

Scientific Measurement in Redistricting Research

Gary King¹

Institute for Quantitative Social Science
Harvard University

The Gerrymandering Project, Princeton University, 5/21/2021

¹GaryKing.org

Principles

Defining and Measuring Compactness

Differential Privacy

Fairness in Redistricting

Requirements for Scientific Measurement

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 - **E.g.:** Margins of error (CIs), SEs, hypothesis tests, etc.
 - **Without uncertainty estimates:** Measures are useless
 - **A scientific statement:** not one that is necessarily correct, but one that comes with accurate uncertainty estimates

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Fairness in Redistricting

What's Compactness?



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- Researchers vs the Law

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- **How do we know if we find it?**

- All know it when they see it & agree on the same dimension

Which is more compact?



Which is more compact? Depends on the standard!



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Convex Hull

4

3

2

1

Which is more compact? Depends on the standard!



Convex Hull	4	3	2	1
Reock	1	2	3	4

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Polsby-Popper	4	1	2	3

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Significant Corners	4	1	3	2

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- 7 measures;

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- ⇒ To determine the best measure:

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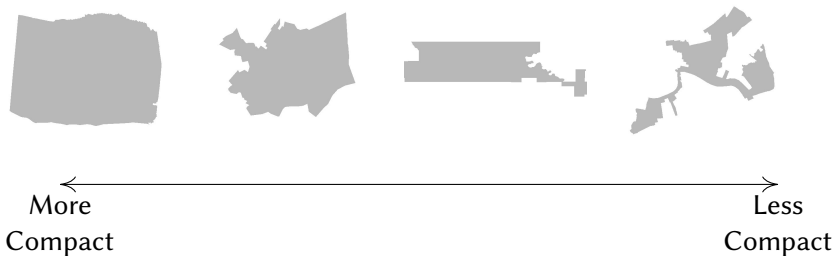
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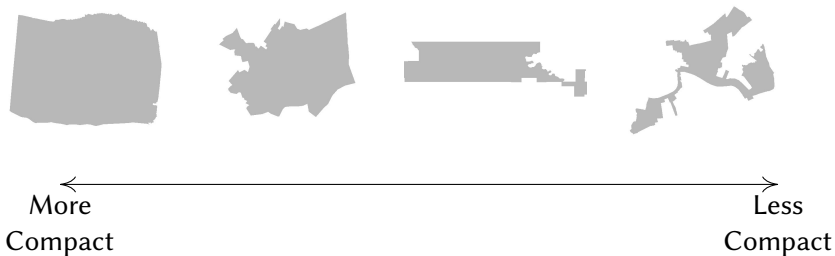
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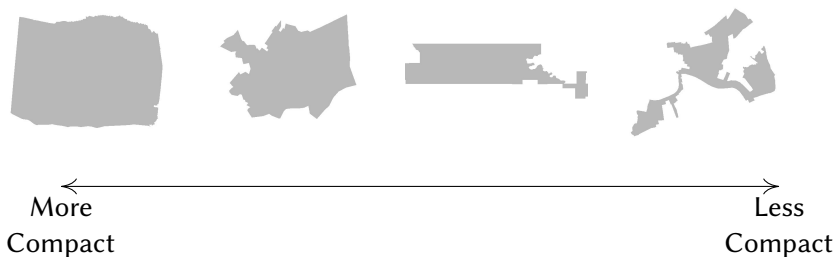
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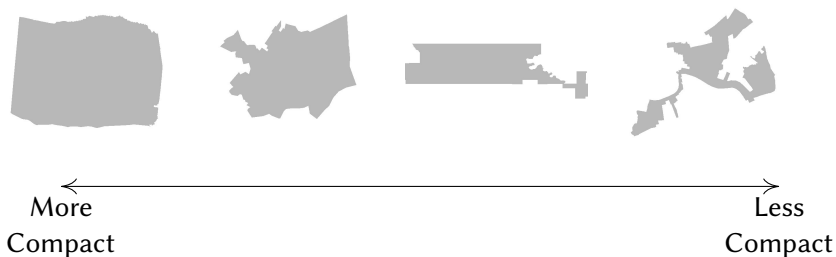
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- The **QOI** is intuitive
- We give **measure = $f(\text{shape})$** with uncertainty estimates
- **All agree**: Very high intra- and inter-coder reliability

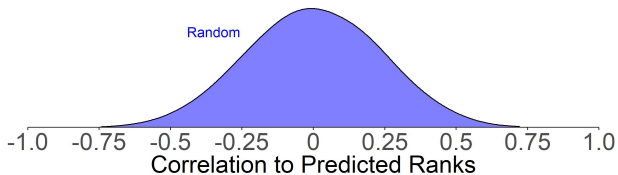
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Respondents ranging from ordinary citizens to those responsible for redistricting

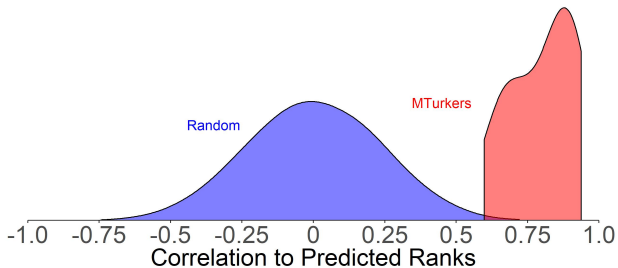
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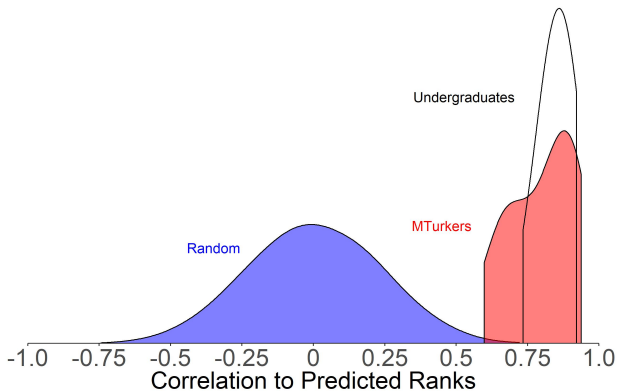
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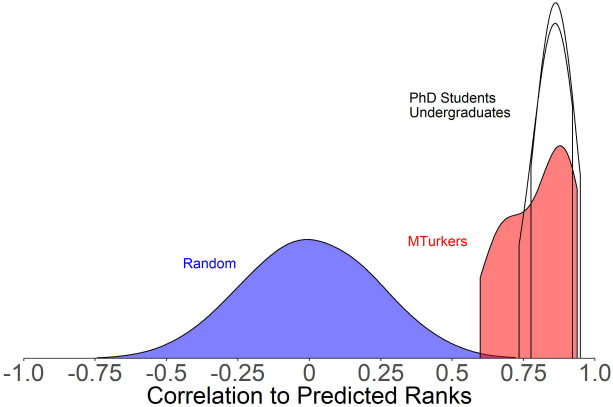
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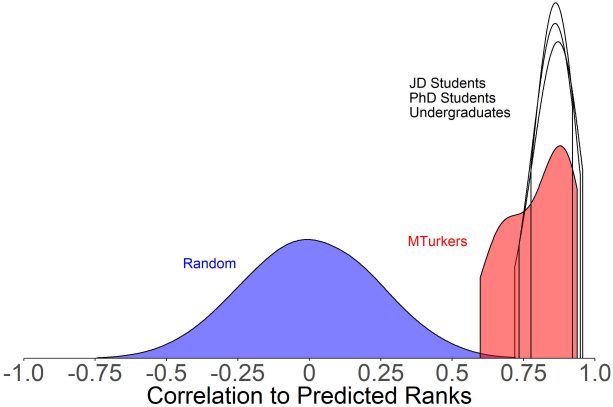
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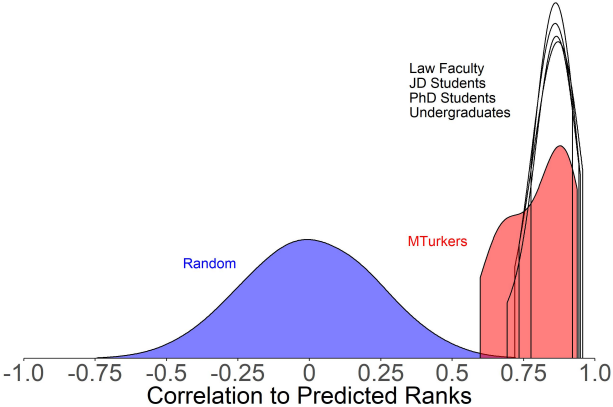
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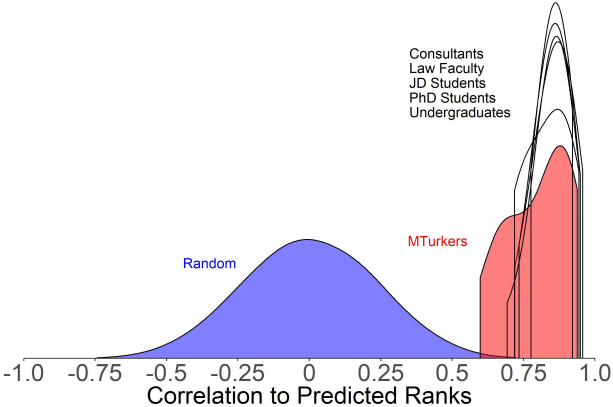
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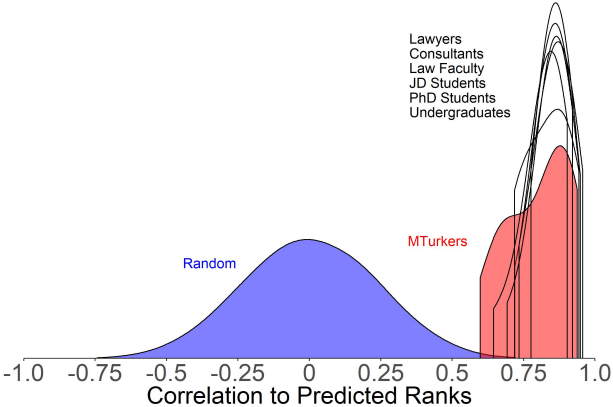
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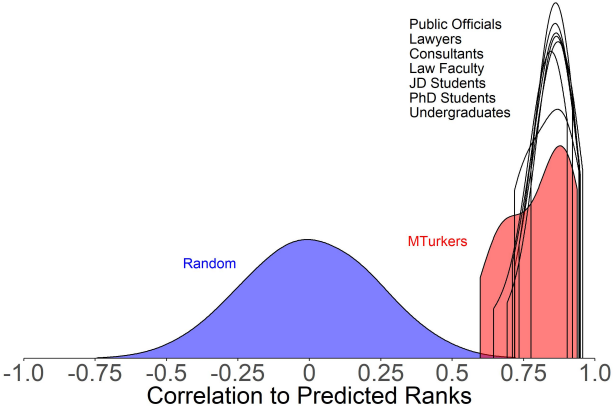
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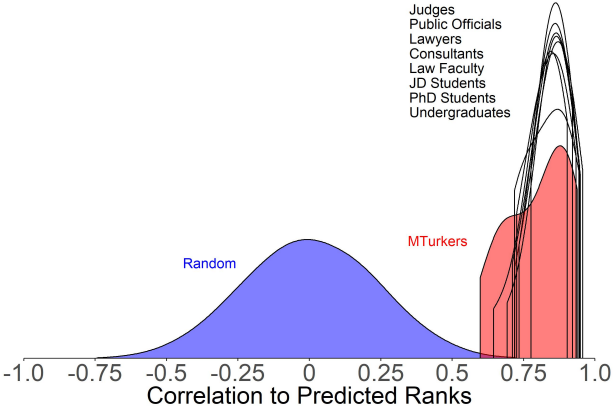
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What do you think?

What do you think?

Our measure:	COMPACT	noncompact	noncompact	COMPACT
Existing measure:	COMPACT	noncompact	COMPACT	noncompact

What do you think?

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COMPACT

noncompact

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COMPACT

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Reock



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Boyce-Clark



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Boyce-Clark



Length/Width



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Population

:

Sam

Adam

Jason

Rick

Zachariah

Sandra

Amanda

Ari

Hannah

Indraneel

\$48

Mean
income:

Quantity
of Interest

Theories of Inference: Statistics vs. CS

Population	Sample
:	X
Sam	✓
Adam	✓
Jason	✓
Rick	✓
Zachariah	✓
Sandra	✓
Amanda	✓
Ari	✓
Hannah	✓
Indraneel	✓
<hr/>	
\$48	

Mean
income:

Quantity
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Theories of Inference: Statistics vs. CS

Population	Sample	\$
:	X	?
Sam	✓	122
Adam	✓	76
Jason	✓	145
Rick	✓	96
Zachariah	✓	86
Sandra	✓	127
Amanda	✓	72
Ari	✓	132
Hannah	✓	95
Indraneel	✓	134

Mean
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Classical
Inference

\$108

Quantity
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Usually
no direct
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Theories of Inference: Statistics vs. CS

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Theories of Inference: Statistics vs. CS

Population	Sample	\$	+Privacy
:	X	?	
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Jason	✓	145	
Rick	✓	96	
Zachariah	✓	86	
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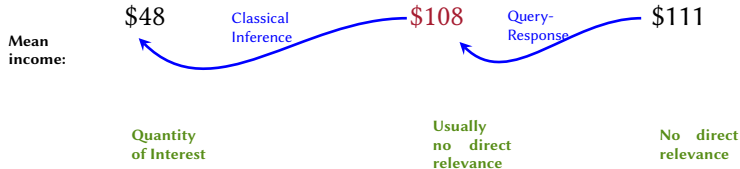
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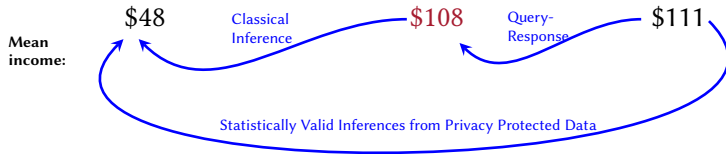
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Jason	✓	145		75
Rick	✓	96		113
Zachariah	✓	86		125
Sandra	✓	127		97
Amanda	✓	72		101
Ari	✓	132		128
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Hannah	✓	95		83
Indraneel	✓	134		201



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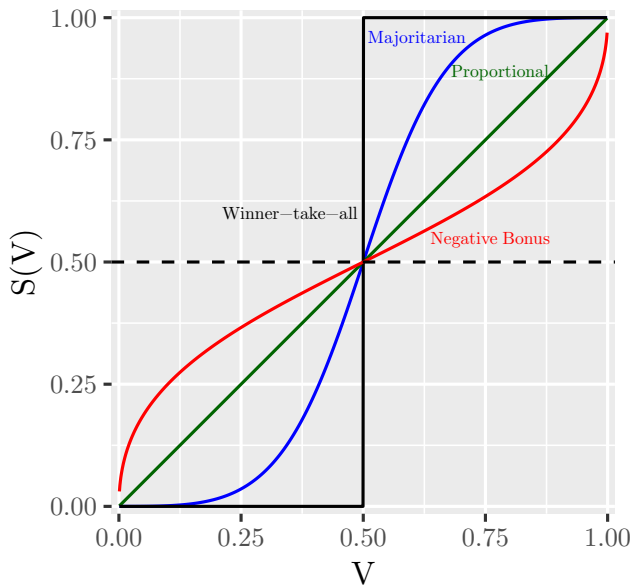
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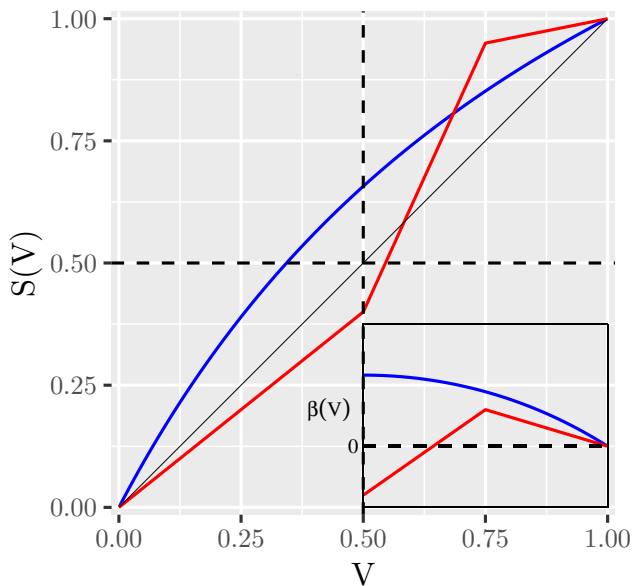
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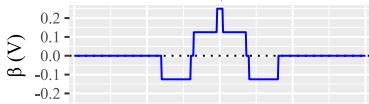
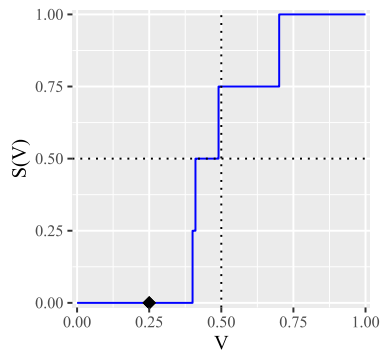
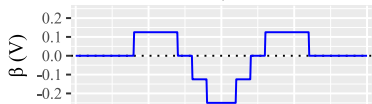
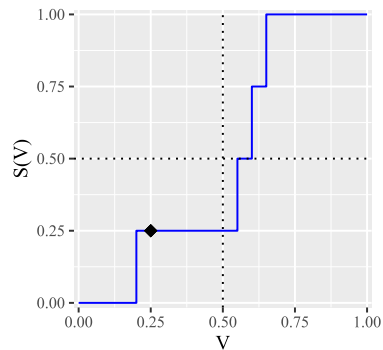
Symmetric (“Fair”) Seats-Votes Curves



Asymmetric (“Biased”) Seats-Votes Curves



Limited QOIs Revealed by Full Seats-Votes Curves



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For more information

GaryKing.org