Comparative Effectiveness of Matching Methods for Causal Inference

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joint work with

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Harvard, IQSS

(for a talk at Princeton University, 10/15/10)

Overview

- Problem: Model dependence (review)

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- Solution: Matching to preprocess data (review)

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- Solution: The Space Graph helps us compare
- Problem: The most commonly used method can increase imbalance!
- Solution: Other methods do not share this problem
- $\rightsquigarrow$ Lots of insights revealed in the process

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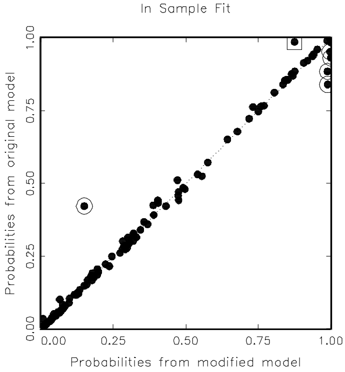
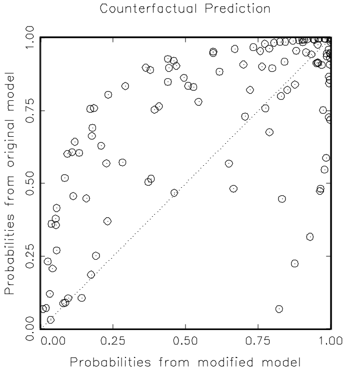
- Data analysis: Logit model
- The question: How *model dependent* are the results?

Two Logit Models, Apparently Similar Results

	Original "Interactive" Model			Modified Model		
Variables	Coeff	SE	P-val	Coeff	SE	P-val
Wartype	−1.742	.609	.004	−1.666	.606	.006
Logdead	−.445	.126	.000	−.437	.125	.000
Wardur	.006	.006	.258	.006	.006	.342
Factnum	−1.259	.703	.073	−1.045	.899	.245
Factnum2	.062	.065	.346	.032	.104	.756
Trnsfcap	.004	.002	.010	.004	.002	.017
Develop	.001	.000	.065	.001	.000	.068
Exp	−6.016	3.071	.050	−6.215	3.065	.043
Decade	−.299	.169	.077	−0.284	.169	.093
Treaty	2.124	.821	.010	2.126	.802	.008
UNOP4	3.135	1.091	.004	.262	1.392	.851
Wardur*UNOP4	—	—	—	.037	.011	.001
Constant	8.609	2.157	0.000	7.978	2.350	.000
N		122			122	
Log-likelihood		-45.649			-44.902	
Pseudo R^2		.423			.433	

Doyle and Sambanis: Model Dependence





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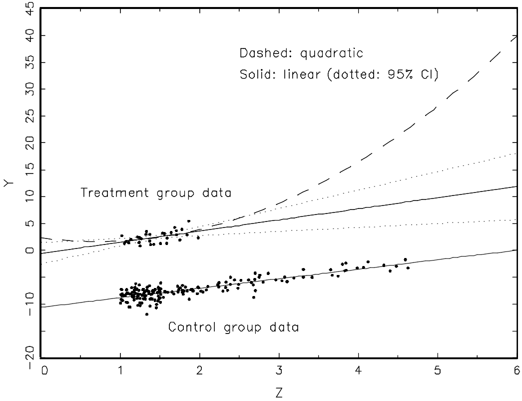

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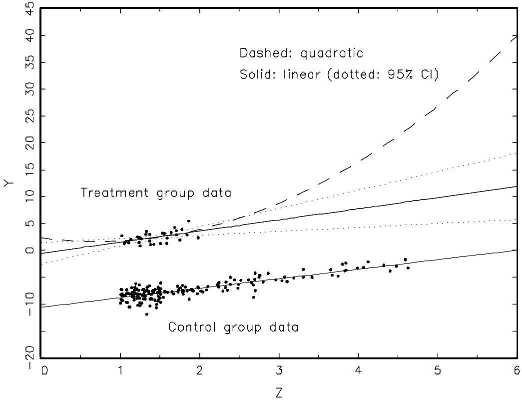

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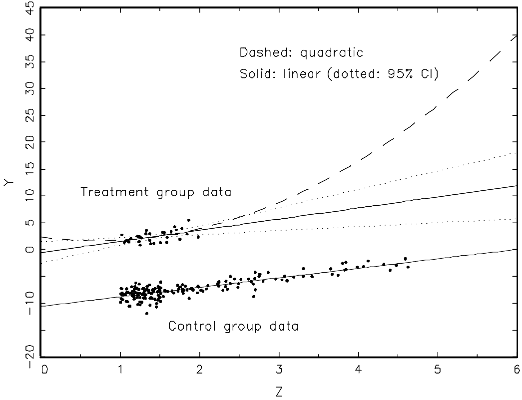

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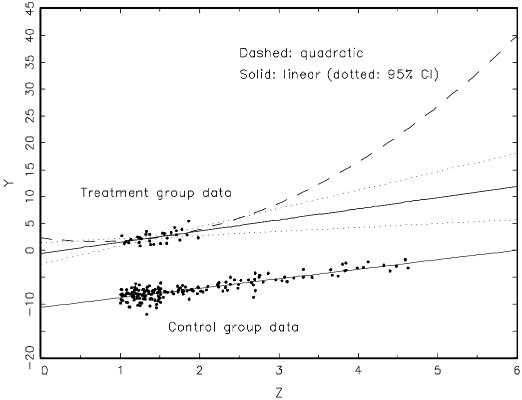

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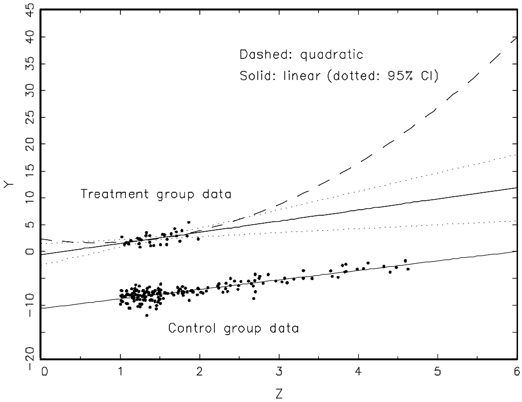

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- Model remaining imbalance

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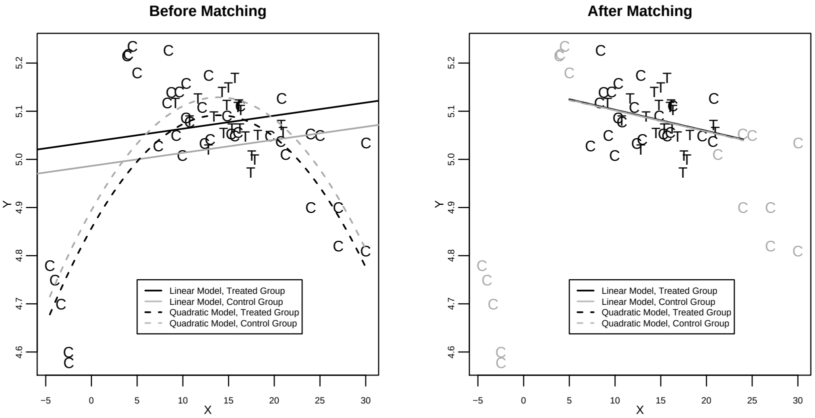

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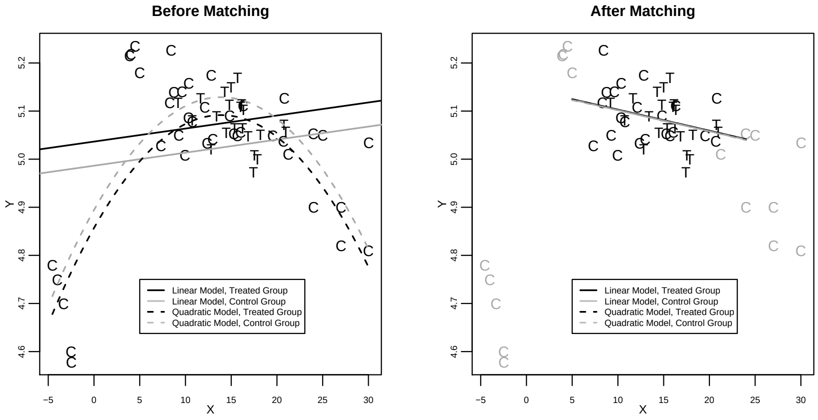

Matching reduces model dependence, bias, and variance

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   - Can apply other matching methods within CEM strata (inherit CEM's properties)

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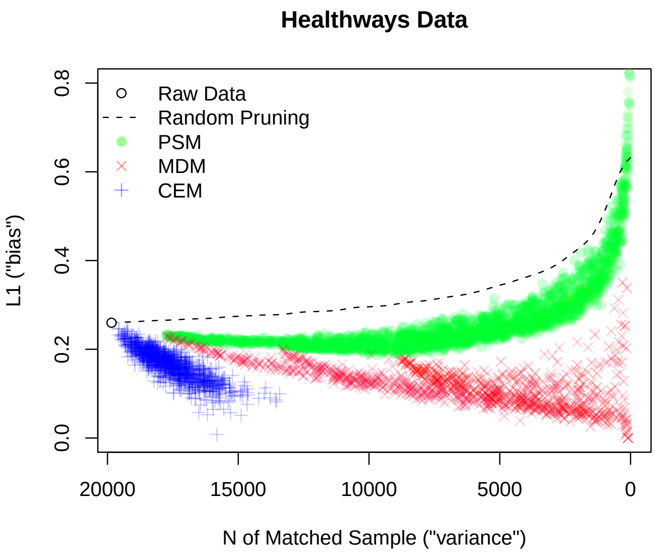
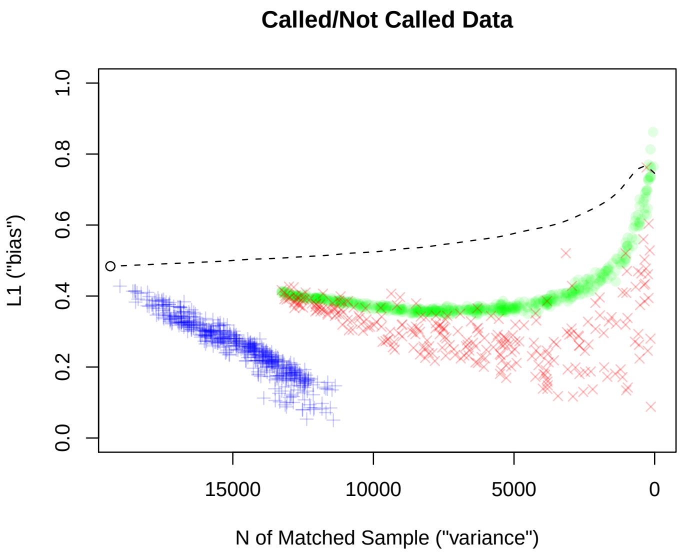
- Goal of Matching: reduce imbalance
- Classic measure: Difference of means (for each variable)
- Better measure (difference of multivariate histograms):

$$\mathcal{L}_1(f, g; H) = \frac{1}{2} \sum_{\ell_1 \cdots \ell_k \in H(\mathbf{X})} |f_{\ell_1 \cdots \ell_k} - g_{\ell_1 \cdots \ell_k}|$$


Healthways Data
Raw Data
Random Pruning
PSM
MDM
CEM
L1 ("bias")
0.0
0.2
0.4
0.6
0.8
20000
15000
10000
5000
0
N of Matched Sample ("variance")

A Space Graph: Real Data



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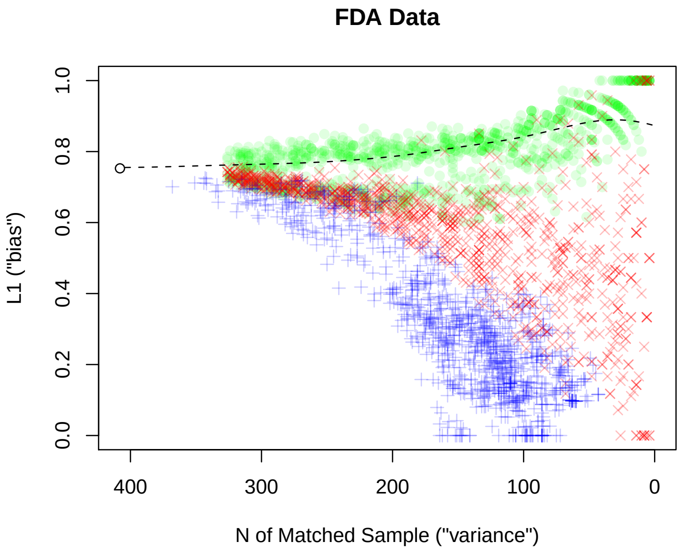

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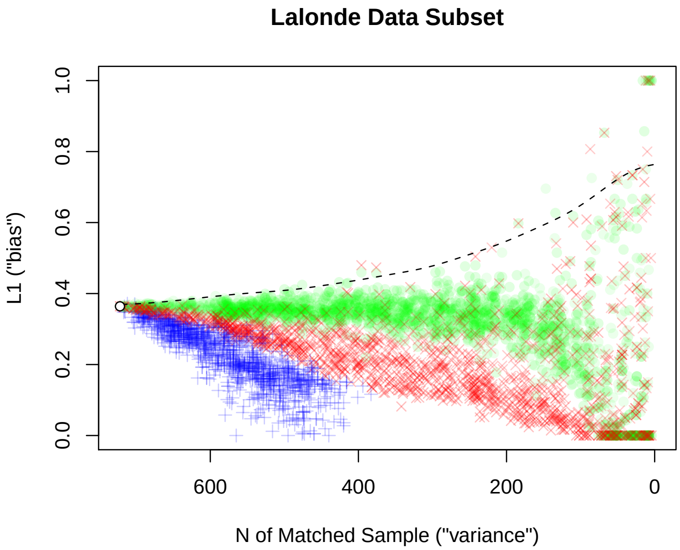
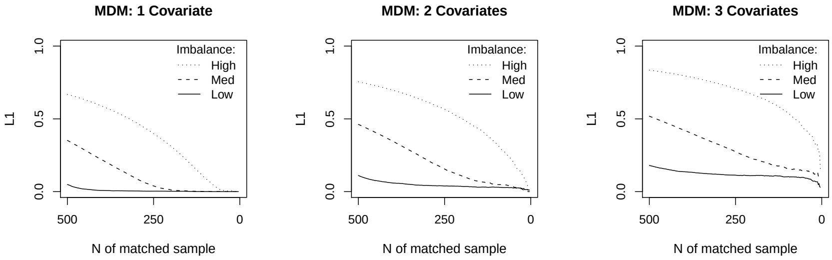
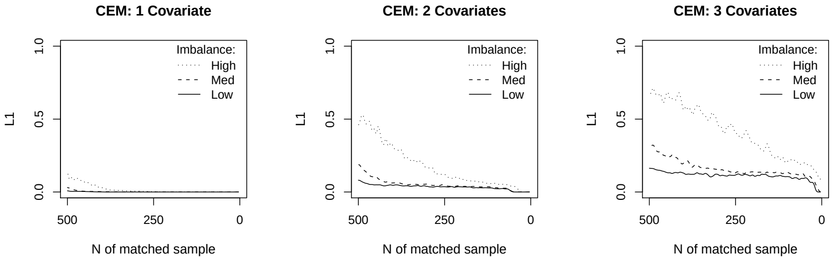


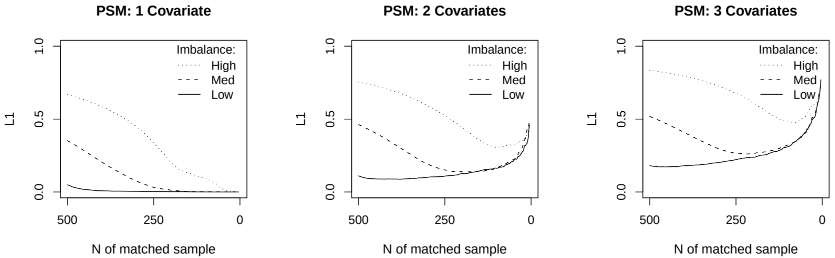
A Space Graph: Simulated Data — Mahalanobis



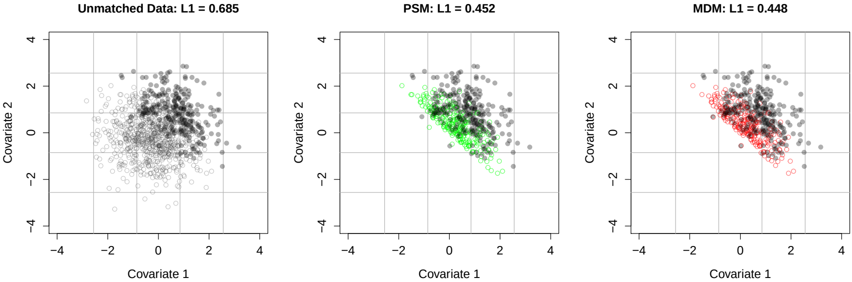
A Space Graph: Simulated Data — CEM



A Space Graph: Simulated Data — Propensity Score



An Example where PSM Works Reasonably Well



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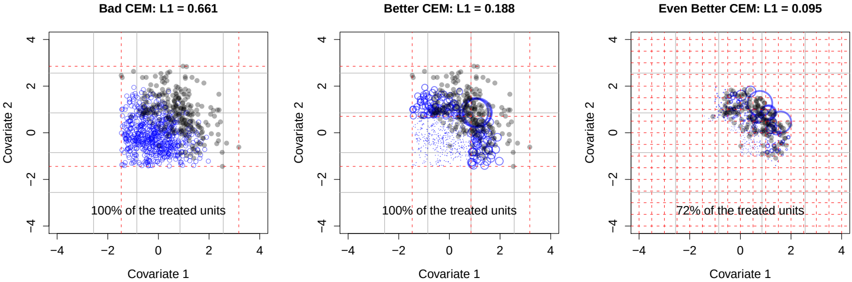
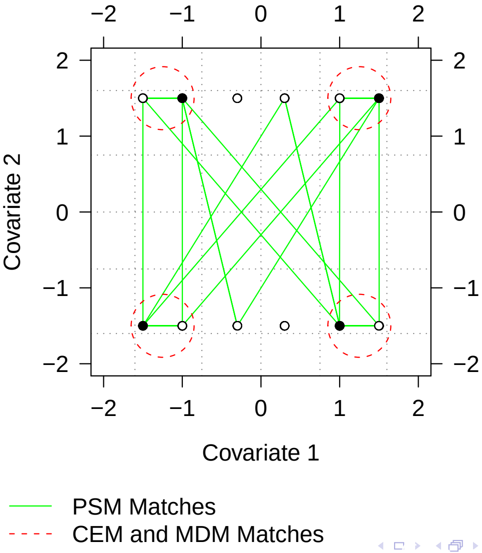


CEM Weights and Nonparametric Propensity Score

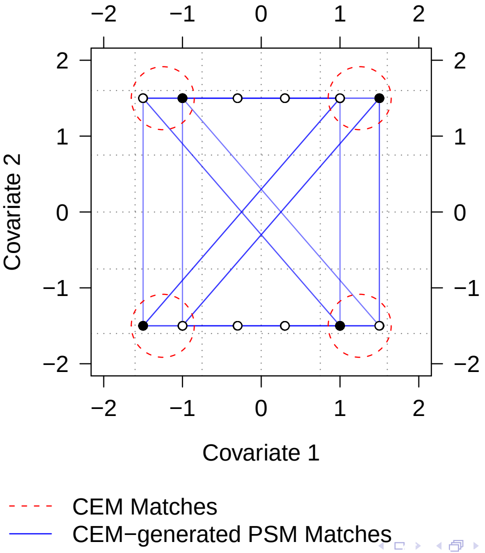
CEM Pscore: $$\widehat{\Pr}(T_i = 1|X_i) = \frac{m_i^T}{m_i^T + m_i^C}$$

CEM Weight: $$w_i = \frac{m_i^T}{m_i^C} \quad \text{(Unnormalized)}$$

PSM Approximates Random Matching in Balanced Data



Destroying CEM by using PSM's Two Step Approach



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    - Imbalance can be worse than original data
    - Can increase imbalance when removing the worst matches
    - Approximates random matching in well-balanced data (Random matching increases imbalance)

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For papers, software (for R and Stata), tutorials, etc.

http://GKing.Harvard.edu/cem