# Matching Methods for Causal Inference 

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(Talk at University of Rochester, 11/4/2011)

## Overview

- Problem: Model dependence (review)


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- Solution: The Space Graph helps us choose
- Problem: The most commonly used method can increase imbalance!
- Solution: Other methods do not share this problem
- (Coarsened Exact Matching is simple, easy, and powerful)
- $\rightsquigarrow$ Lots of insights revealed in the process


## Model Dependence Example

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- Counterfactual question: UN intervention switched for each war
- Data analysis: Logit model
- The question: How model dependent are the results?


## Two Logit Models, Apparently Similar Results

|  | Original "Interactive" Model |  | Modified Model |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Variables | Coeff | SE | P-val | Coeff | SE | P-val |
| Wartype | -1.742 | .609 | .004 | -1.666 | .606 | .006 |
| Logdead | -.445 | .126 | .000 | -.437 | .125 | .000 |
| Wardur | .006 | .006 | .258 | .006 | .006 | .342 |
| Factnum | -1.259 | .703 | .073 | -1.045 | .899 | .245 |
| Factnum2 | .062 | .065 | .346 | .032 | .104 | .756 |
| Trnsfcap | .004 | .002 | .010 | .004 | .002 | .017 |
| Develop | .001 | .000 | .065 | .001 | .000 | .068 |
| Exp | -6.016 | 3.071 | .050 | -6.215 | 3.065 | .043 |
| Decade | -.299 | .169 | .077 | -0.284 | .169 | .093 |
| Treaty | 2.124 | .821 | .010 | 2.126 | .802 | .008 |
| UNOP4 | 3.135 | 1.091 | .004 | .262 | 1.392 | .851 |
| Wardur*UNOP4 | - | - | - | .037 | .011 | .001 |
| Constant | 8.609 | 2.157 | 0.000 | 7.978 | 2.350 | .000 |
| N |  |  |  |  |  |  |
| Log-likelihood | 122 |  |  | 122 |  |  |
| Pseudo $R^{2}$ | -45.649 |  |  | -44.902 |  |  |

## Doyle and Sambanis: Model Dependence

In Sample Fit


Counterfactual Prediction


## Model Dependence: A Simpler Example

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What to do?

- Preprocess I: Eliminate extrapolation region
- Preprocess II: Match (prune bad matches) within interpolation region
- Model remaining imbalance


## Matching within the Interpolation Region

(Ho, Imai, King, Stuart, 2007: fig.1, Political Analysis)


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Matching reduces model dependence, bias, and variance

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- or Feasible Average Treatment effect on the Treated: FSATT


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## Education (years)

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- Can apply other matching methods within CEM strata (inherit CEM's properties)


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Education

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- Measuring Imbalance
- Classic measure: Difference of means (for each variable)
- Better measure (difference of multivariate histograms):

$$
\mathcal{L}_{1}(f, g ; H)=\frac{1}{2} \sum_{\ell_{1} \cdots \ell_{k} \in H(\mathbf{X})}\left|f_{\ell_{1} \cdots \ell_{k}}-g_{\ell_{1} \cdots \ell_{k}}\right|
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- CEM: Choose imbalance, match, check matched n
- Best practice: iterate
- Choose matched solution \& matching method becomes irrelevant
- Our idea: Compute lots of matching solutions, identify the frontier of lowest imbalance for each given $n$, and choose a matching solution


## A Space Graph: Real Data

King, Nielsen, Coberley, Pope, and Wells (2011)

Healthways Data


## A Space Graph: Real Data

## Called/Not Called Data



## A Space Graph: Real Data

## Lalonde Data Subset



## Space Graphs: Different Imbalance Metrics



Aid Shocks (Average Mahalanobis Discrepancy)


## A Space Graph: Simulated Data - Mahalanobis



MDM: 2 Covariates


MDM: 3 Covariates


## A Space Graph: Simulated Data - CEM

CEM: 1 Covariate


CEM: 2 Covariates


CEM: 3 Covariates


## A Space Graph: Simulated Data - Propensity Score

PSM: 1 Covariate


PSM: 2 Covariates


PSM: 3 Covariates


## PSM Approximates Random Matching in Balanced Data



- PSM Matches
---- CEM and MDM Matches


## CEM Weights and Nonparametric Propensity Score

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$\rightsquigarrow$ CEM:

- Gives a better pscore than PSM
- Doesn't match based on crippled information


## Destroying CEM with PSM's Two Step Approach


--- CEM Matches

- CEM-generated PSM Matches


## Data where PSM Works Reasonably Well - PSM \& MDM



## Data where PSM Works Reasonably Well - CEM

Bad CEM: L1 = 0.661


Better CEM: L1 $=0.188$


Even Better CEM: L1 $=0.095$


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- You can easily check with the Space Graph


## For papers, software (for R, Stata, \& SPSS), tutorials, etc.


http://GKing.Harvard.edu/cem

