Demographic Forecasting

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joint work with

Federico Girosi, with contributions from Kevin Quinn and Greg Wawro

(talk at Graduate Methods and Models Seminar, IQSS, Harvard University, 12/5/08)

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- Mortality forecasts, which are studied in:
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- forecasts and farcasts based on much more information

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  - Resolves analogous issues in predicting mortality by age, sex, and cause

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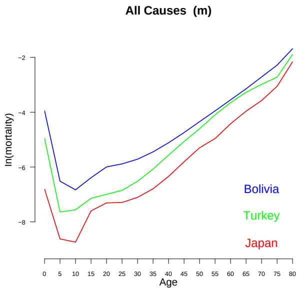
Existing Method 1: Parameterize the Age Profile



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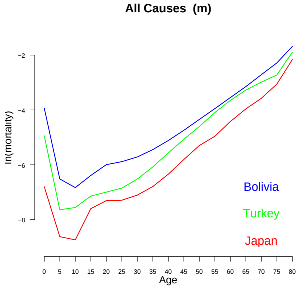

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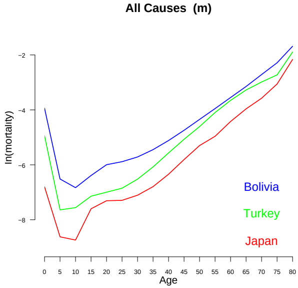

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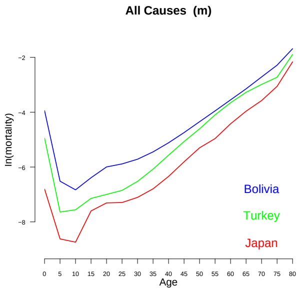

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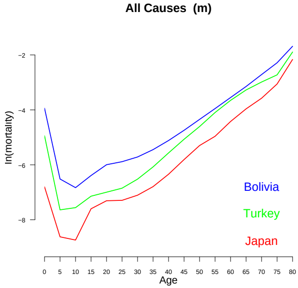

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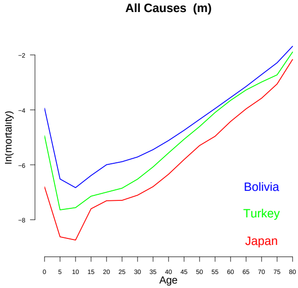

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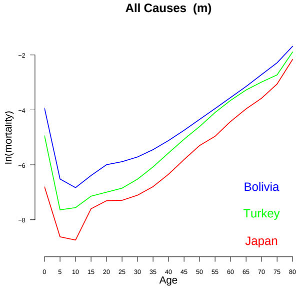

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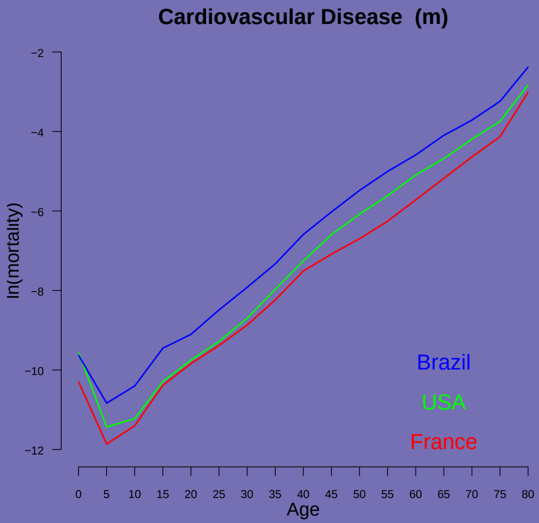

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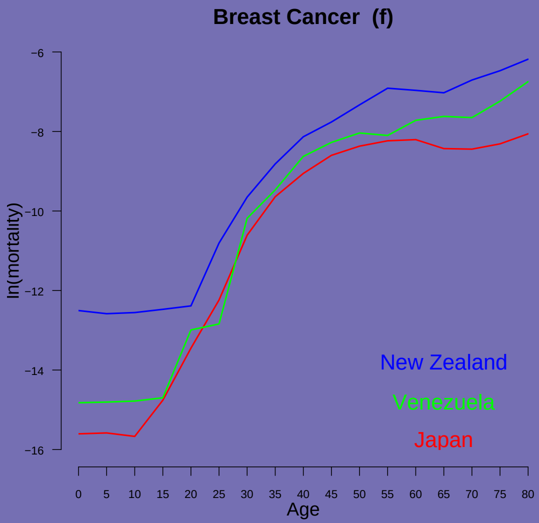

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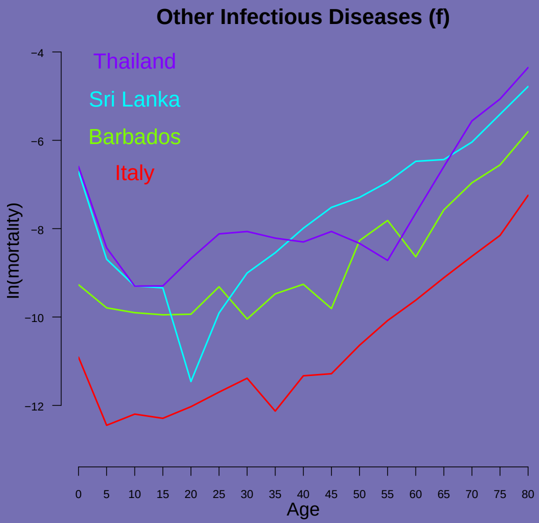

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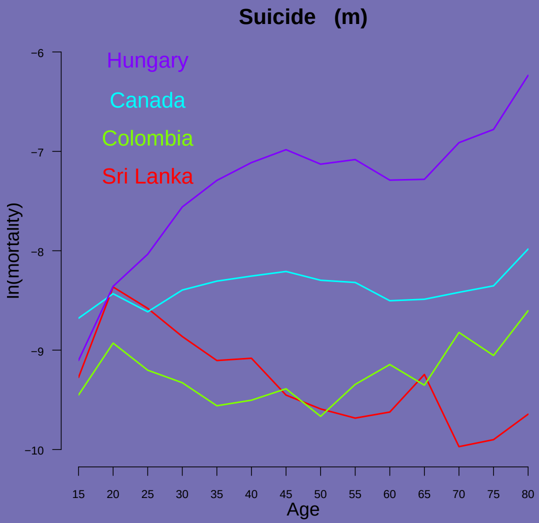

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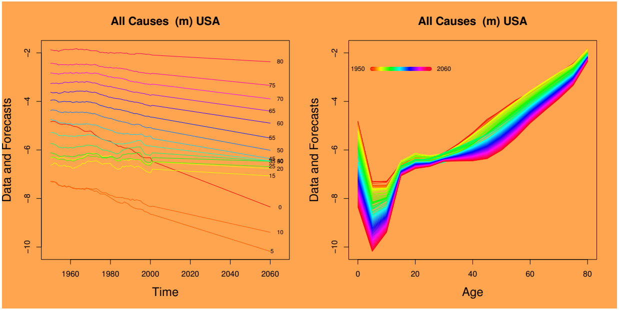

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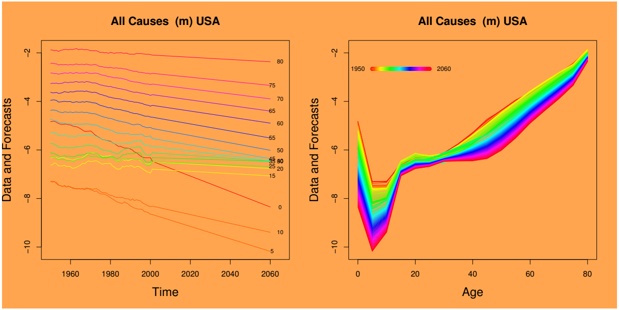

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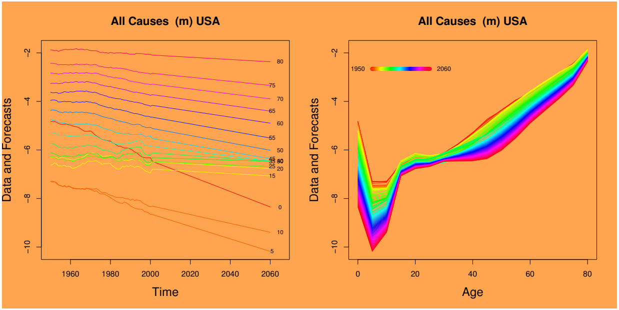

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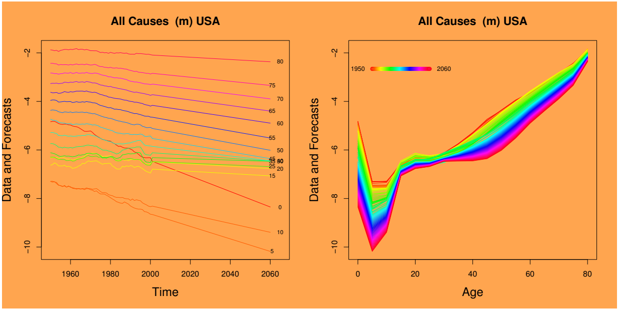

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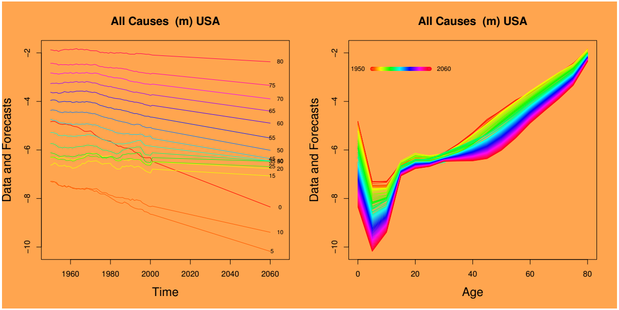

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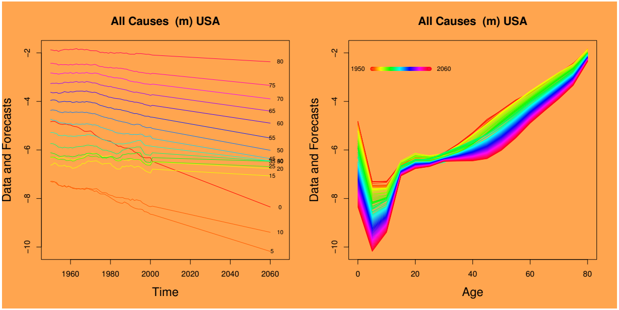

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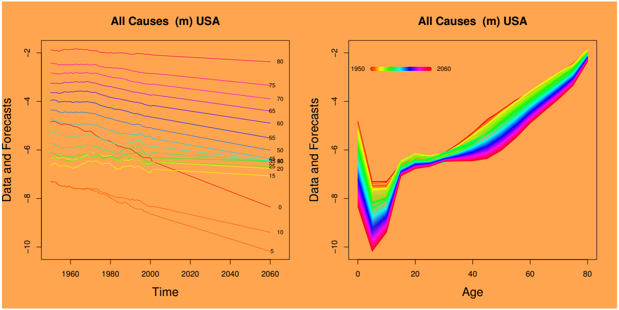

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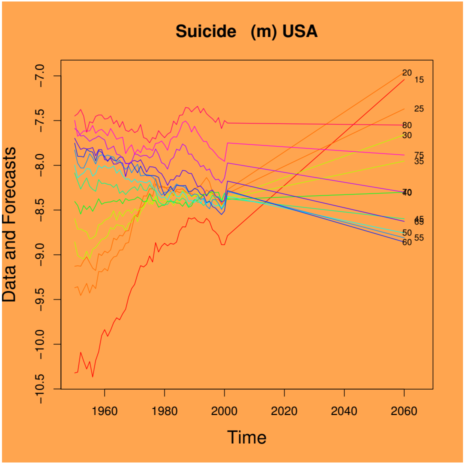

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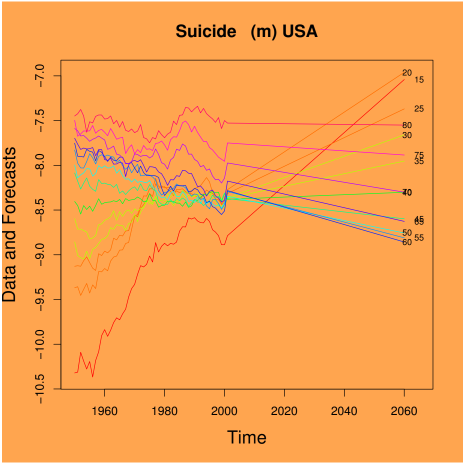

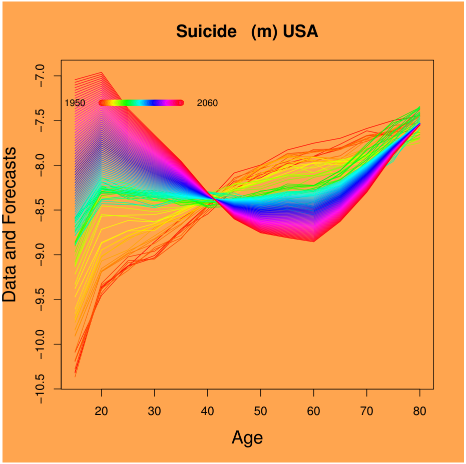

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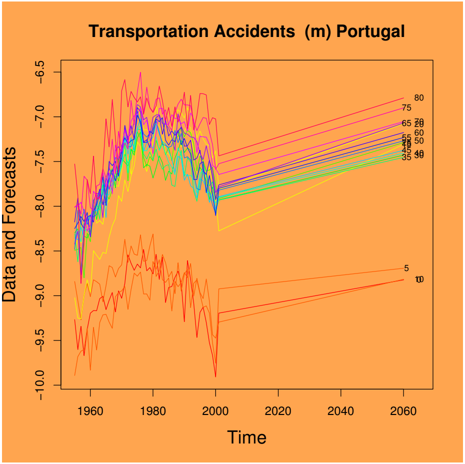

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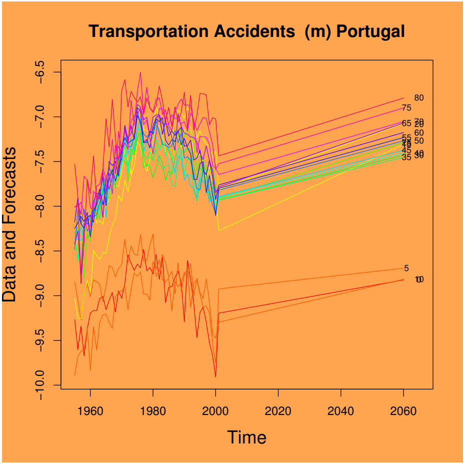

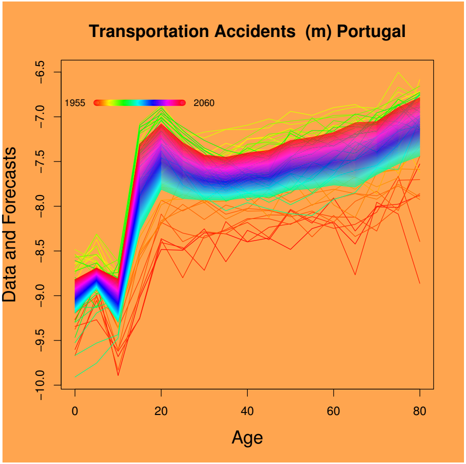

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- (It always seems ok to pool over variables outside your own field.)

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$$\mathcal{P}(m \mid \boldsymbol{\beta}_i, \sigma_i) = \prod_t \mathcal{N}\left(m_{it} \mid \mathbf{Z}_{it}\boldsymbol{\beta}_i, \sigma_i^2\right)$$

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- The easy part: *easy-to-use software* to implement everything we discuss today.

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Natural choice for the prior:

$$\mathcal{P}(\boldsymbol{\beta} \mid \Phi) \propto \exp\left(-\frac{1}{2}\sum_{ij} s_{ij}\|\boldsymbol{\beta}_i - \boldsymbol{\beta}_j\|^2_\Phi\right)$$

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3. In the subspace, we can invert $\mu = \mathbf{Z}\boldsymbol{\beta}$ as $\boldsymbol{\beta} = (\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\mu$, giving:

$$\mathcal{P}(\boldsymbol{\beta} \mid \theta) \propto \exp\left(-\frac{1}{2}H[\mu,\theta]\right) = \exp\left(-\frac{1}{2}H[\mathbf{Z}\boldsymbol{\beta},\theta]\right)$$

   the same prior on μ, expressed as a function of $\boldsymbol{\beta}$ (with constant Jacobian).

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- We start with one-dimensional $P(\mu_{cat})$, and treat it as the multidimensional $P(\boldsymbol{\beta}_{ca})$, constant in all directions but $Z_{cat}\boldsymbol{\beta}_{ca}$.

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- The normalization matrix Φ is unnecessary (normalization is performed by $\mathbf{Z}$, which is known)

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- where W^n is a matrix uniquely determined by n and θ

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where we have defined:

$$\mathbf{C}_{aa'} \equiv \frac{1}{T}\mathbf{Z}'_a\mathbf{Z}_{a'} \qquad \mathbf{Z}_a \text{ is a } T \times d_a \text{ data matrix for age group } a$$

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- θ: the "strength" of the prior

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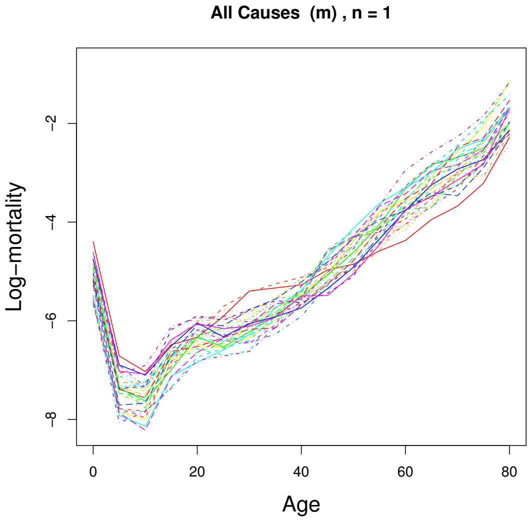
$$\mathcal{P}(\boldsymbol{\beta} \mid \theta) \propto \exp\left(-\theta \sum_{aa'} W^n_{aa'} \boldsymbol{\beta}'_a \mathbf{C}_{aa'} \boldsymbol{\beta}_{a'}\right)$$

- The prior is normal (and improper)
- n: defines the prior through the "interaction" matrix W^n.
- θ: the "strength" of the prior
- Different age groups can have different covariates: the matrices $\mathbf{C}_{aa'}$ are rectangular ($d_a \times d_{a'}$).

Samples From Age Prior



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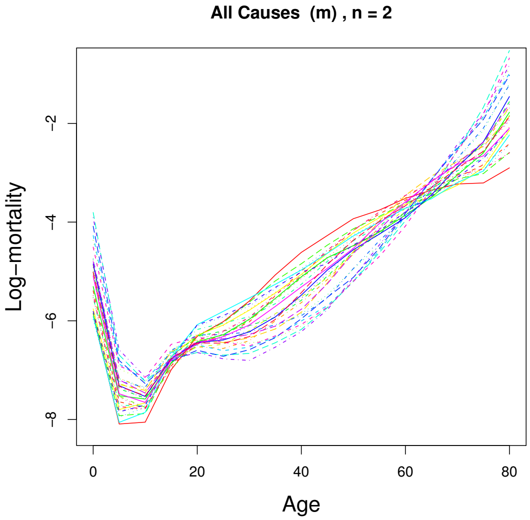

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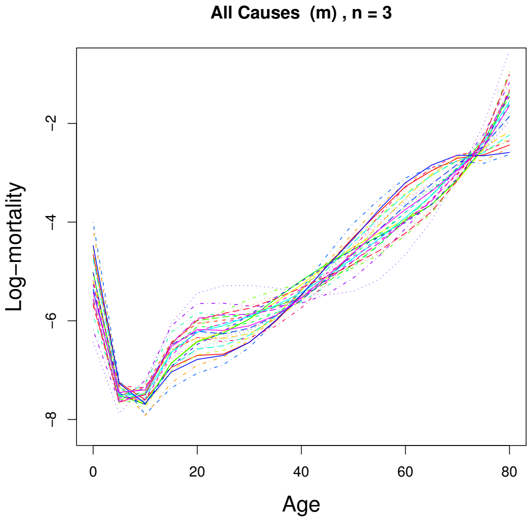

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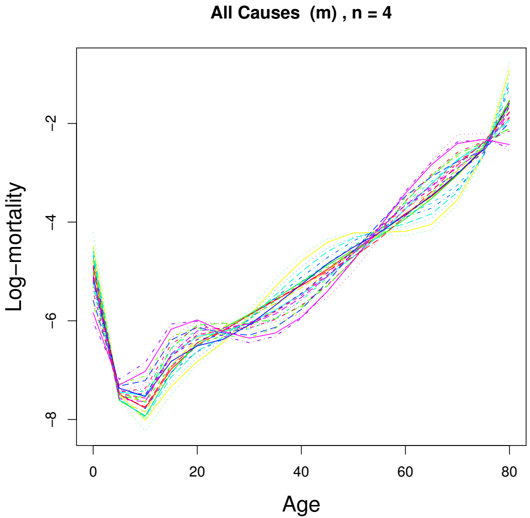

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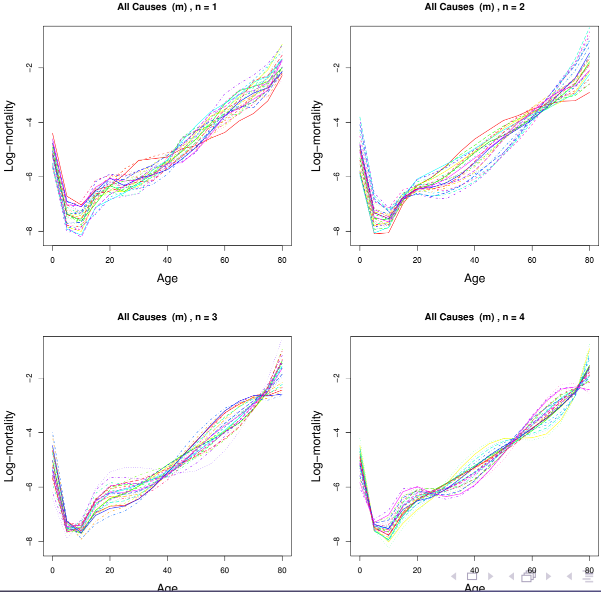

Formalizing (Prior) Indifference

equal color = equal probability

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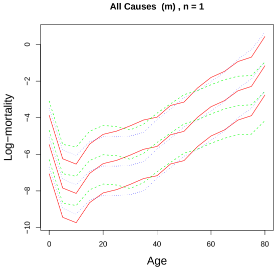
Level indifference



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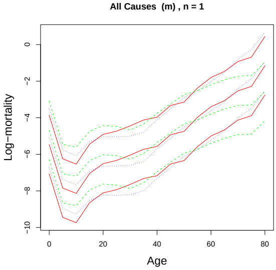
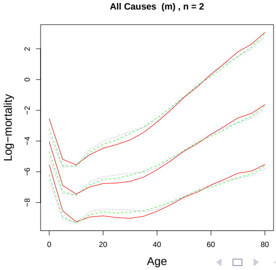


Level and slope indifference



Smoothness Parameter

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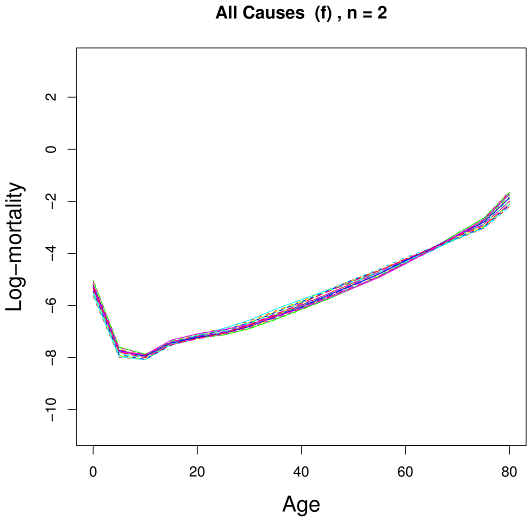

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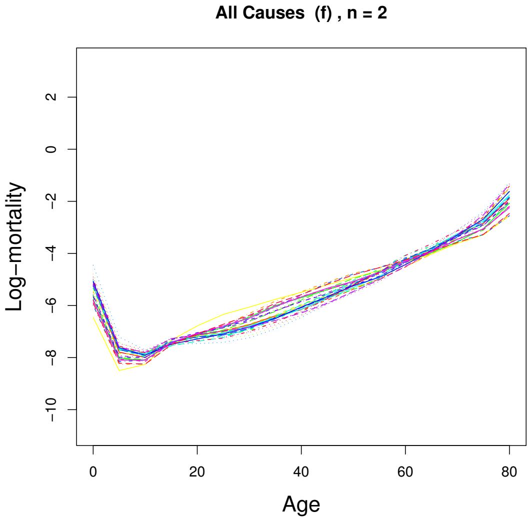

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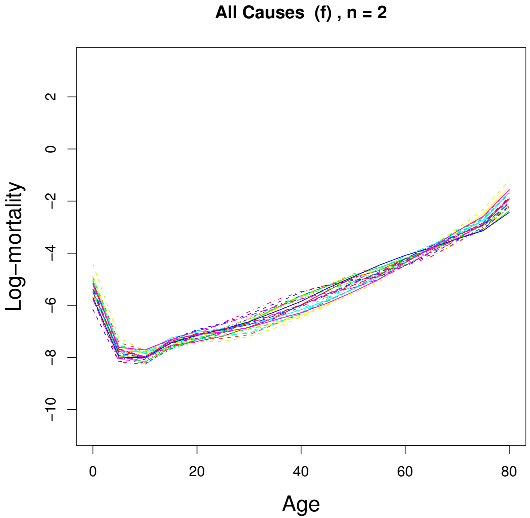

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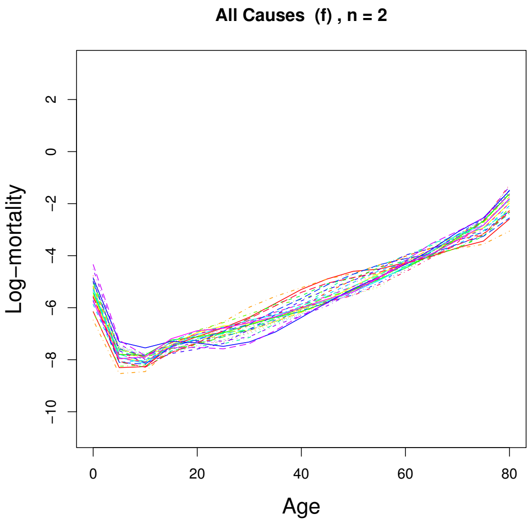

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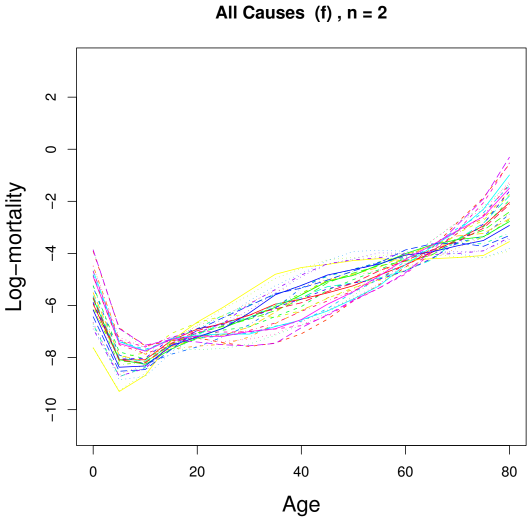

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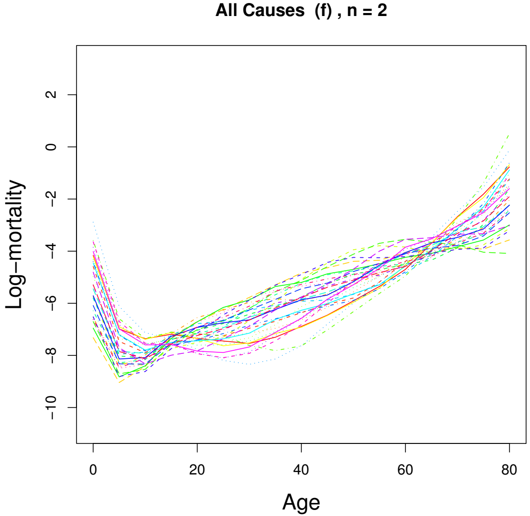

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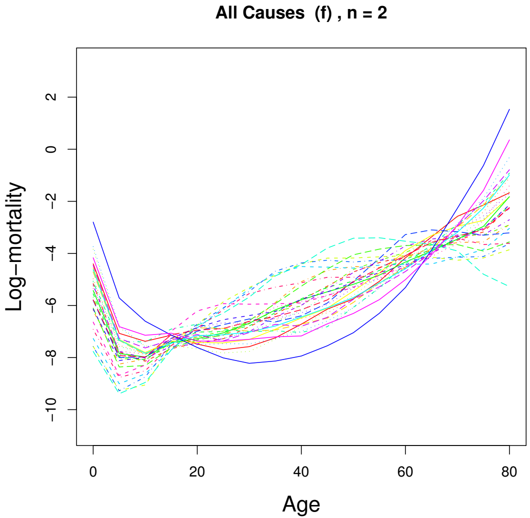

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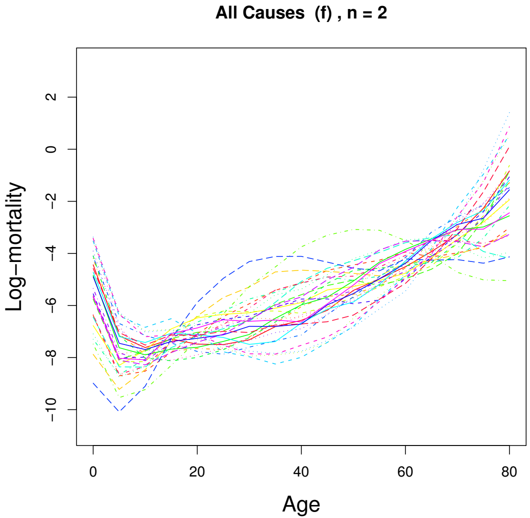

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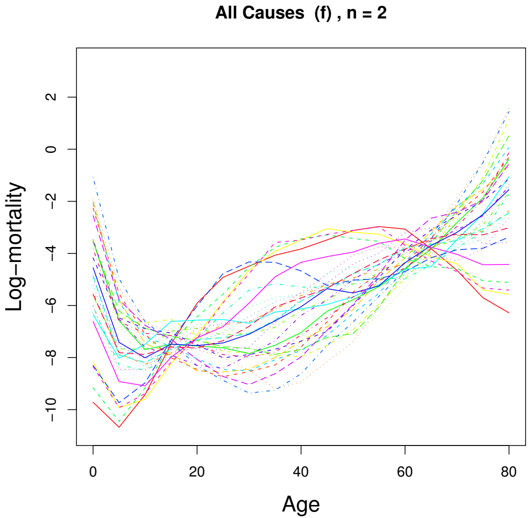

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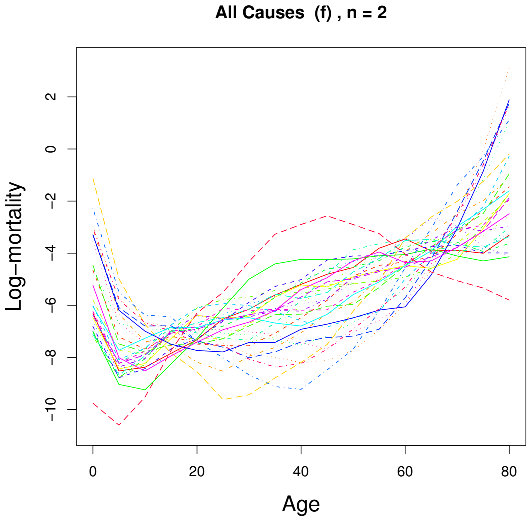

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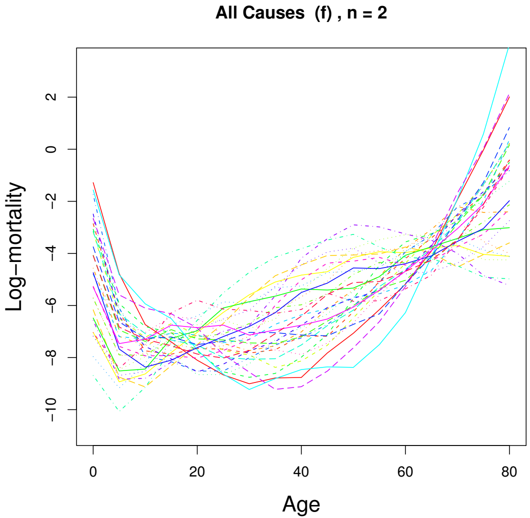

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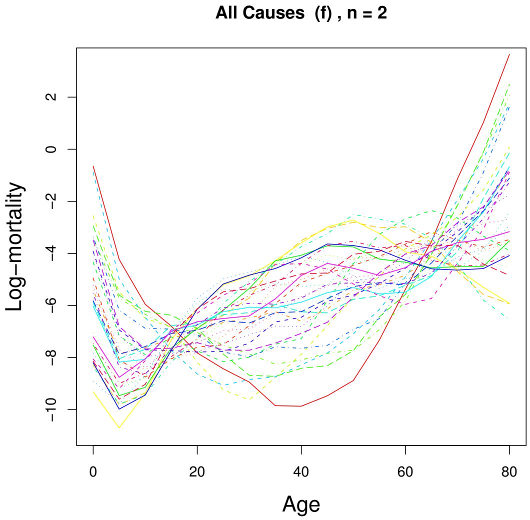

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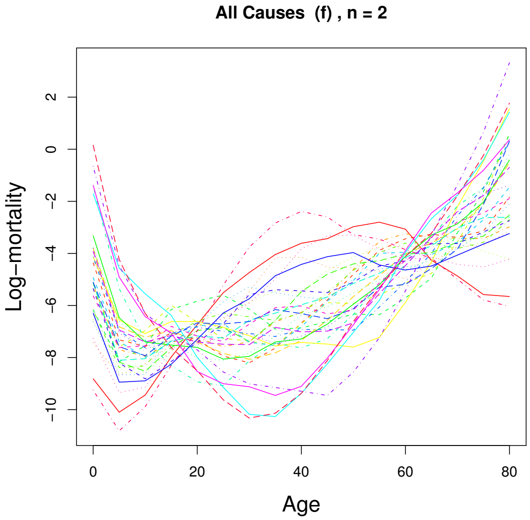

Generalizations

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- We can smooth interactions:
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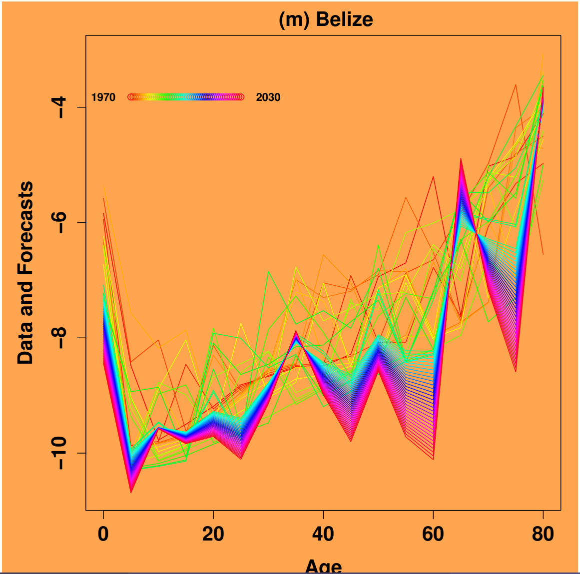
  - Smoothing trends over age groups as they vary across countries, etc.
- The mathematical form for *all* these (separately or together) turns out to be the same:

$$\mathcal{P}(\beta \mid \theta) \propto \exp\left(-\frac{\theta}{2}\sum_{ij} W_{ij}\beta_i' \mathbf{C}_{ij}\beta_j\right), \qquad \mathbf{C}_{aa'} \equiv \frac{1}{T}\mathbf{Z}_a\mathbf{Z}_{a'}$$

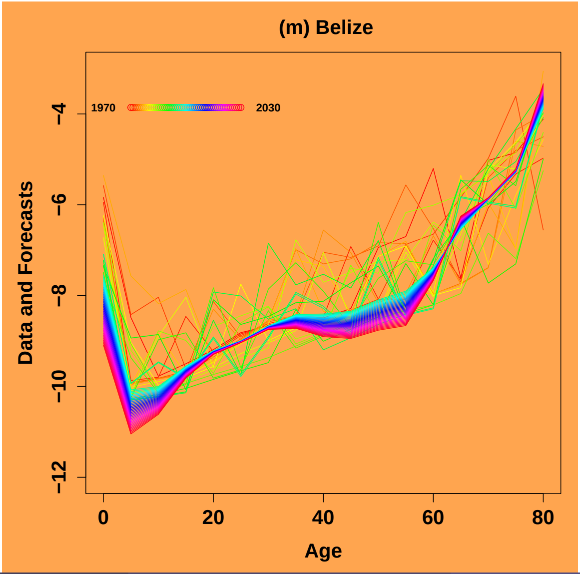
Mortality from Respiratory Infections, Males

Least Squares



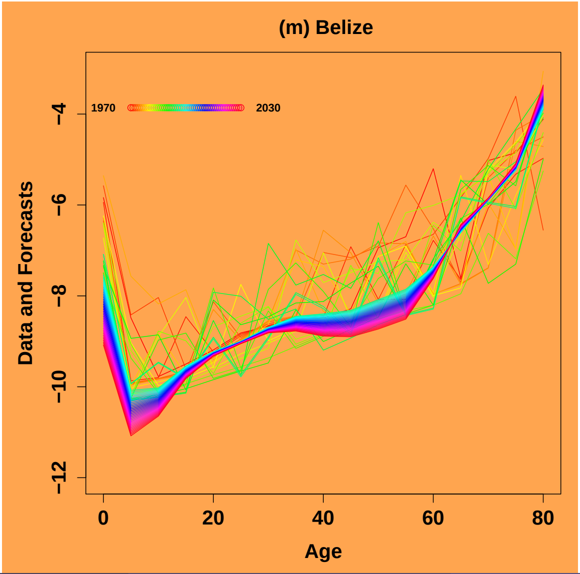
Mortality from Respiratory Infections, males, $\sigma = 2.00$

Smoothing over Age Groups



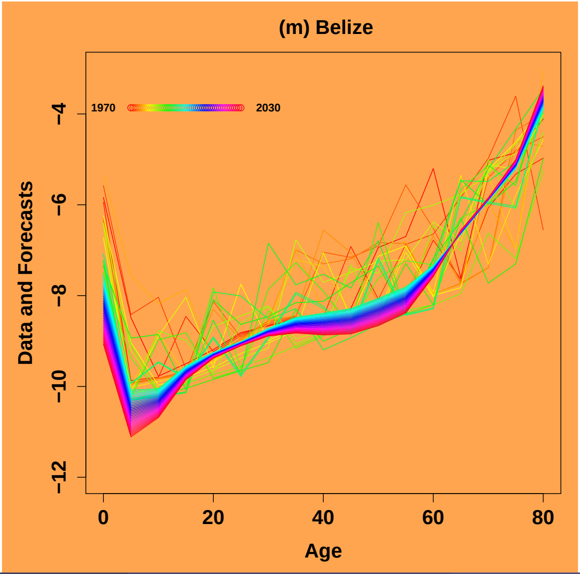
Mortality from Respiratory Infections, males, $\sigma = 1.51$

Smoothing over Age Groups



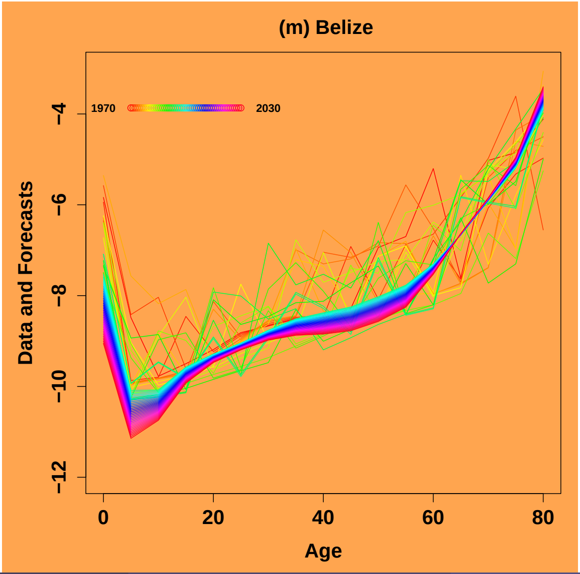
Mortality from Respiratory Infections, males, $\sigma = 1.15$

Smoothing over Age Groups



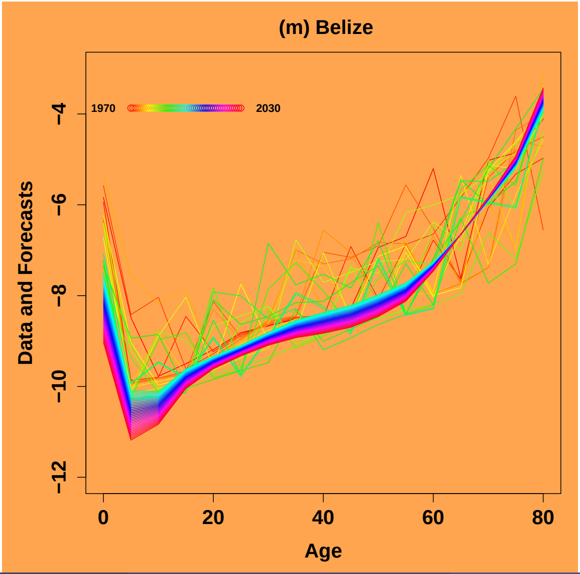
Mortality from Respiratory Infections, males, $\sigma = 0.87$

Smoothing over Age Groups



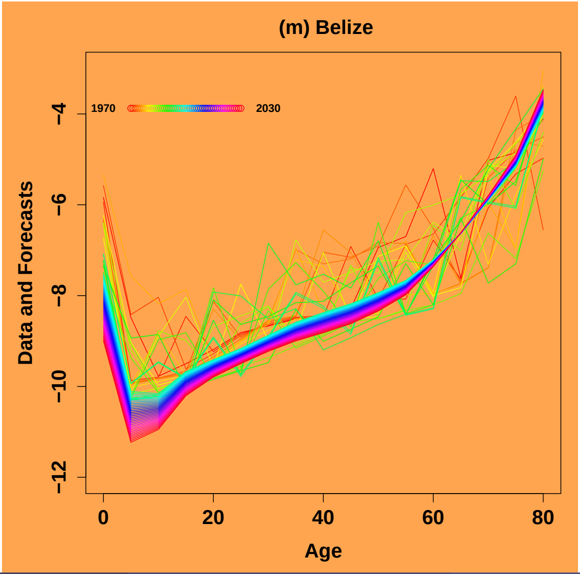
Mortality from Respiratory Infections, males, $\sigma = 0.66$

Smoothing over Age Groups



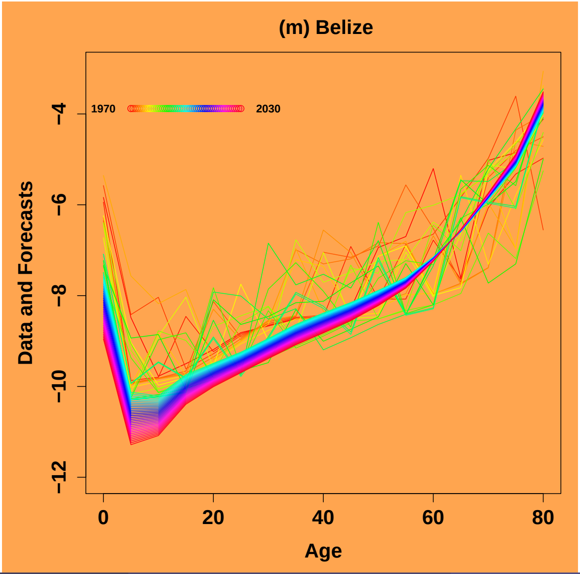
Mortality from Respiratory Infections, males, $\sigma = 0.50$

Smoothing over Age Groups



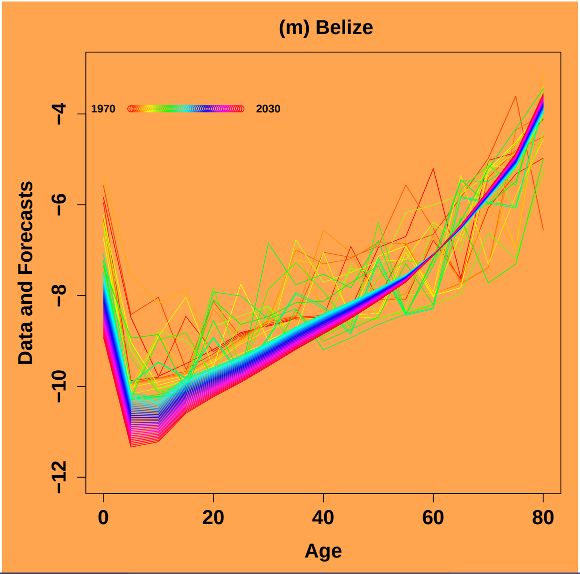
Mortality from Respiratory Infections, males, $\sigma = 0.38$

Smoothing over Age Groups



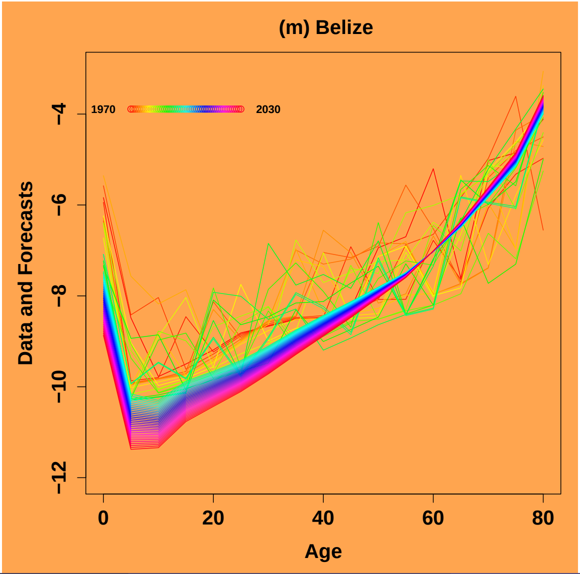
Mortality from Respiratory Infections, males, $\sigma = 0.28$

Smoothing over Age Groups



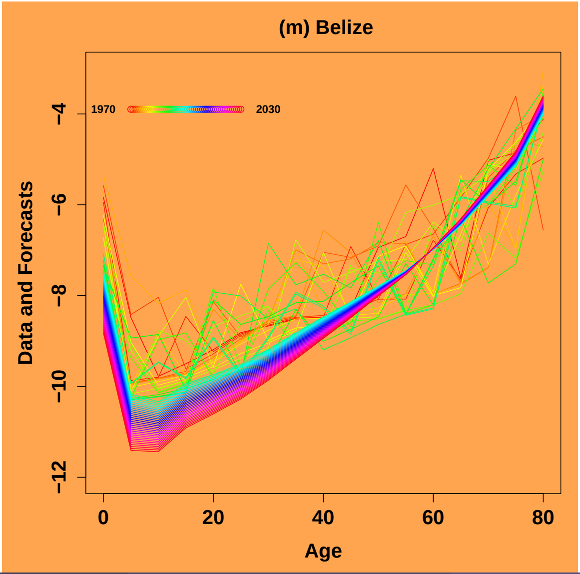
Mortality from Respiratory Infections, males, $\sigma = 0.21$

Smoothing over Age Groups



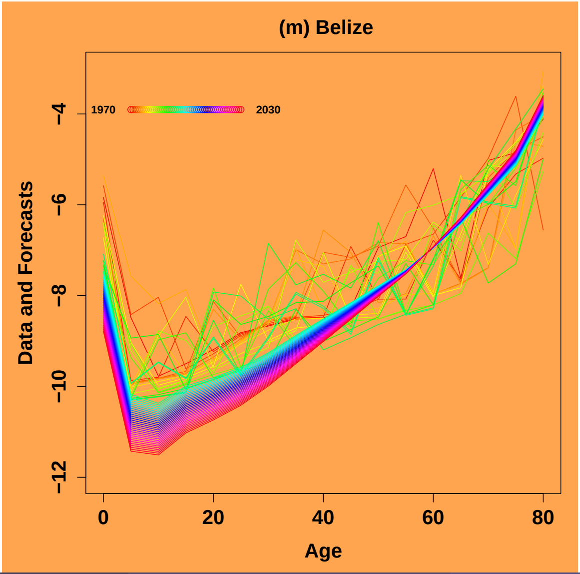
Mortality from Respiratory Infections, males, $\sigma = 0.16$

Smoothing over Age Groups



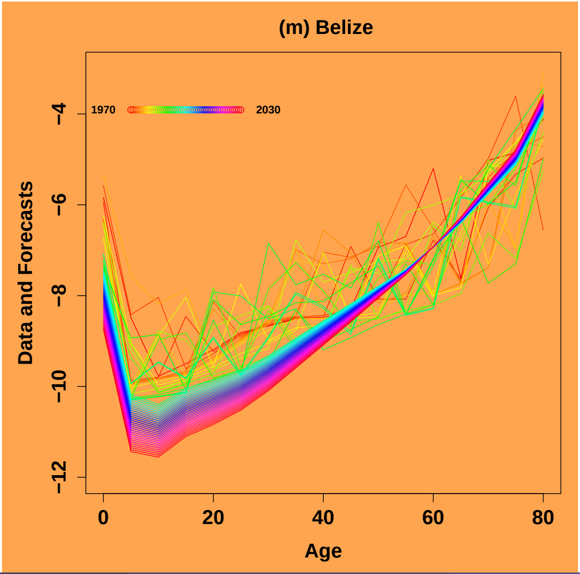
Mortality from Respiratory Infections, males, $\sigma = 0.12$

Smoothing over Age Groups



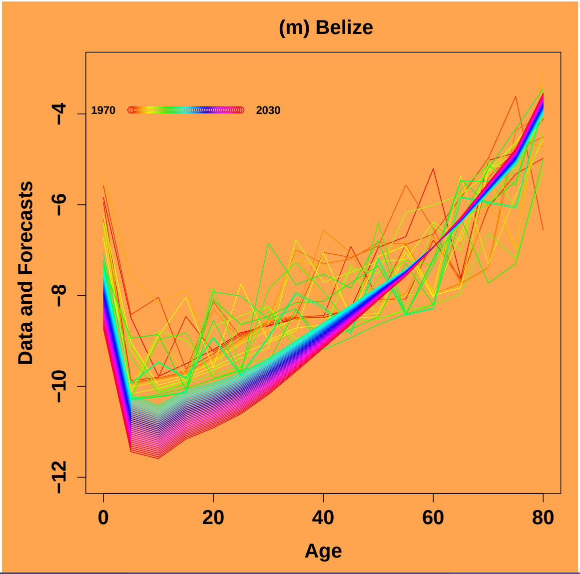
Mortality from Respiratory Infections, males, $\sigma = 0.09$

Smoothing over Age Groups



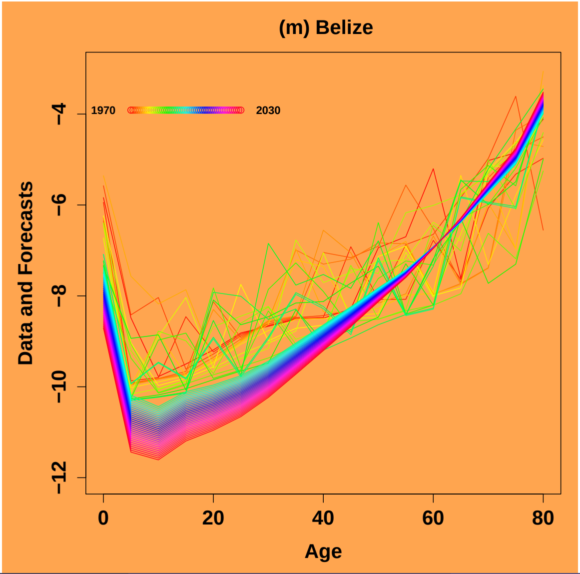
Mortality from Respiratory Infections, males, $\sigma = 0.07$

Smoothing over Age Groups



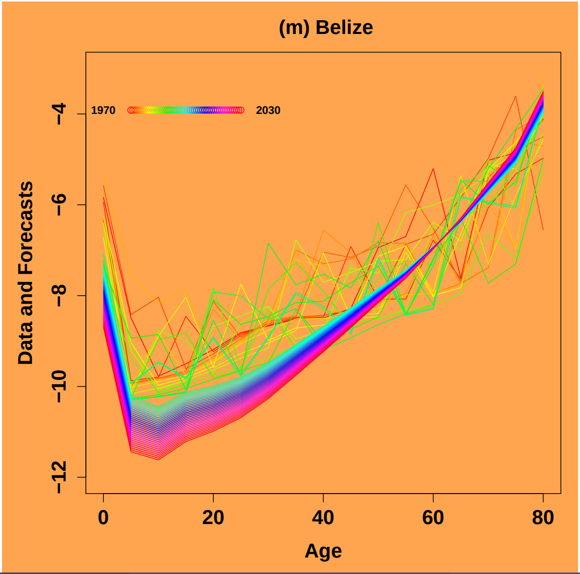
Mortality from Respiratory Infections, males, $\sigma = 0.05$

Smoothing over Age Groups



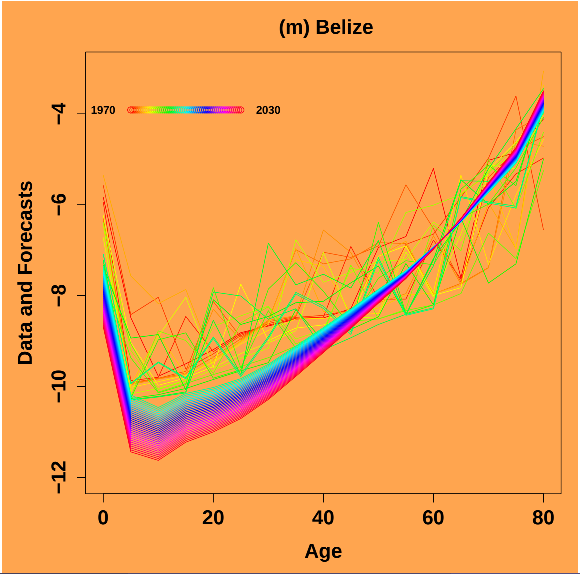
Mortality from Respiratory Infections, males, $\sigma = 0.04$

Smoothing over Age Groups



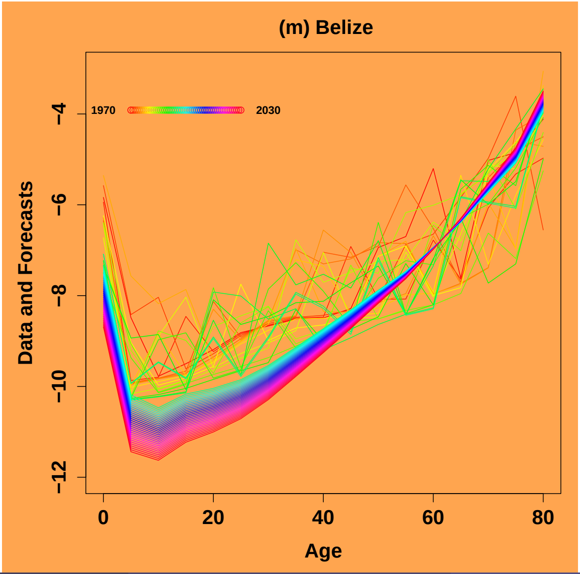
Mortality from Respiratory Infections, males, $\sigma = 0.03$

Smoothing over Age Groups



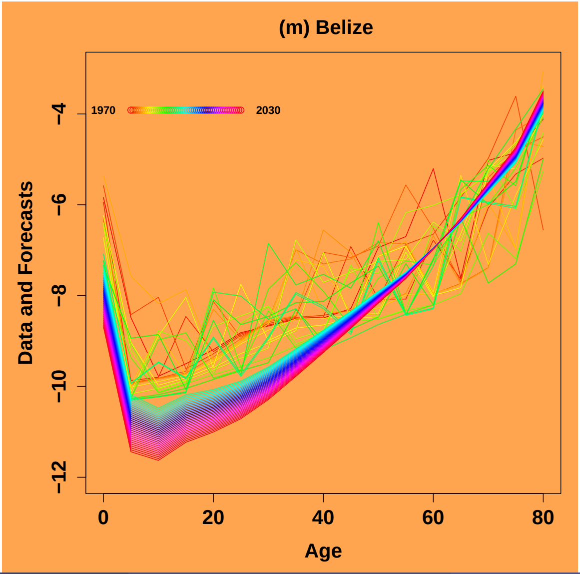
Mortality from Respiratory Infections, males, $\sigma = 0.02$

Smoothing over Age Groups



Mortality from Respiratory Infections, males, $\sigma = 0.01$

Smoothing over Age Groups



Mortality from Respiratory Infections, males

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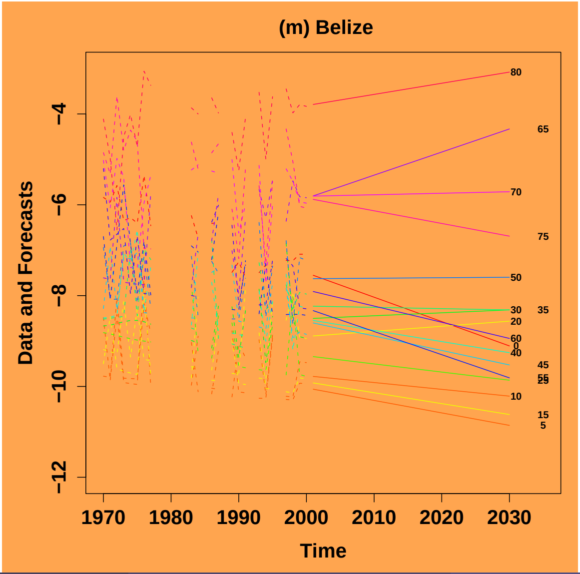
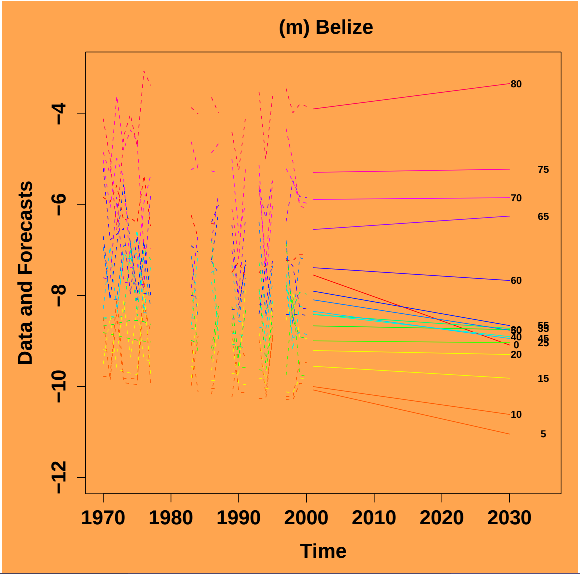


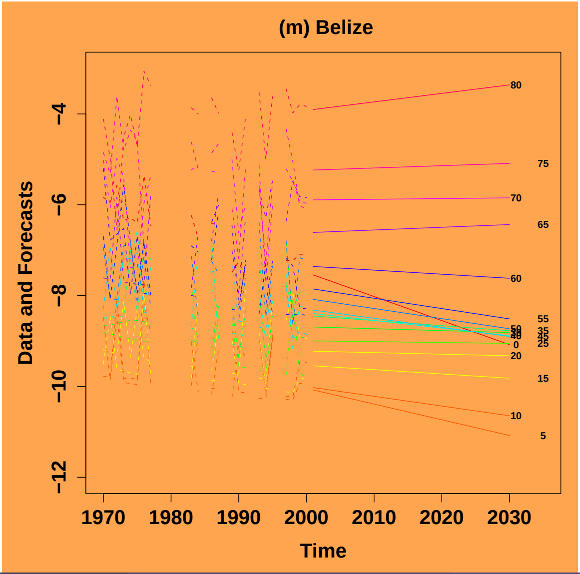
Mortality from Respiratory Infections, males, $\sigma = 2.00$

Smoothing over Age Groups



Mortality from Respiratory Infections, males, $\sigma = 1.51$

Smoothing over Age Groups



Mortality from Respiratory Infections, males, $\sigma = 1.15$

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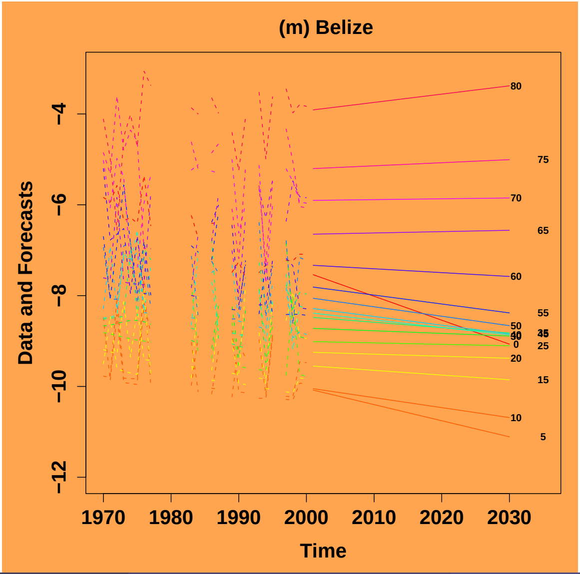

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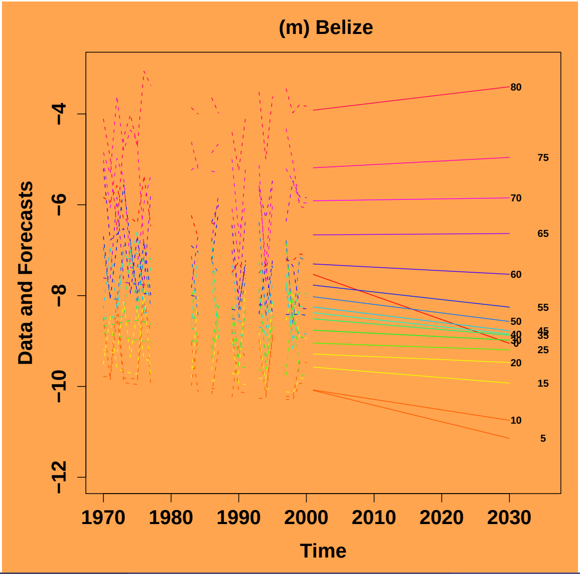

Mortality from Respiratory Infections, males, $\sigma = 0.66$

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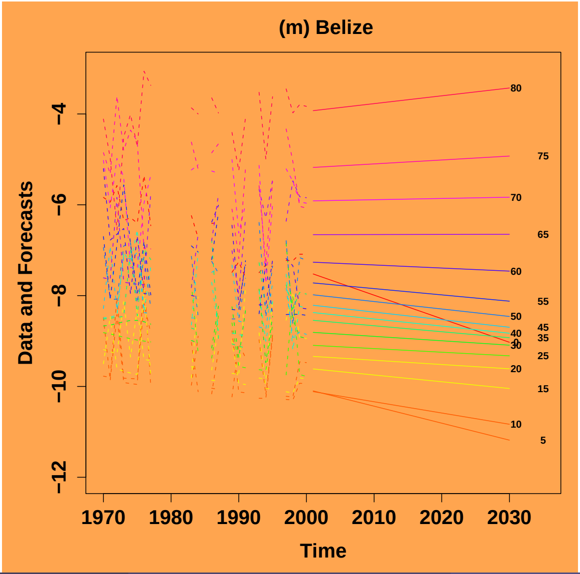

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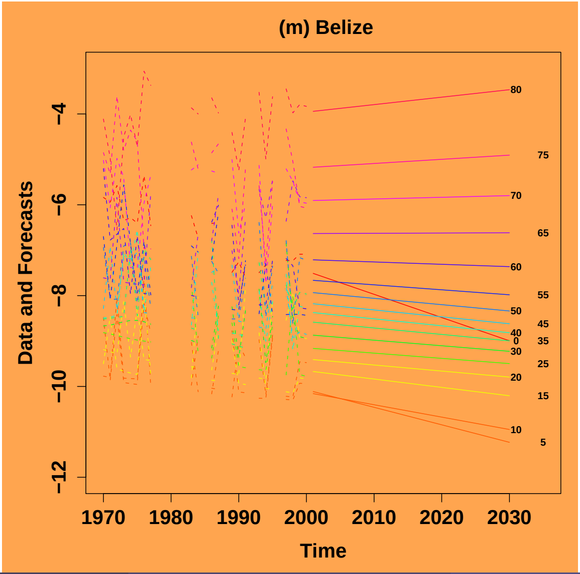

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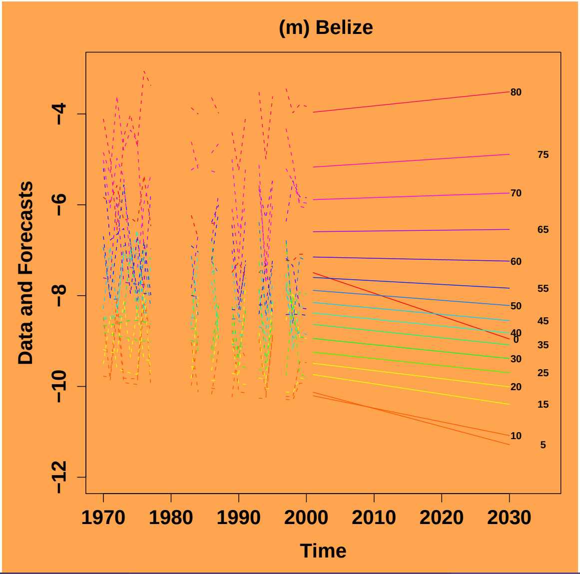

Mortality from Respiratory Infections, males, $\sigma = 0.28$

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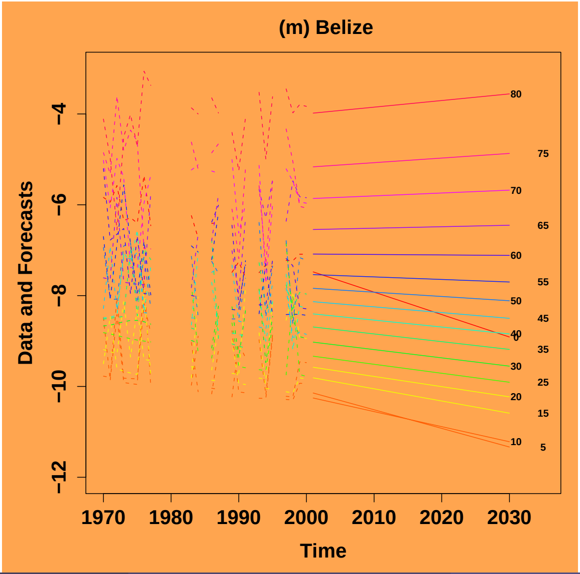

Mortality from Respiratory Infections, males, $\sigma = 0.21$

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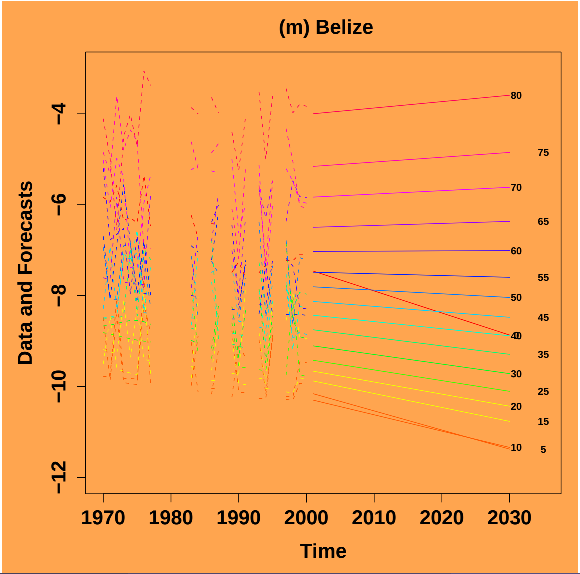

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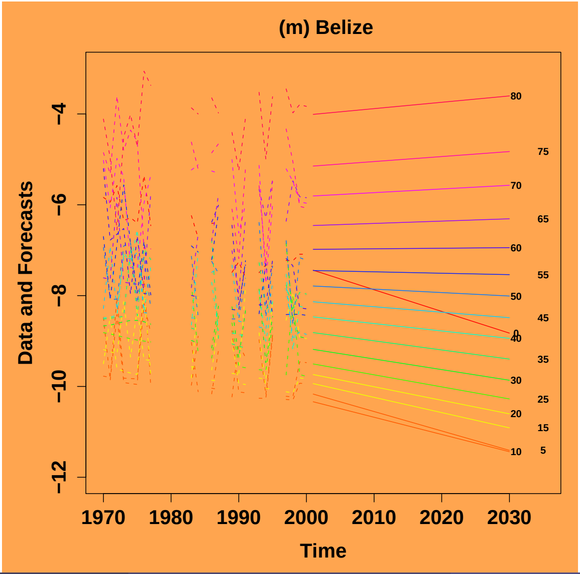

Mortality from Respiratory Infections, males, $\sigma = 0.12$

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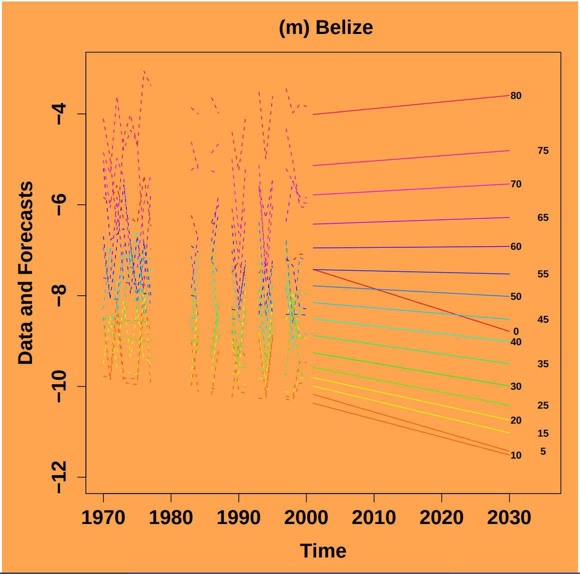

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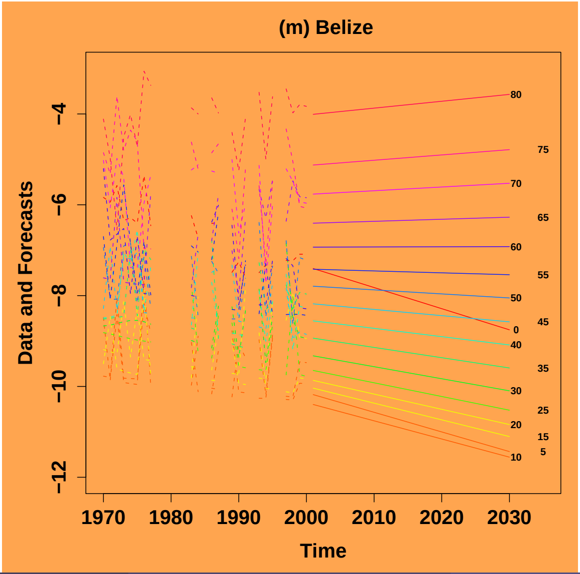

Mortality from Respiratory Infections, males, $\sigma = 0.07$

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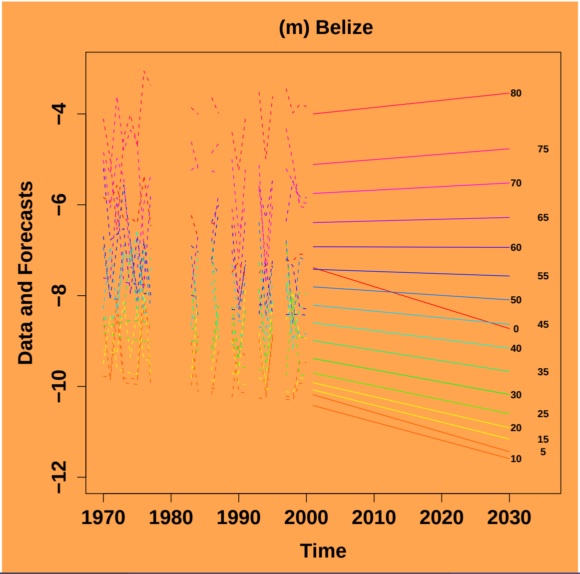
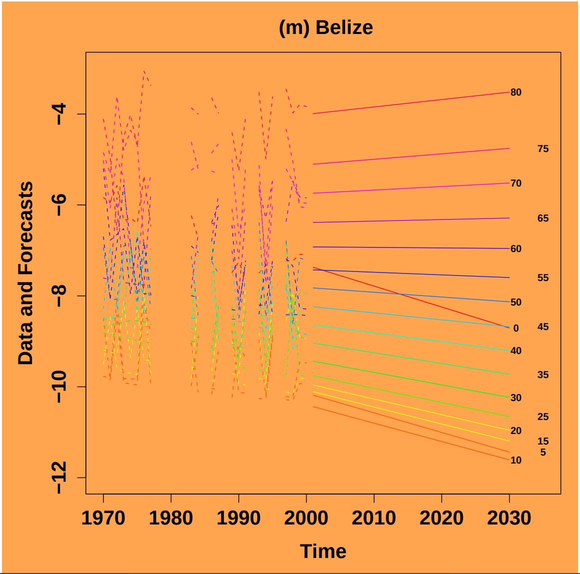


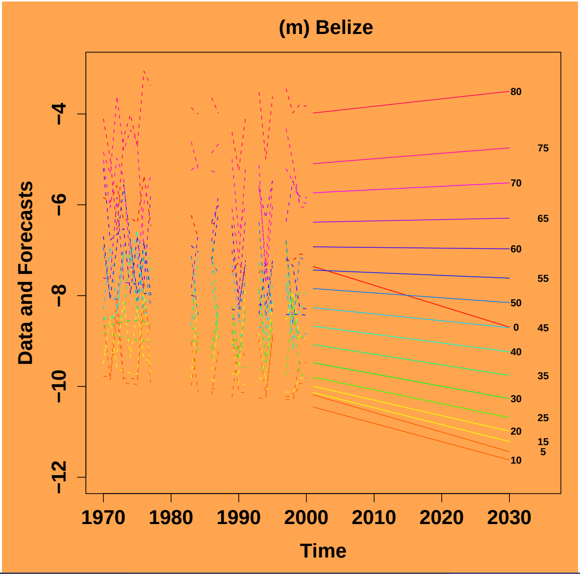
Mortality from Respiratory Infections, males, $\sigma = 0.05$

Smoothing over Age Groups



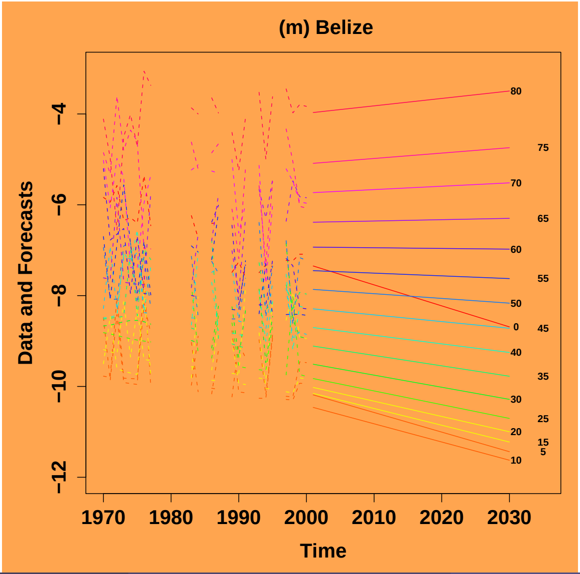
Mortality from Respiratory Infections, males, $\sigma = 0.04$

Smoothing over Age Groups



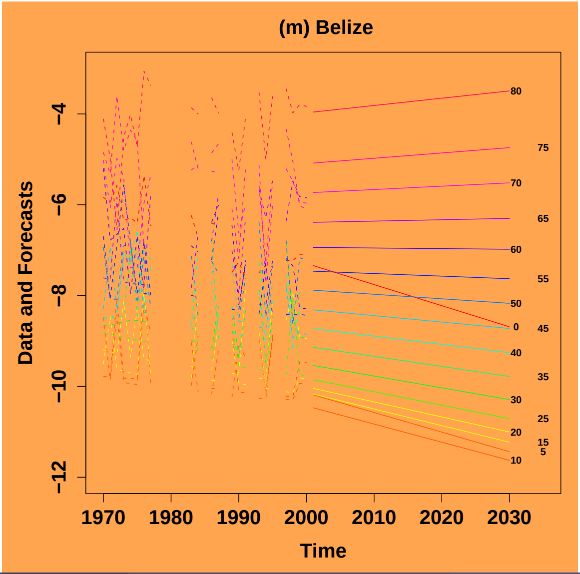
Mortality from Respiratory Infections, males, $\sigma = 0.03$

Smoothing over Age Groups



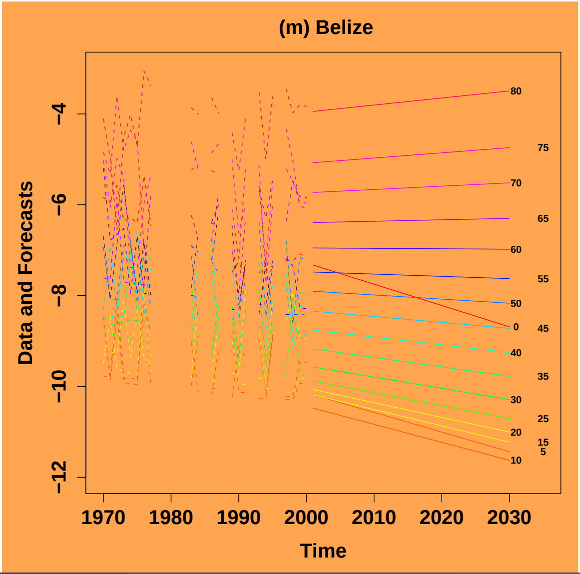
Mortality from Respiratory Infections, males, $\sigma = 0.02$

Smoothing over Age Groups



Mortality from Respiratory Infections, males, $\sigma = 0.01$

Smoothing over Age Groups



Smoothing Trends over Age Groups

Smoothing Trends over Age Groups

Log-mortality in Belize males from respiratory infections

Least Squares

Smoothing Trends over Age Groups

Log-mortality in Belize males from respiratory infections

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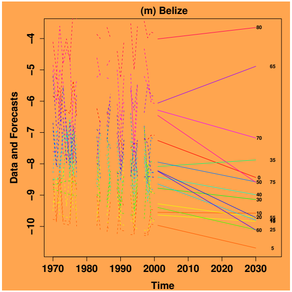

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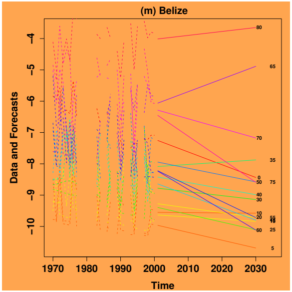

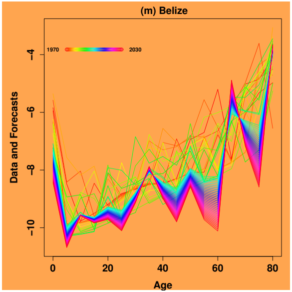

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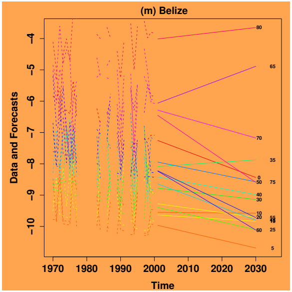

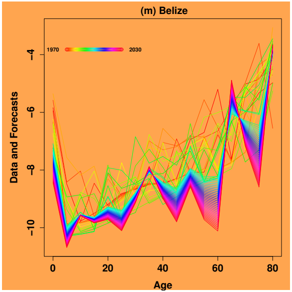

Smoothing
Age Groups

Smoothing Trends over Age Groups

Log-mortality in Belize males from respiratory infections

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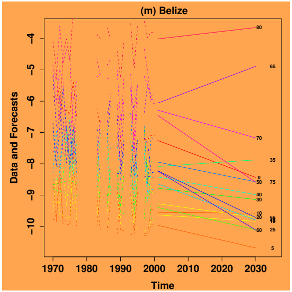

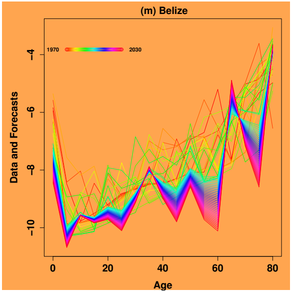

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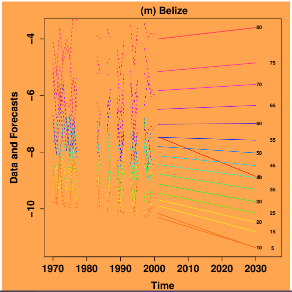

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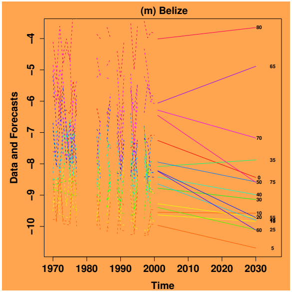

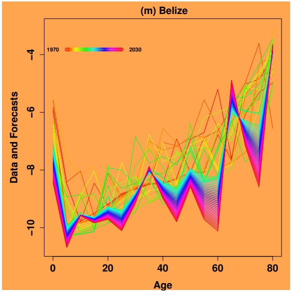

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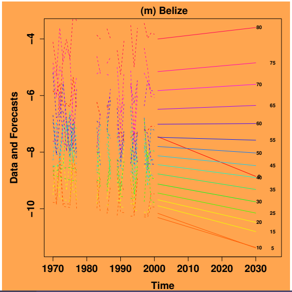
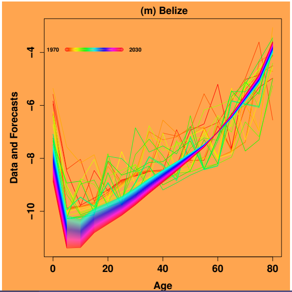




Smoothing Trends over Age Groups and Time

Smoothing Trends over Age Groups and Time

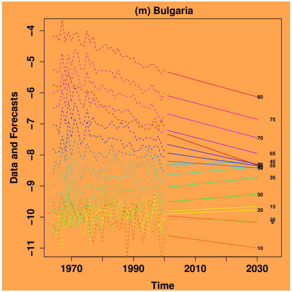
Log-Mortality in Bulgarian males from respiratory infections

Least Squares

Smoothing Trends over Age Groups and Time

Log-Mortality in Bulgarian males from respiratory infections

Least Squares



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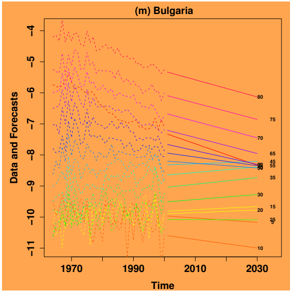
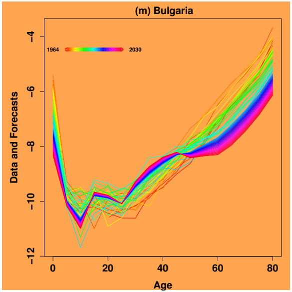
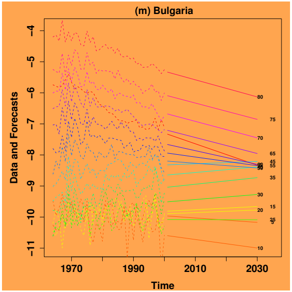


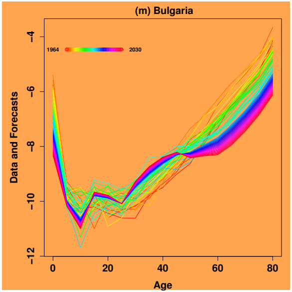


Smoothing Trends over Age Groups and Time

Log-Mortality in Bulgarian males from respiratory infections

Least Squares



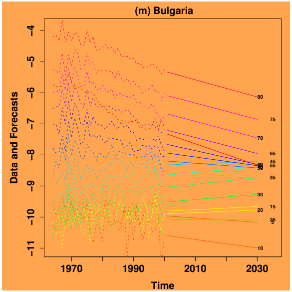


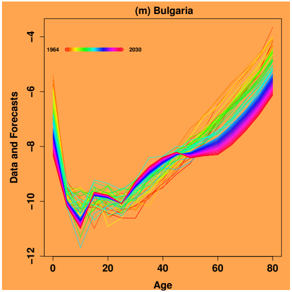
Smoothing
Age and Time

Smoothing Trends over Age Groups and Time

Log-Mortality in Bulgarian males from respiratory infections

Least Squares





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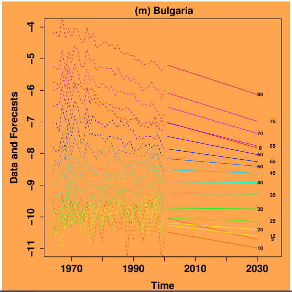

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Log-Mortality in Bulgarian males from respiratory infections

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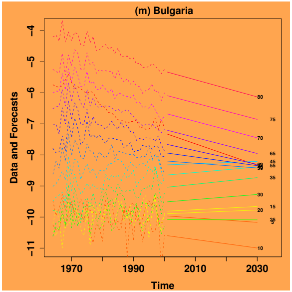

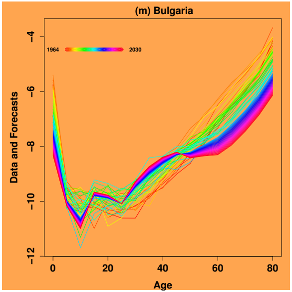

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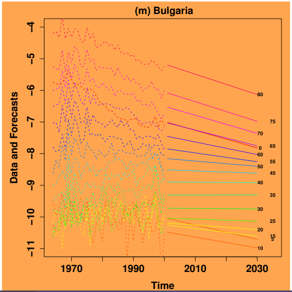
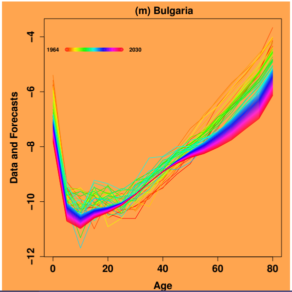
Age and Time





Using Covariates (GDP, tobacco, trend, log trend)

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Lung cancer in Korean Males

Using Covariates (GDP, tobacco, trend, log trend)

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Least Squares

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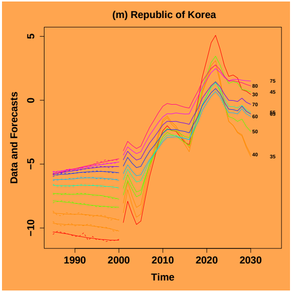

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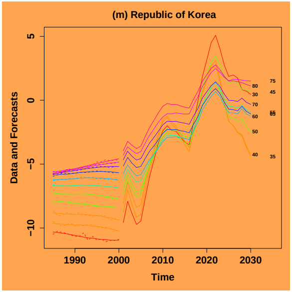

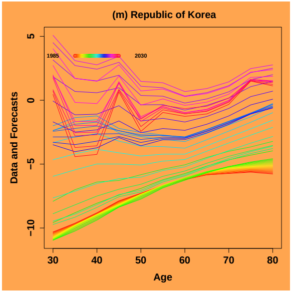

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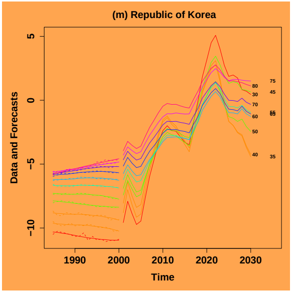
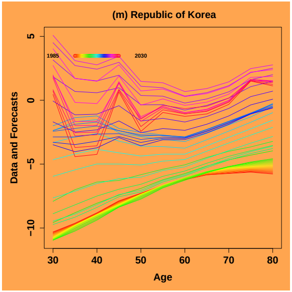
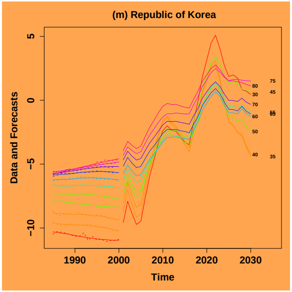
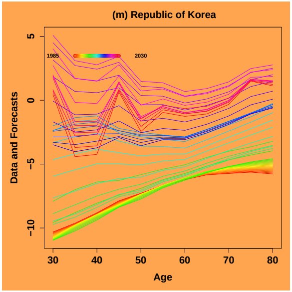




Smooth over age, time, age/time

Least Squares





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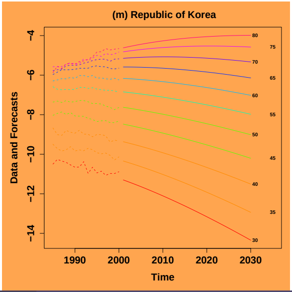
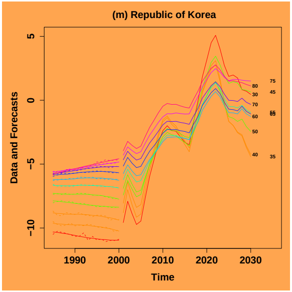
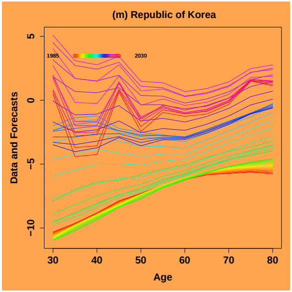


Using Covariates (GDP, tobacco, trend, log trend)

Lung cancer in Korean Males

Least Squares





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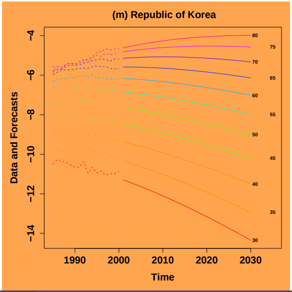
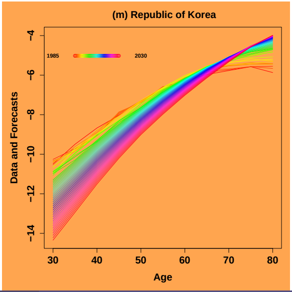




Using Covariates (GDP, tobacco, trend, log trend)

Using Covariates (GDP, tobacco, trend, log trend)

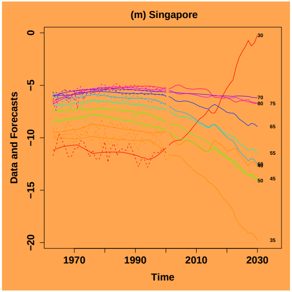
Lung cancer in Males, Singapore

Least Squares

Using Covariates (GDP, tobacco, trend, log trend)

Lung cancer in Males, Singapore

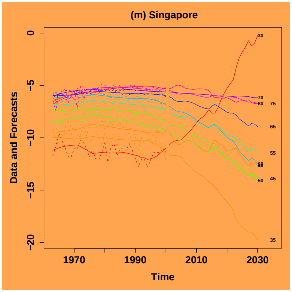
Least Squares

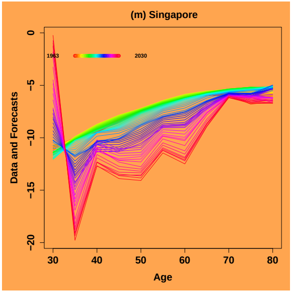


Using Covariates (GDP, tobacco, trend, log trend)

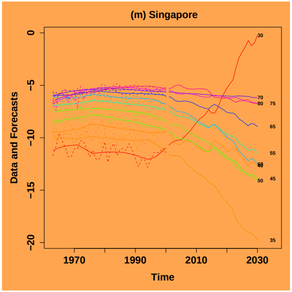
Lung cancer in Males, Singapore

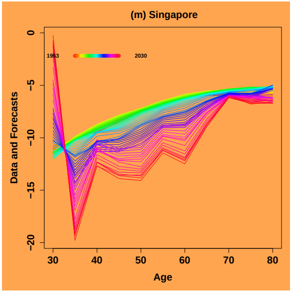
Least Squares





Least Squares



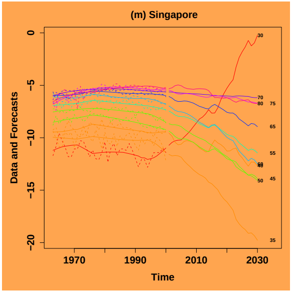


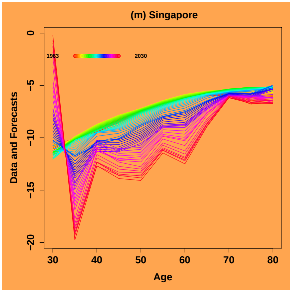
Smooth over age, time, age/time

Using Covariates (GDP, tobacco, trend, log trend)

Lung cancer in Males, Singapore

Least Squares





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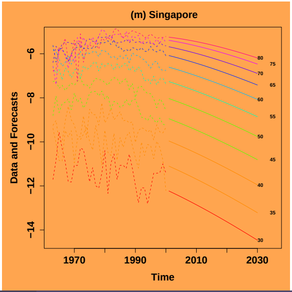

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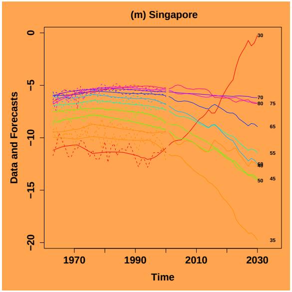
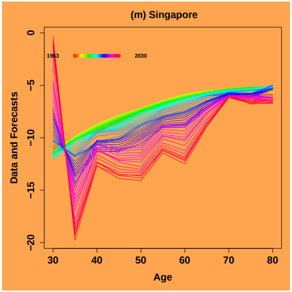
Lung cancer in Males, Singapore

Least Squares





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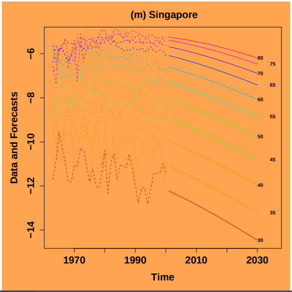
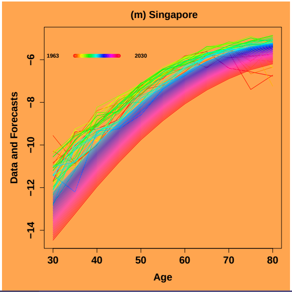
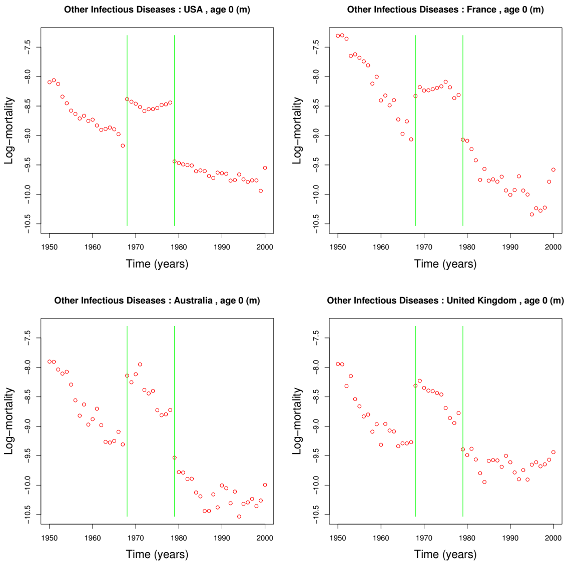
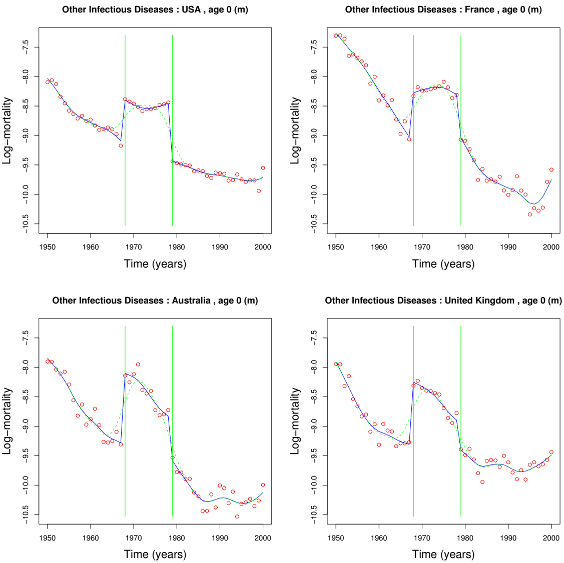




What about ICD Changes?



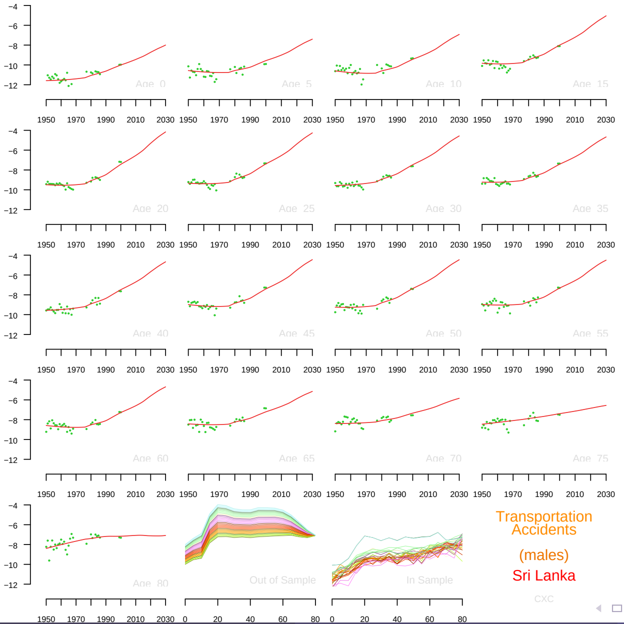
Fixing ICD Changes



A book manuscript, YourCast software, etc.

http://GKing.Harvard.edu

Without Country Smoothing



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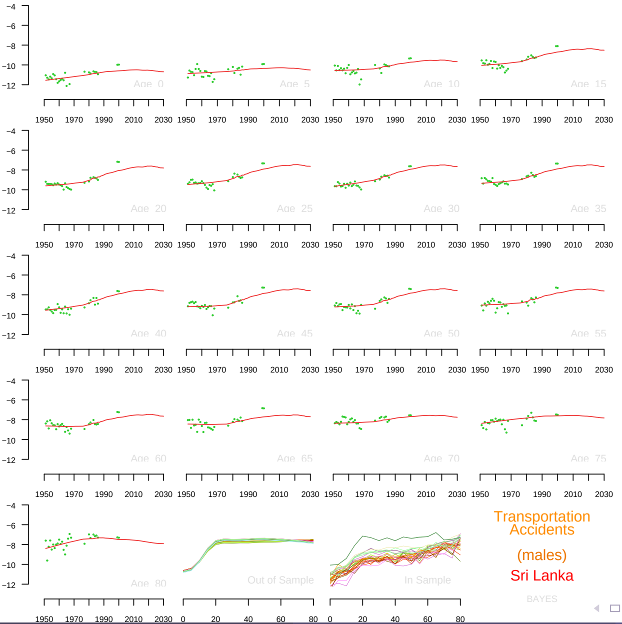
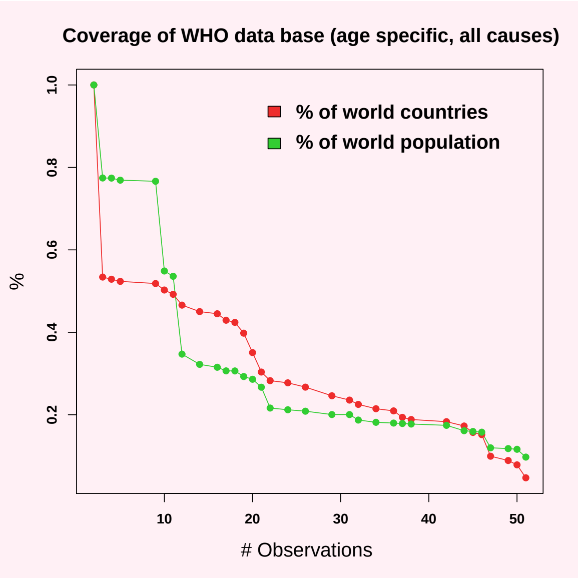


Many Short Time Series



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- Prior information is about relative (not absolute) levels of log-mortality

Preview of Results: Out-of-Sample Evaluation

Preview of Results: Out-of-Sample Evaluation

Mean Absolute Error in Males (over age and country)

Preview of Results: Out-of-Sample Evaluation

Mean Absolute Error in Males (over age and country)

	% Improvement	
	Over Best Previous	to Best Conceivable
Cardiovascular	22	49
Lung Cancer	24	47
Transportation	16	31
Respiratory Chronic	13	30
Other Infectious	12	30
Stomach Cancer	8	24
All-Cause	12	22
Suicide	7	17
Respiratory Infectious	3	7

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- Each row averages 6,800 forecast errors (17 age groups, 40 countries, and 10 out-of-sample years).

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- The new method out-performs with the same covariates, for most countries, causes, sexes, and age groups.
- Does *considerably* better with more informative covariates

Preview of Results: Out-of-Sample Evaluation

Preview of Results: Out-of-Sample Evaluation

Mean Absolute Error in Males (over age and country)

Preview of Results: Out-of-Sample Evaluation

Mean Absolute Error in Males (over age and country)

	Mean Absolute Error			% Improvement	
	Best Previous	Our Method	Best Conceivable	Over Best Previous	to Best Conceivable
Cardiovascular	0.34	0.27	0.19	22	49
Lung Cancer	0.36	0.27	0.17	24	47
Transportation	0.37	0.31	0.18	16	31
Respiratory Chronic	0.45	0.39	0.26	13	30
Other Infectious	0.55	0.48	0.32	12	30
Stomach Cancer	0.30	0.27	0.20	8	24
All-Cause	0.17	0.15	0.08	12	22
Suicide	0.31	0.29	0.18	7	17
Respiratory Infectious	0.49	0.47	0.28	3	7

Preview of Results: Out-of-Sample Evaluation

Mean Absolute Error in Males (over age and country)

	Mean Absolute Error			% Improvement	
	Best Previous	Our Method	Best Conceivable	Over Best Previous	to Best Conceivable
Cardiovascular	0.34	0.27	0.19	22	49
Lung Cancer	0.36	0.27	0.17	24	47
Transportation	0.37	0.31	0.18	16	31
Respiratory Chronic	0.45	0.39	0.26	13	30
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- Each row averages 6,800 forecast errors (17 age groups, 40 countries, and 10 out-of-sample years).

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- The new method out-performs with the same covariates, for most countries, causes, sexes, and age groups.
- Does much better with better covariates