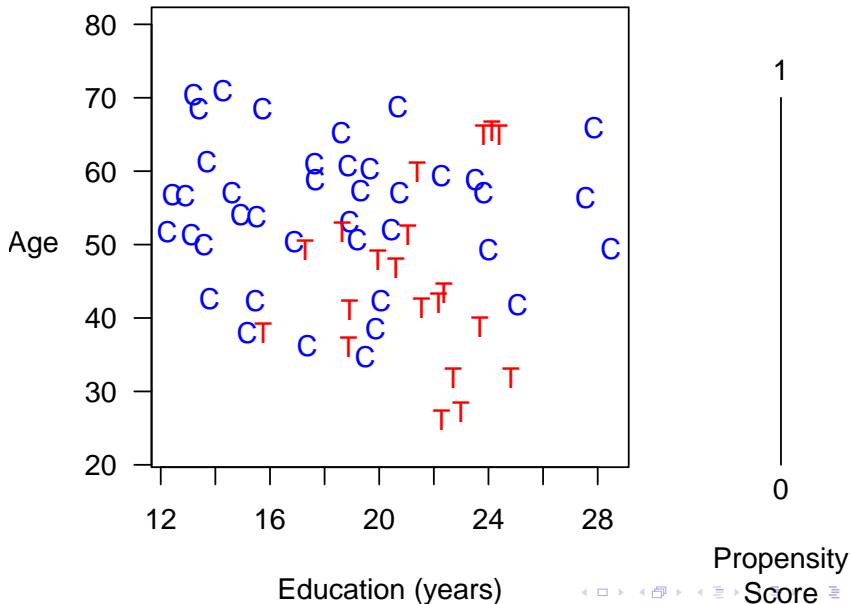
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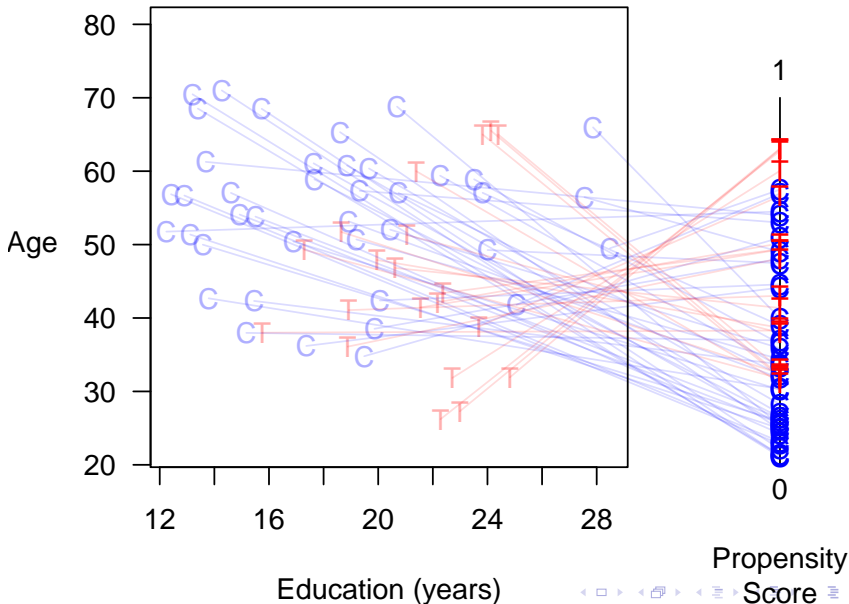
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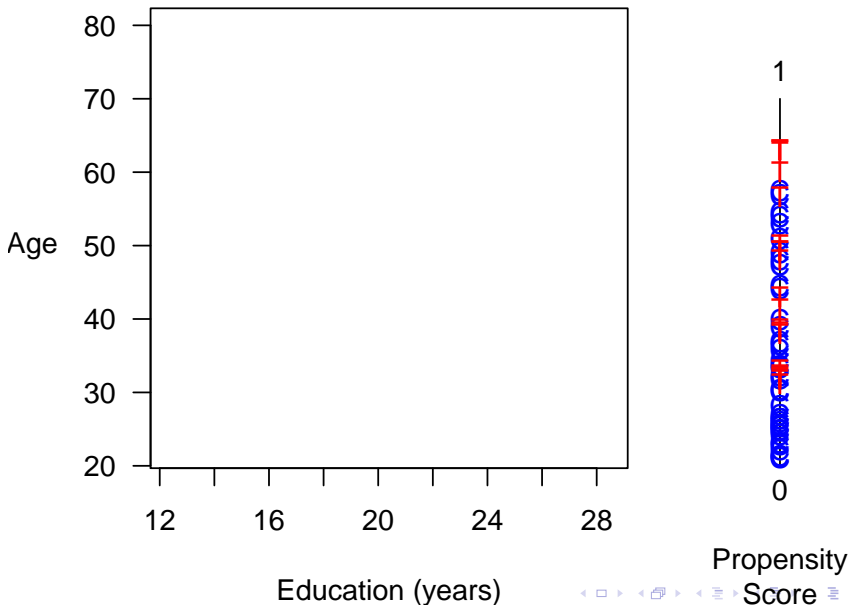
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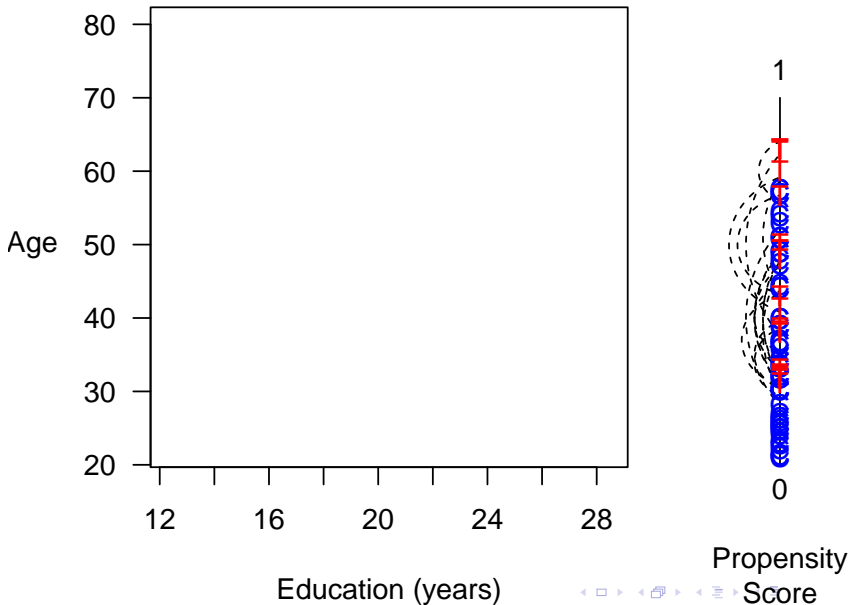


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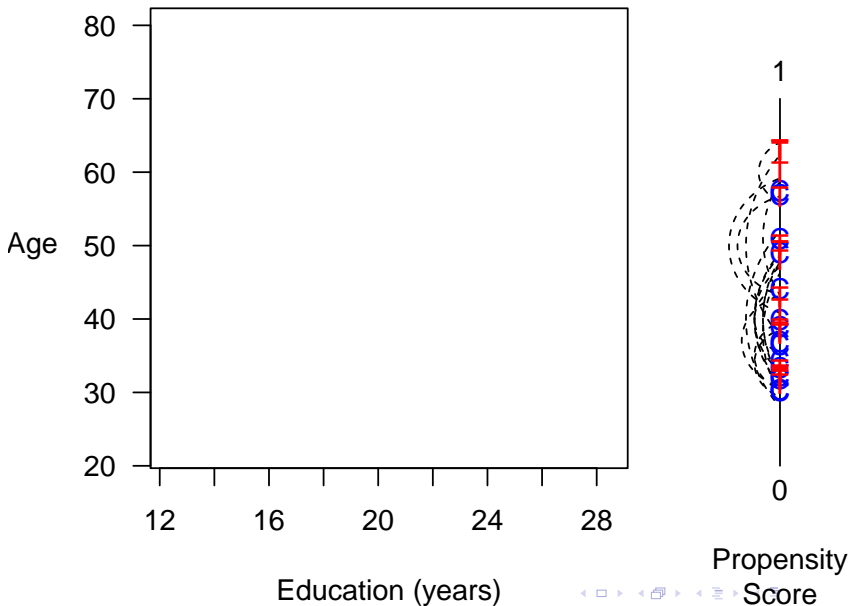




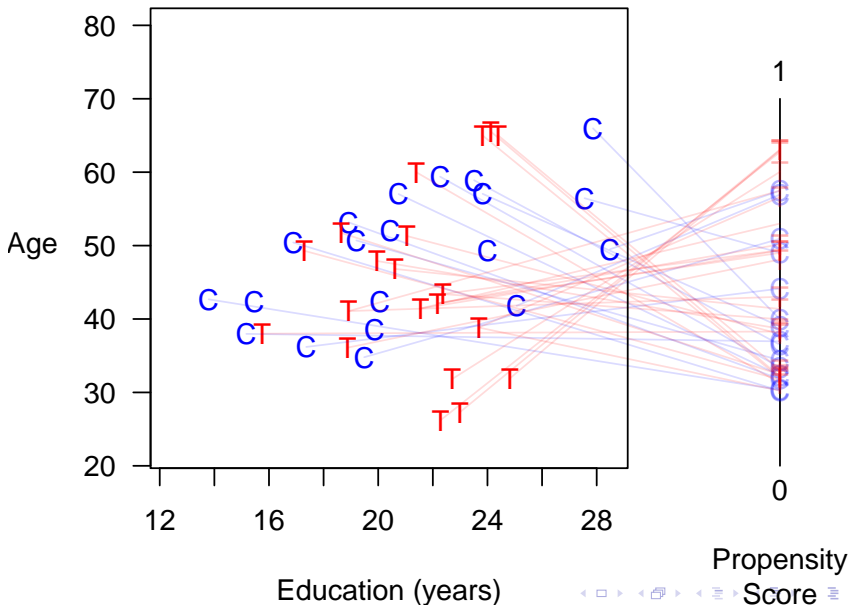
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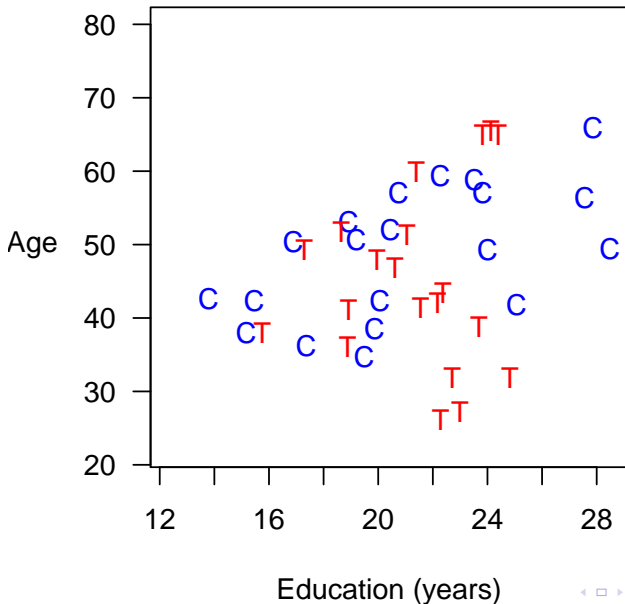
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  - Prune any stratum with 0 treated or 0 control units

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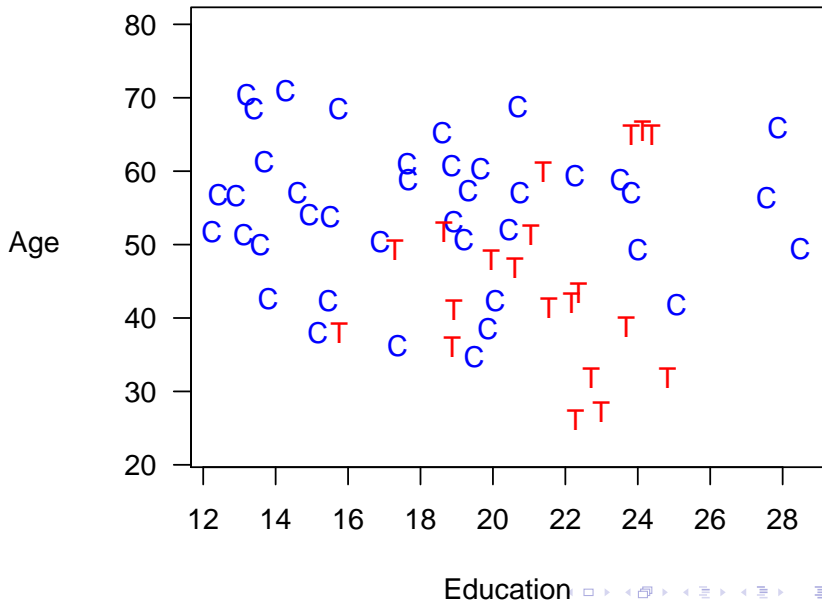
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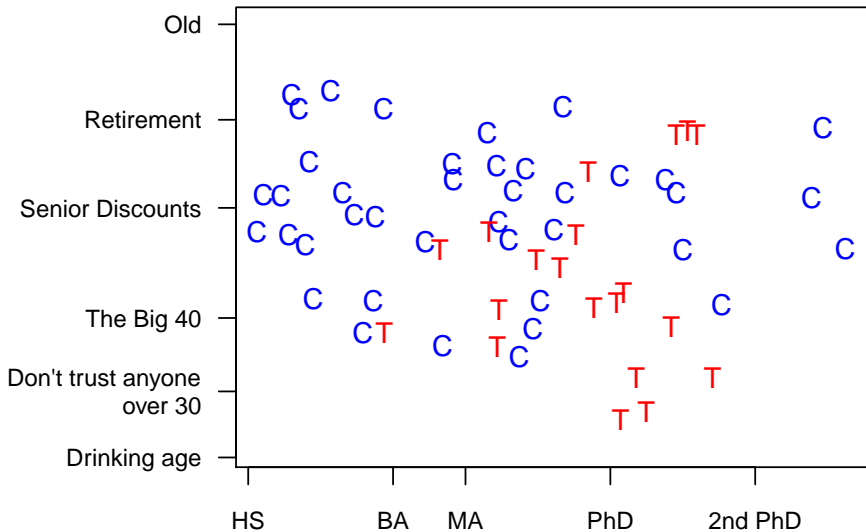
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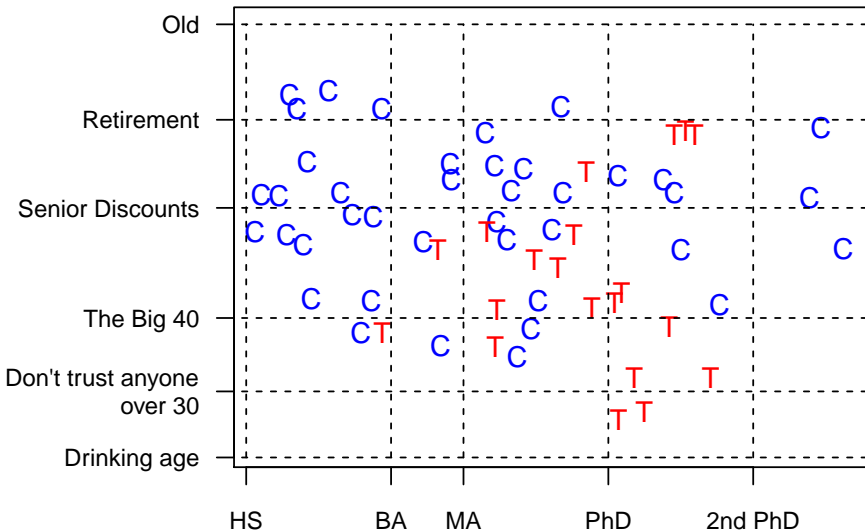
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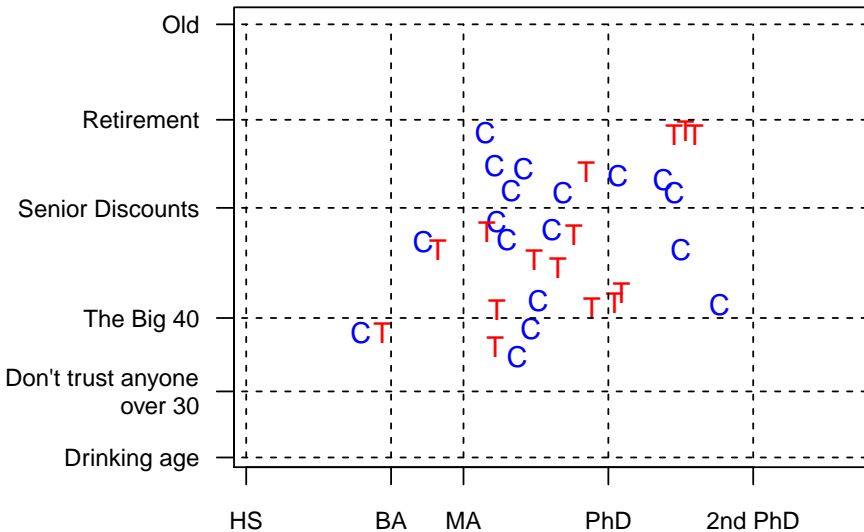
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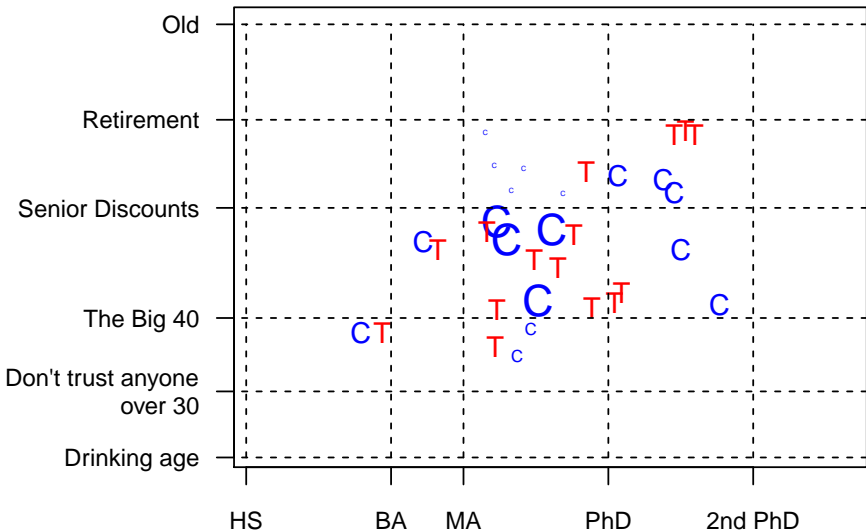
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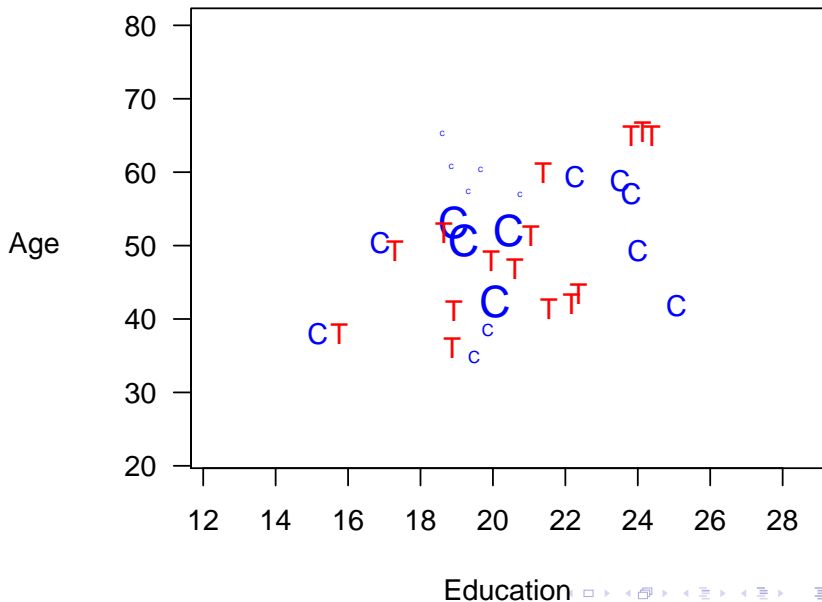
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- Another measure: Mahalanobis distance to closest unit in other group, averaged over each unit

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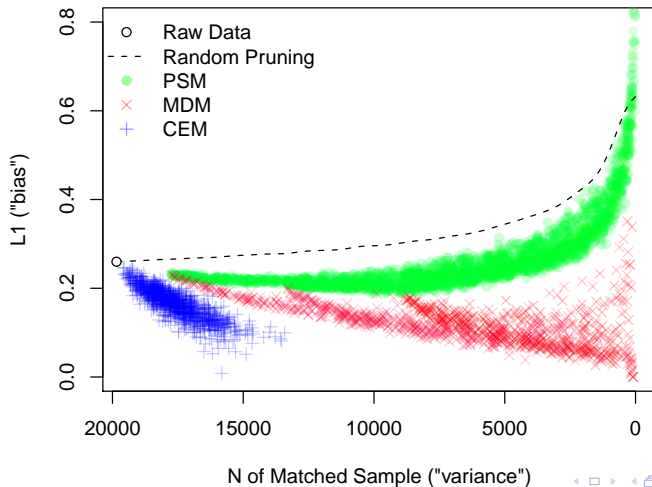
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- Our idea: Compute lots of matching solutions, identify the frontier of lowest imbalance for each given  $n$ , and choose a matching solution

# A Space Graph: Real Data

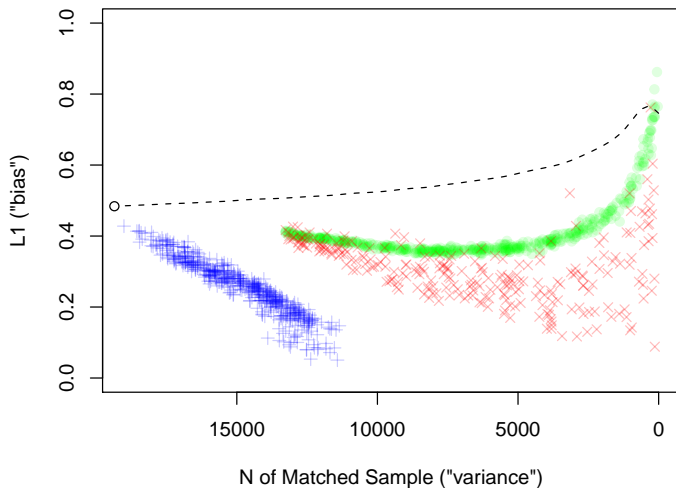
King, Nielsen, Coberley, Pope, and Wells (2011)

## Healthways Data



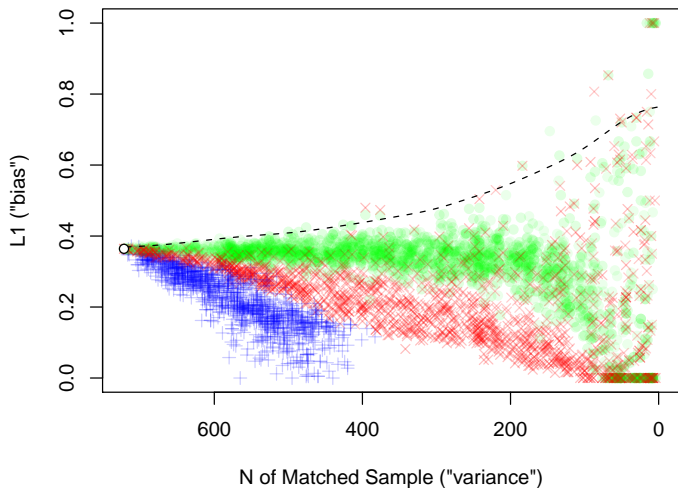
# A Space Graph: Real Data

Called/Not Called Data

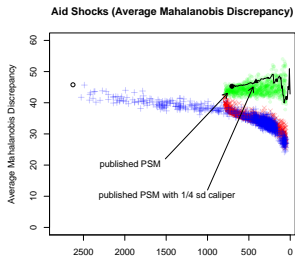
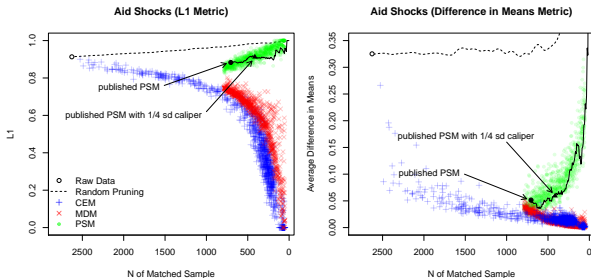


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Lalonde Data Subset

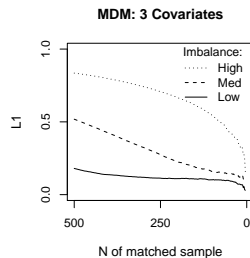
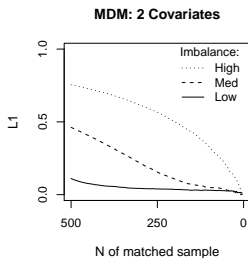
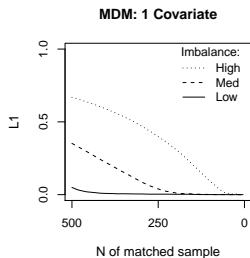


# Space Graphs: Different Imbalance Metrics

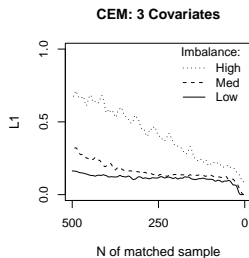
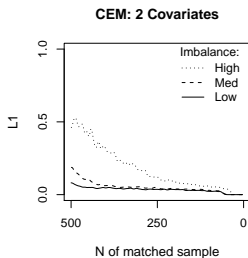
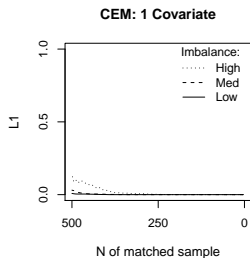




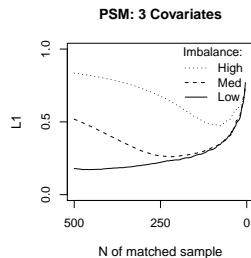
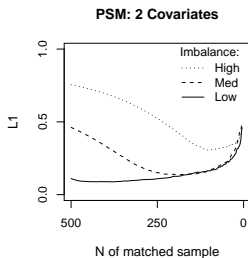
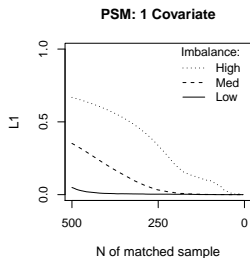
# A Space Graph: Simulated Data — Mahalanobis



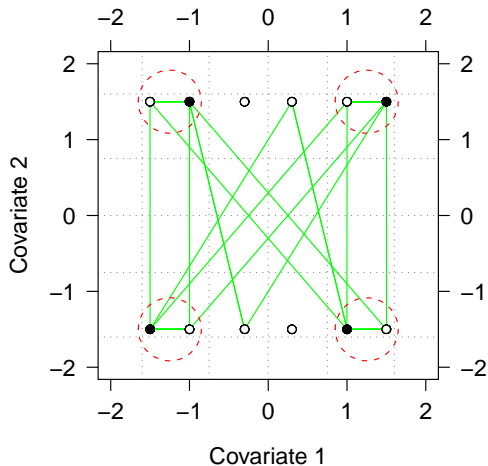
# A Space Graph: Simulated Data — CEM



# A Space Graph: Simulated Data — Propensity Score



# PSM Approximates Random Matching in Balanced Data



- PSM Matches
- - - CEM and MDM Matches

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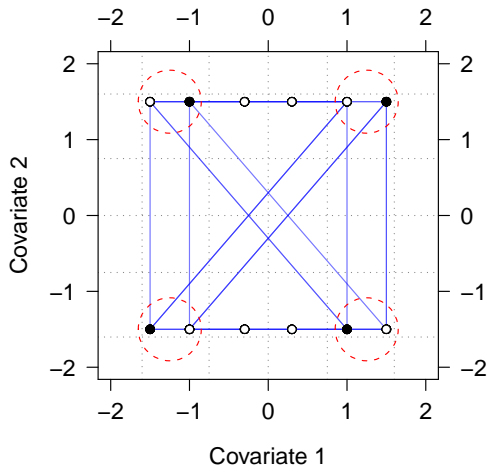
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# Destroying CEM with PSM's Two Step Approach



- CEM Matches
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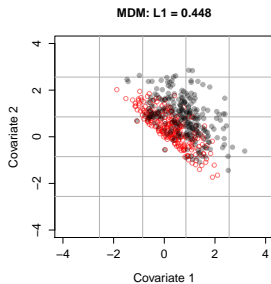
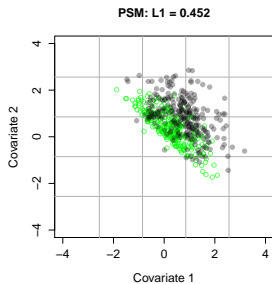
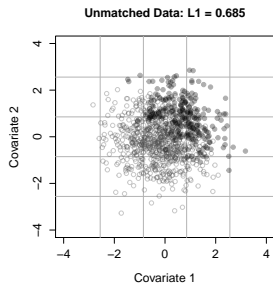
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For papers, software (for R, Stata, & SPSS), tutorials, etc.



<http://GKing.Harvard.edu/cem>

# Data where PSM Works Reasonably Well — PSM & MDM



# Data where PSM Works Reasonably Well — CEM

