Demographic Forecasting

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Harvard University

Joint work with Federico Girosi (RAND)
with contributions from Kevin Quinn and Gregory Wawro

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- Approach: Formalizing qualitative insights in quantitative models

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- Better ways of creating Bayesian priors
- Produces forecasts and farcasts using considerably more information

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	Over Best Previous	to Best Conceivable
Cardiovascular	22	49
Lung Cancer	24	47
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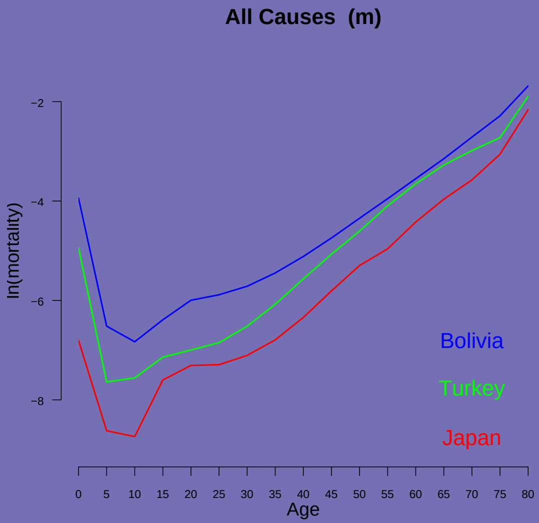
- Does *considerably* better with more informative covariates

All-Cause Mortality Age Profile Patterns



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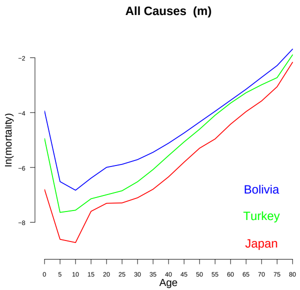

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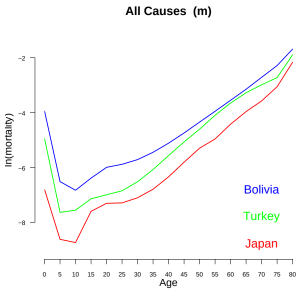

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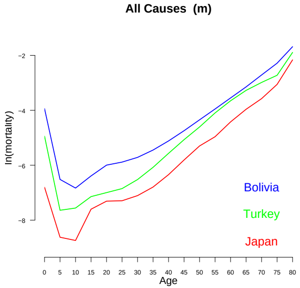

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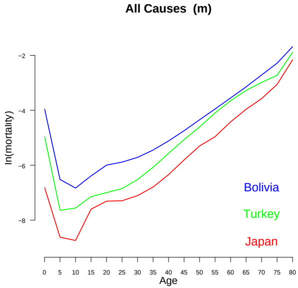

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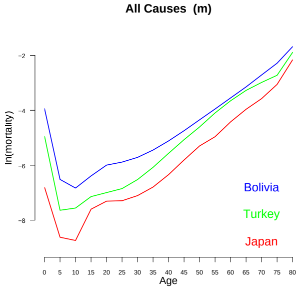

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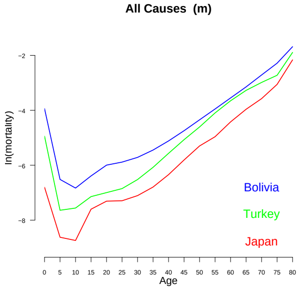

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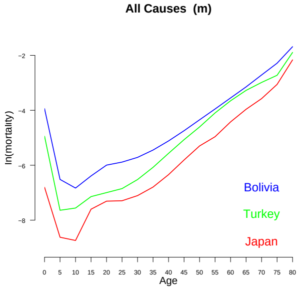

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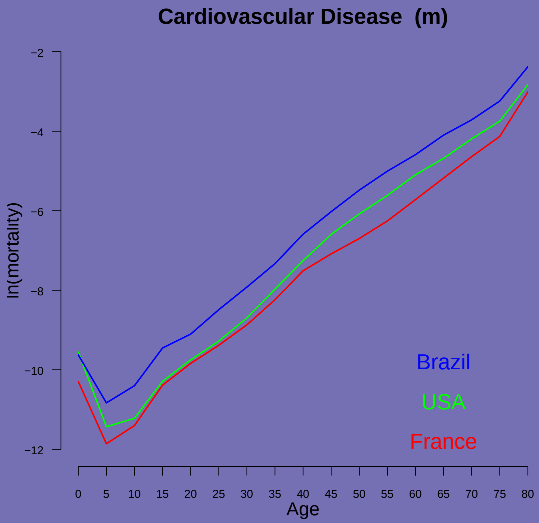
- But does it fit anything else?

Mortality Age Profile: The Same Pattern?



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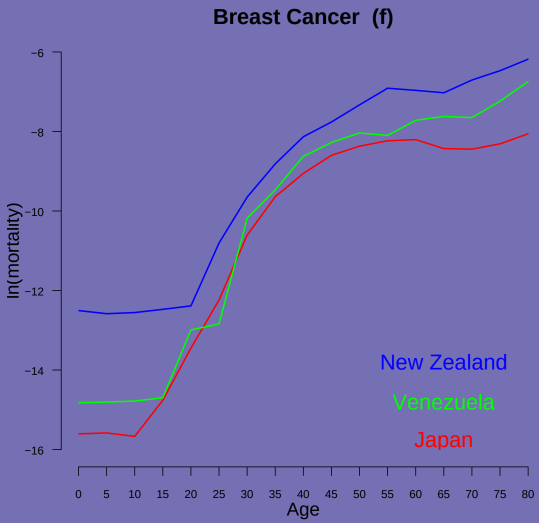

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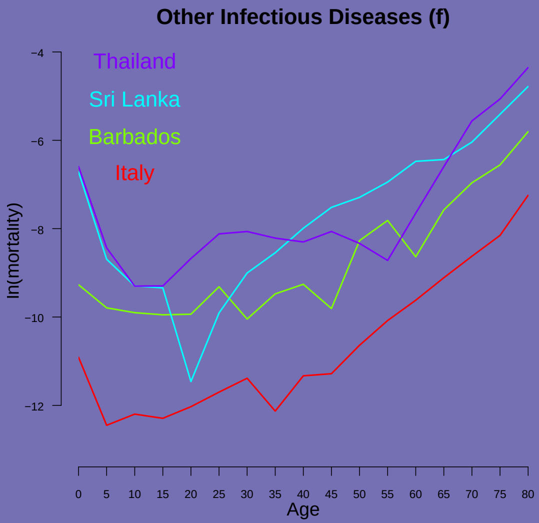

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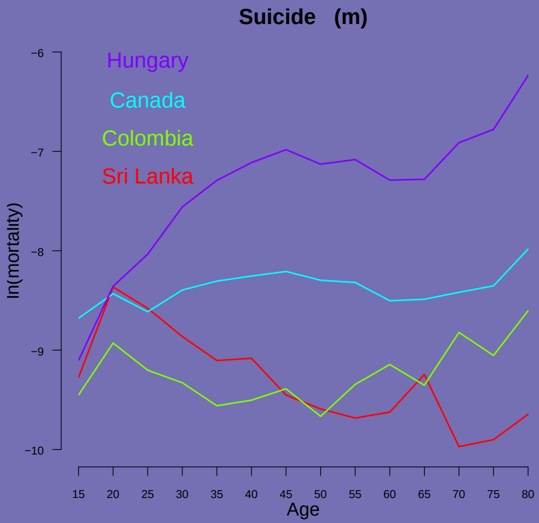

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- Also: Method ignores covariate information; the leading current method (McNown-Rogers) not replicable

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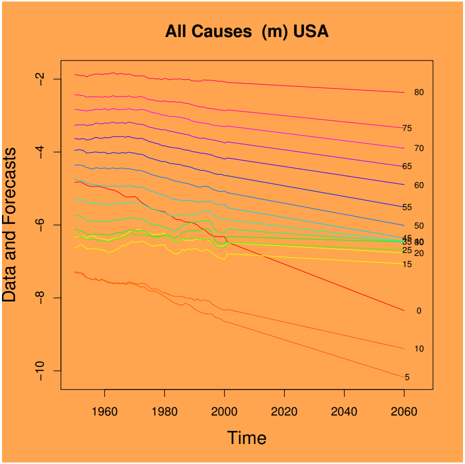

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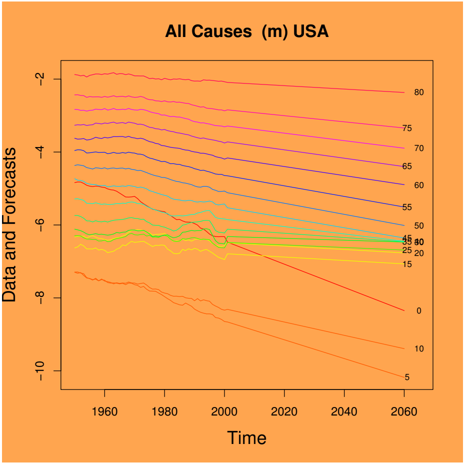

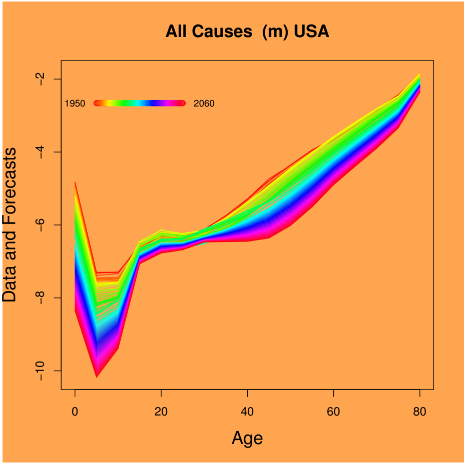

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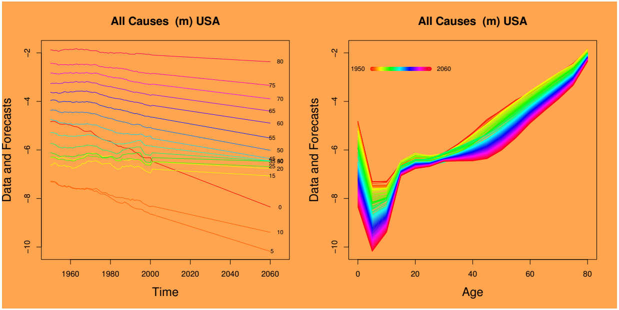

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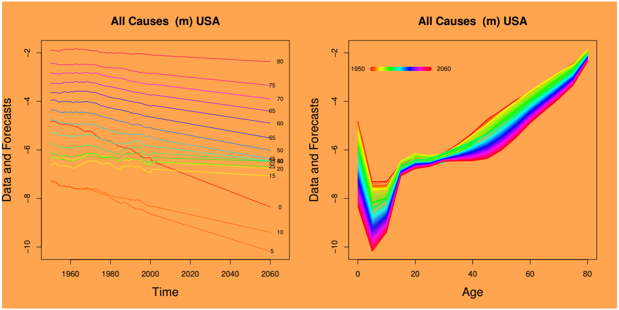

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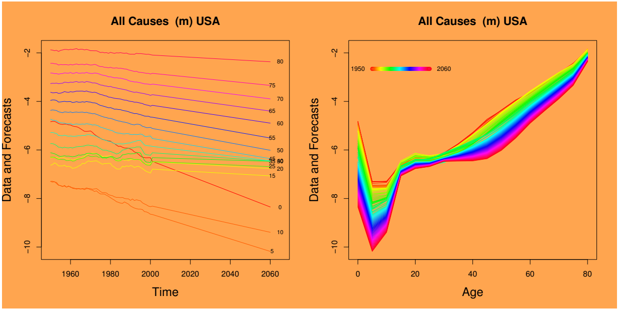

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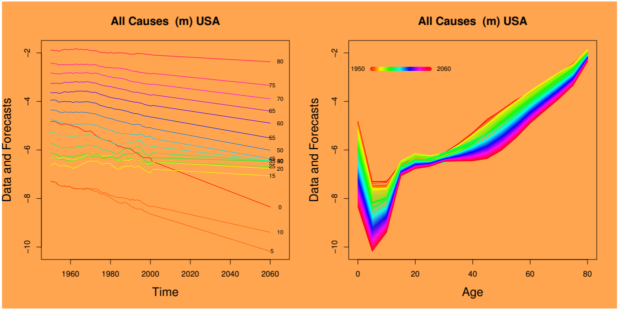

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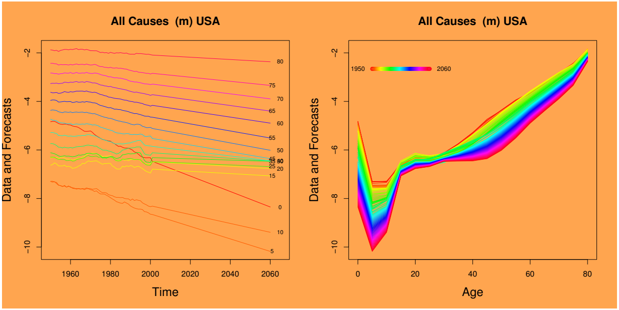

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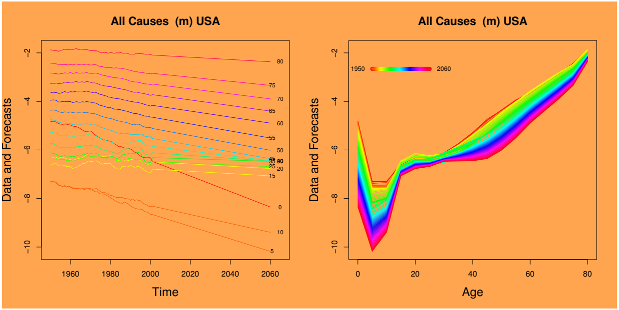

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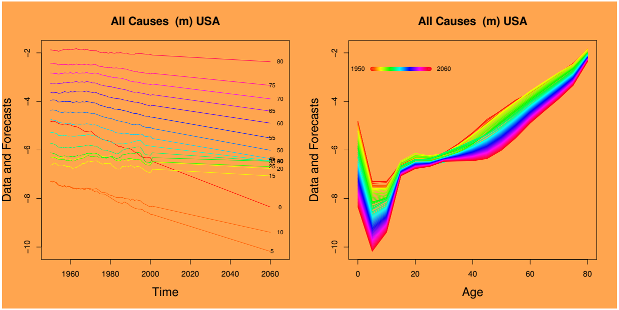

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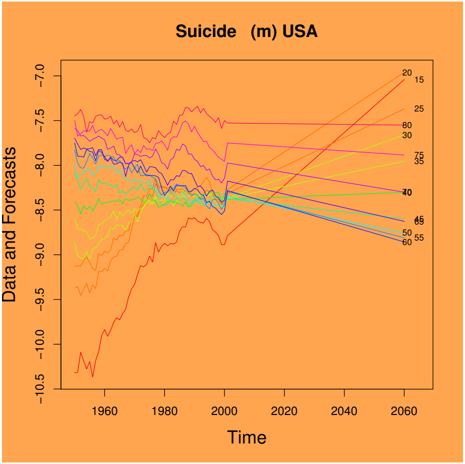

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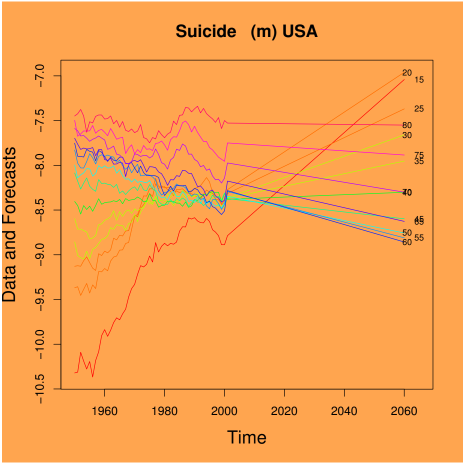

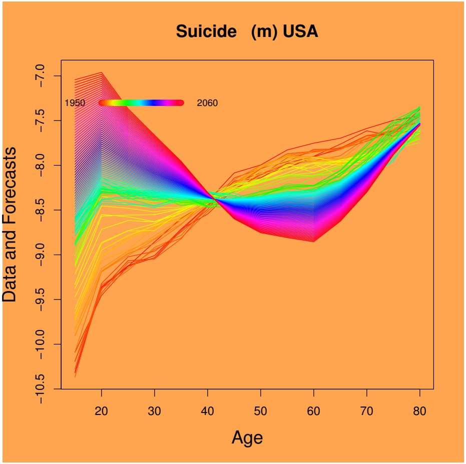

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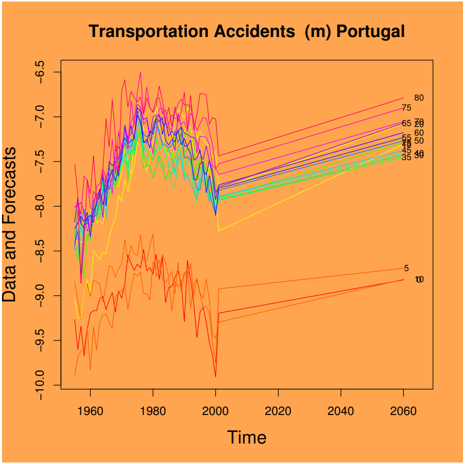

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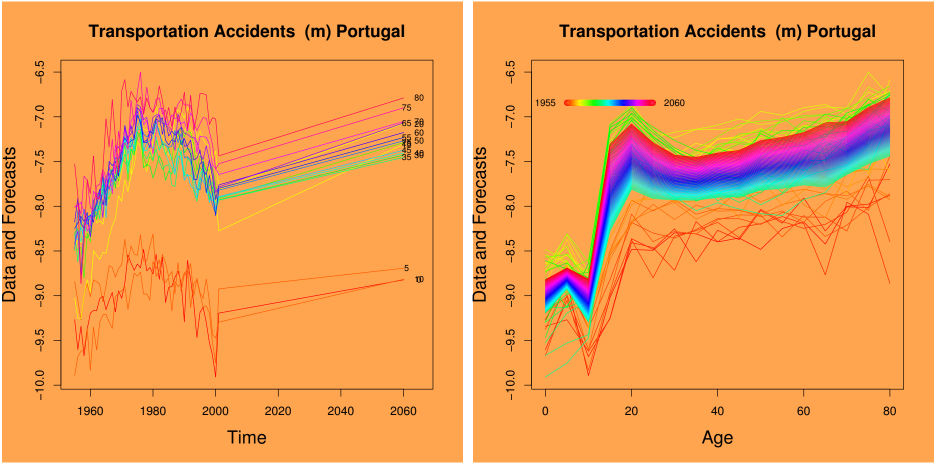

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Partial Pooling via a Bayesian Hierarchical Approach

- Likelihood for equation-by-equation least squares:

$$\mathcal{P}(m \mid \beta_i, \sigma_i) = \prod_t \mathcal{N}\left(m_{it} \mid \mathbf{Z}_{it}\beta_i, \sigma_i^2\right)$$

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- The easy part: *easy-to-use software* to implement everything we discuss today.

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Natural choice for the prior:

$$\mathcal{P}(\boldsymbol{\beta} \mid \Phi) \propto \exp\left(-\frac{1}{2}\sum_{ij} s_{ij}\|\boldsymbol{\beta}_i - \boldsymbol{\beta}_j\|^2_\Phi\right)$$

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- Extensive trial-and-error runs, yielded no useful parameter values.

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3. In the subspace, we can invert $\mu = \mathbf{Z}\boldsymbol{\beta}$ as $\boldsymbol{\beta} = (\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\mu$, giving:

$$\mathcal{P}(\boldsymbol{\beta} \mid \theta) \propto \exp\left(-\frac{1}{2}H[\mu,\theta]\right) = \exp\left(-\frac{1}{2}H[\mathbf{Z}\boldsymbol{\beta},\theta]\right)$$

   the same prior on μ, expressed as a function of $\boldsymbol{\beta}$ (with constant Jacobian).

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- We start with one-dimensional $P(\mu_{cat})$, and treat it as the multidimensional $P(\boldsymbol{\beta}_{ca})$, constant in all directions but $Z_{cat}\boldsymbol{\beta}_{ca}$.

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- Priors are based on knowledge rather than guesses.

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$$\mathcal{P}(\mu \mid \theta) \propto \exp\left(-\frac{1}{2}\theta \sum_{aa't} (\mu_{at} - \bar{\mu}_a)' W^n_{aa'} (\mu_{a't} - \bar{\mu}_{a'}) \right)$$

- where W^n is a matrix uniquely determined by n and θ

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where we have defined:

$\mathbf{C}_{aa'} \equiv \frac{1}{T}\mathbf{Z}'_a\mathbf{Z}_{a'}$ $\quad$ $\mathbf{Z}_a$ is a $T \times d_a$ data matrix for age group a

The Prior on the Coefficients $\boldsymbol{\beta}$

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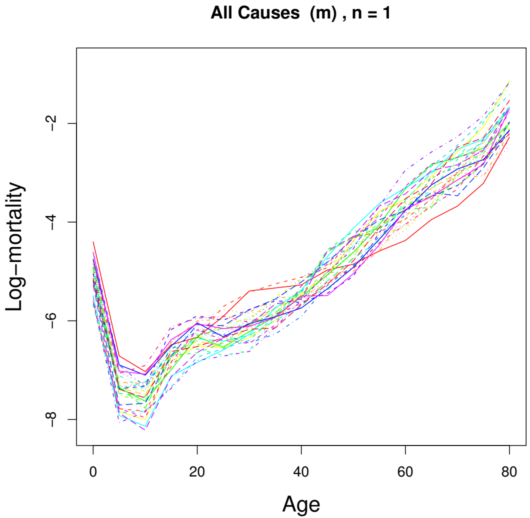
- The variance of the prior is inversely proportional to θ, which controls the "strength" of the prior.

Samples From Age Prior



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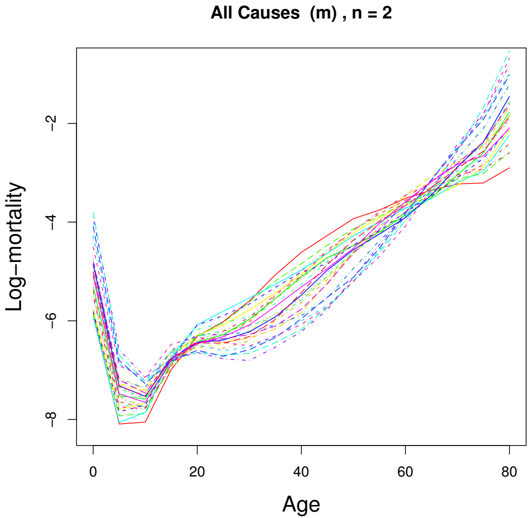

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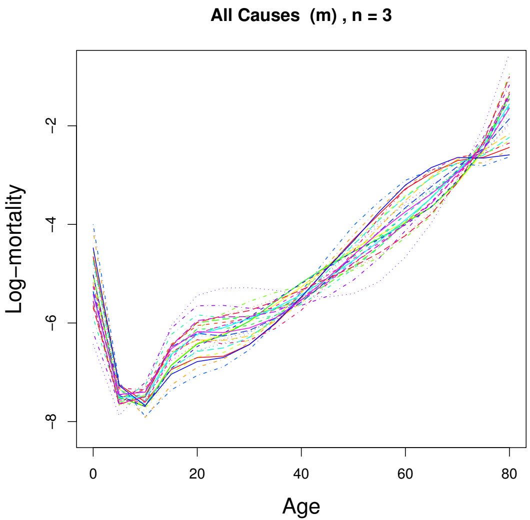

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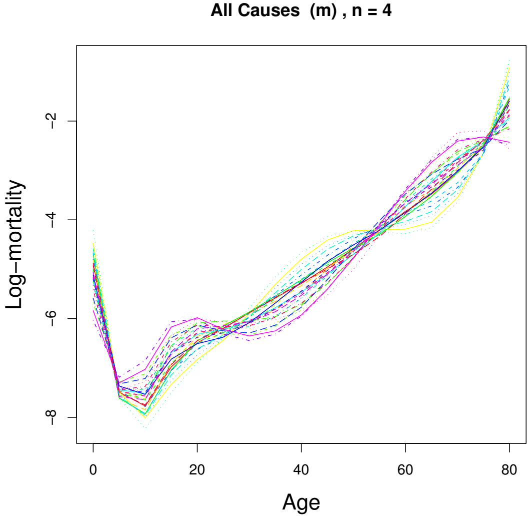

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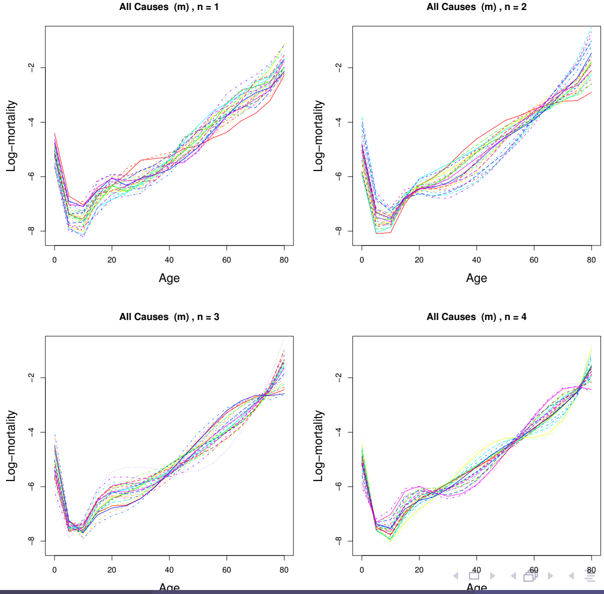

Prior Indifference

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- Prior information is about relative (not absolute) levels of log-mortality

Formalizing (Prior) Indifference

equal color = equal probability

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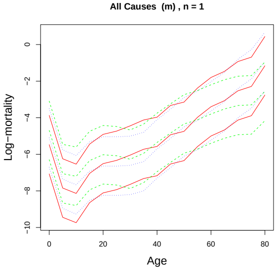
Level indifference



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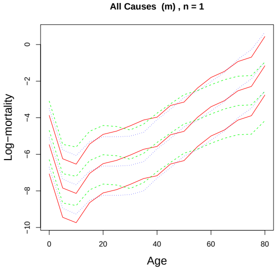
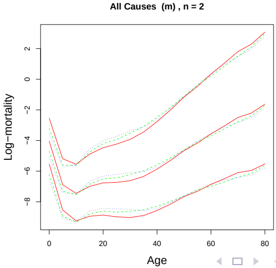


Level and slope indifference



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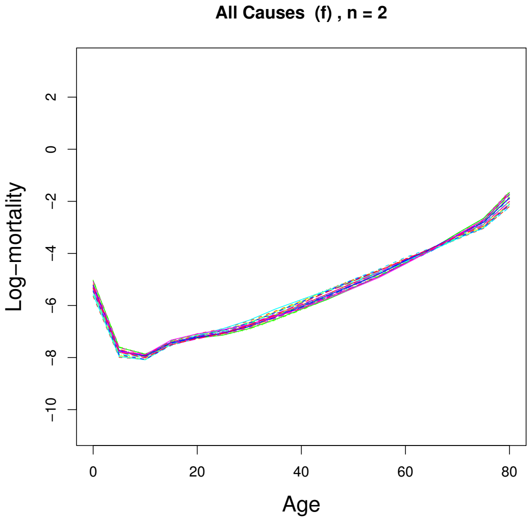
- θ controls the prior standard deviation

Samples from Age Prior



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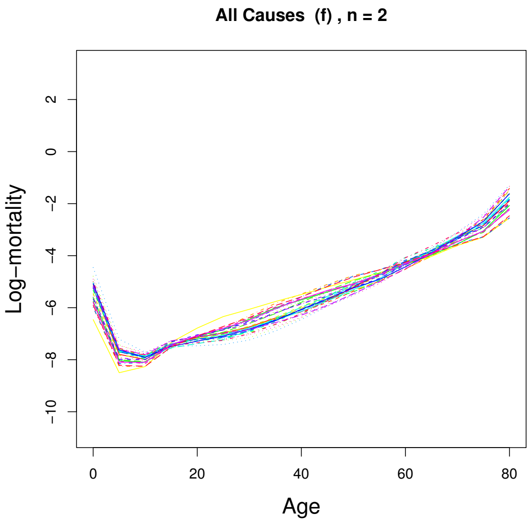

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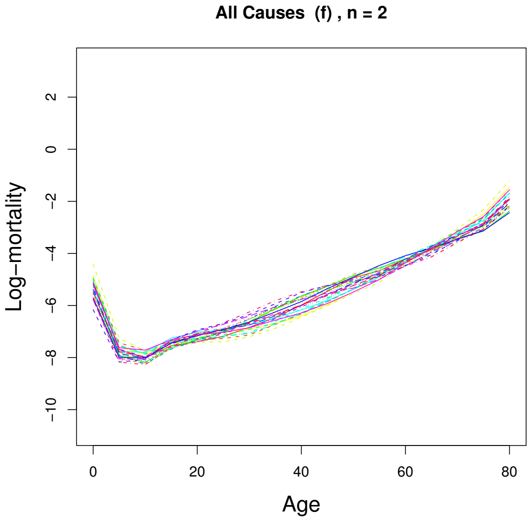

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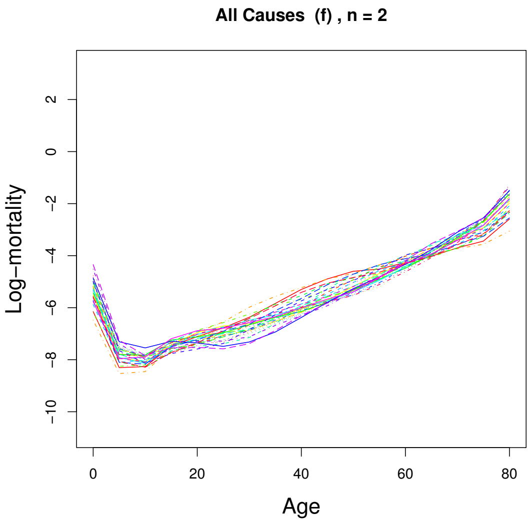

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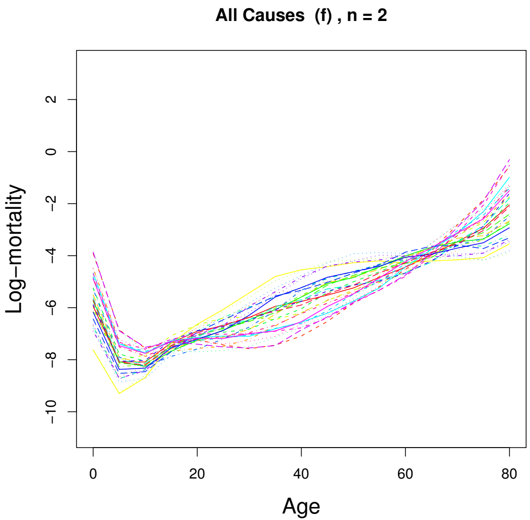

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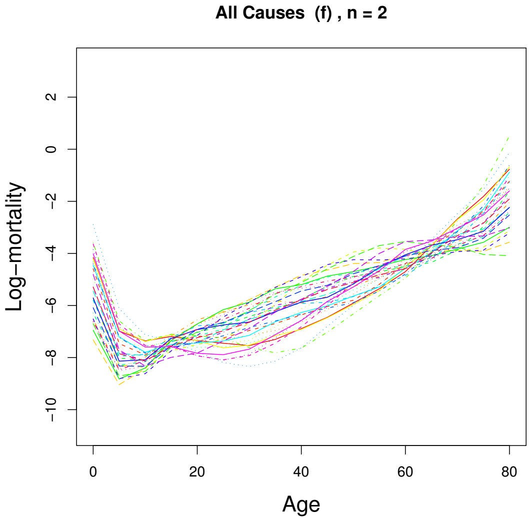

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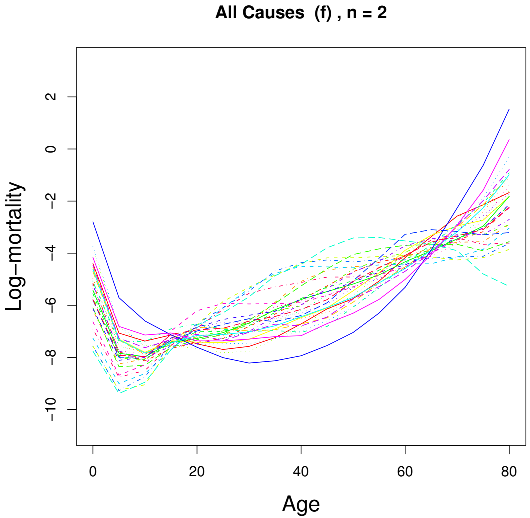

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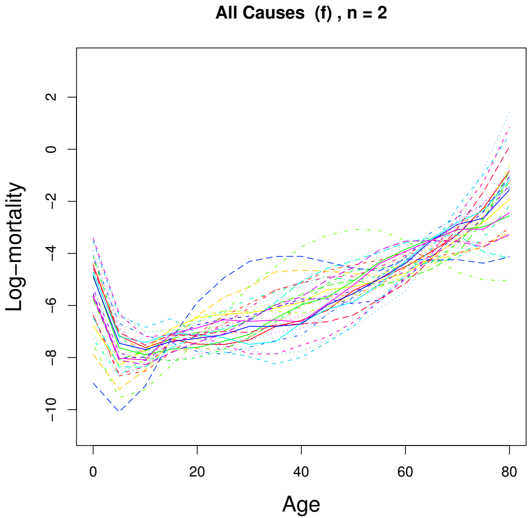

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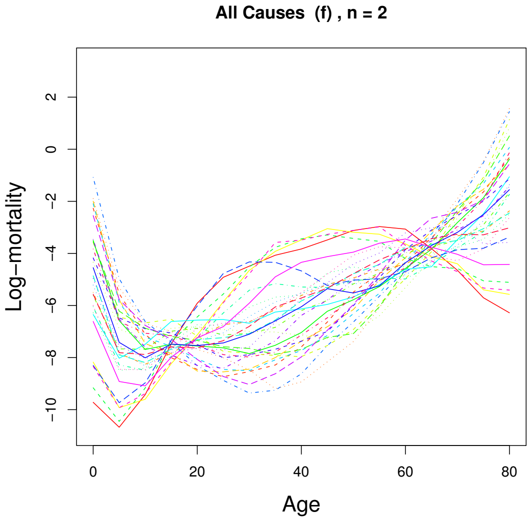

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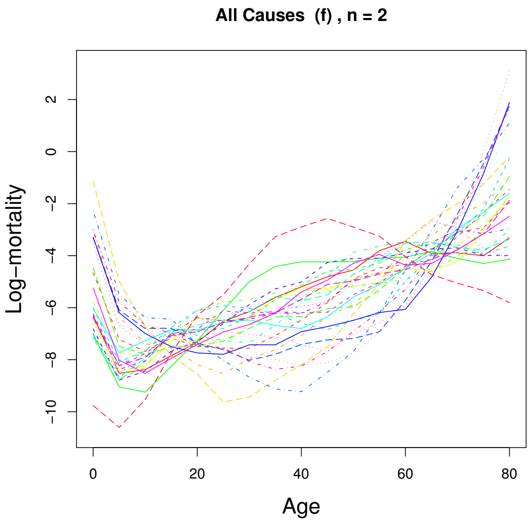

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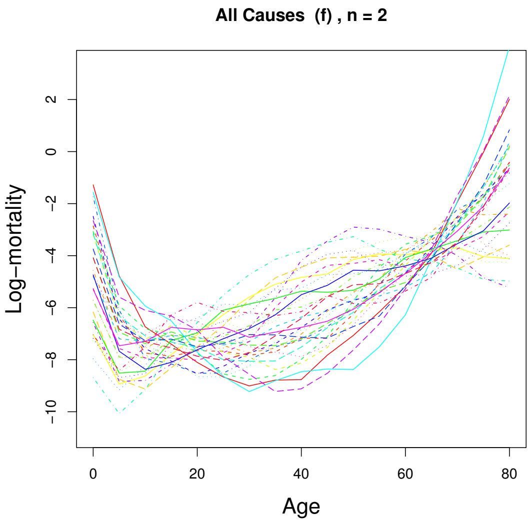

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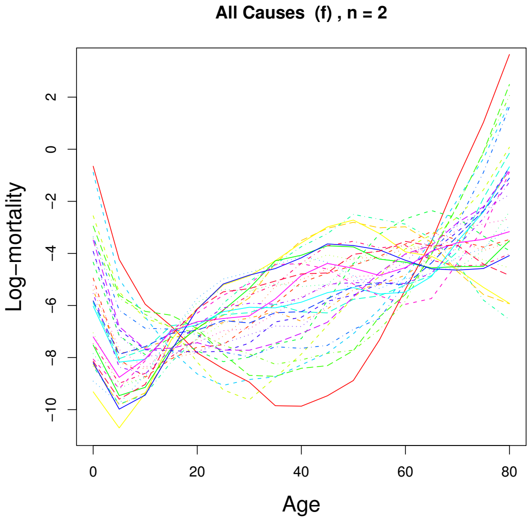

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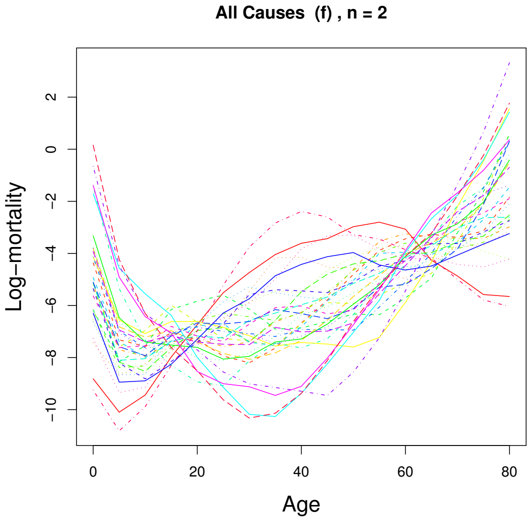

Generalizations

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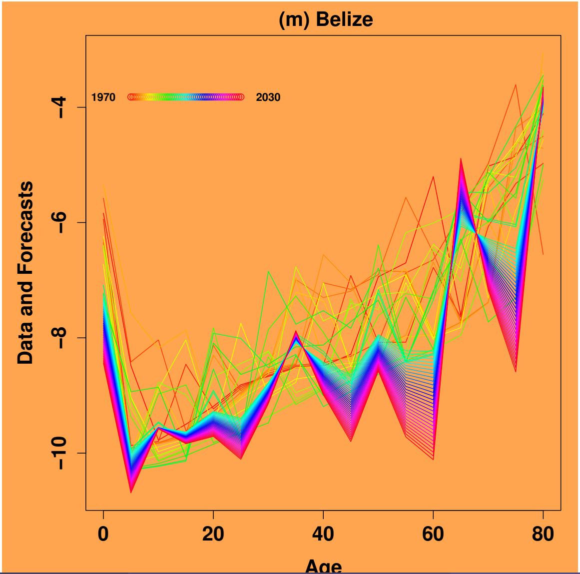
  - Smoothing trends over age groups as they vary across countries, etc.
- The mathematical form for *all* these (separately or together) turns out to be the same:

$$\mathcal{P}(\beta \mid \theta) \propto \exp\left(-\frac{\theta}{2}\sum_{ij} W_{ij}\beta_i'\mathbf{C}_{ij}\beta_j\right), \qquad \mathbf{C}_{aa'} \equiv \frac{1}{T}\mathbf{Z}_a\mathbf{Z}_{a'}$$

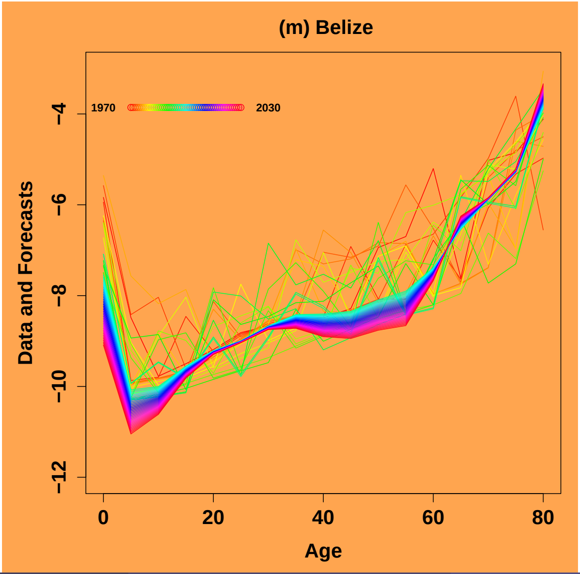
Mortality from Respiratory Infections, Males

Least Squares



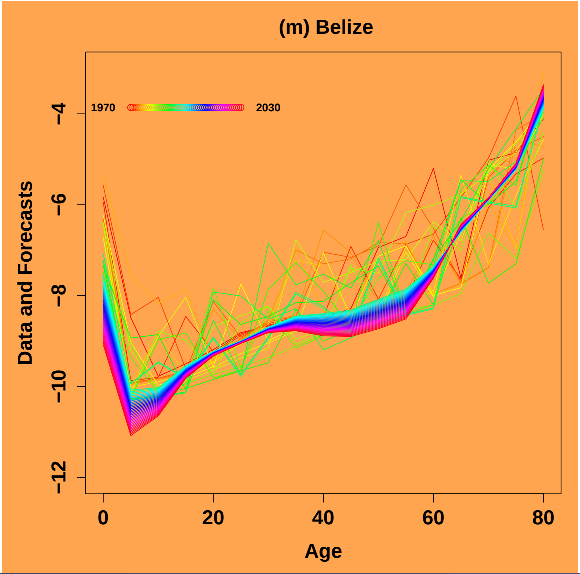
Mortality from Respiratory Infections, males, $\sigma = 2.00$

Smoothing over Age Groups



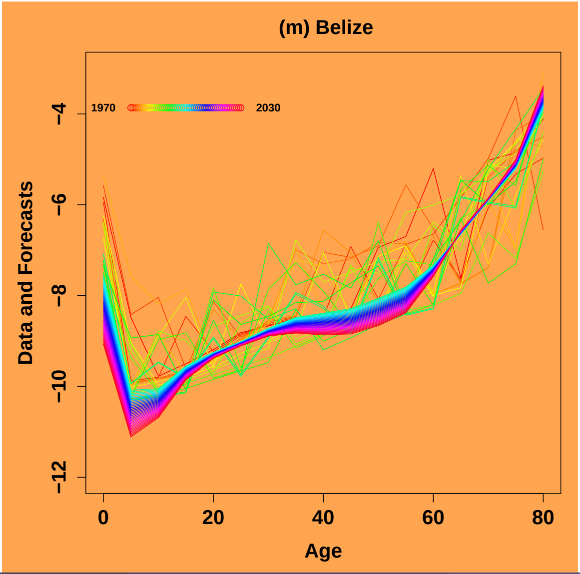
Mortality from Respiratory Infections, males, $\sigma = 1.51$

Smoothing over Age Groups



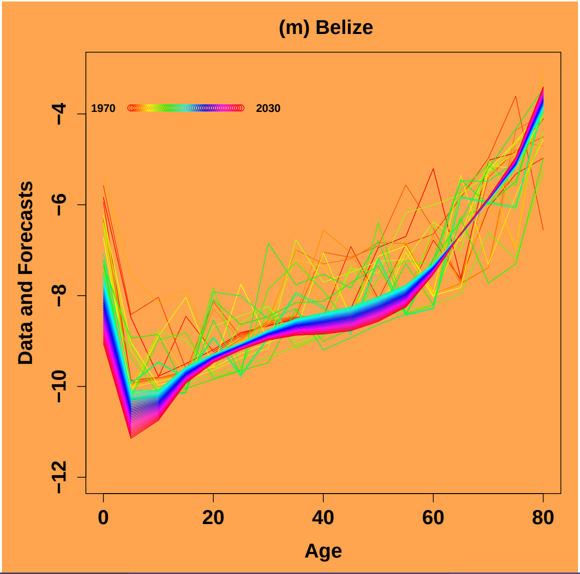
Mortality from Respiratory Infections, males, $\sigma = 1.15$

Smoothing over Age Groups



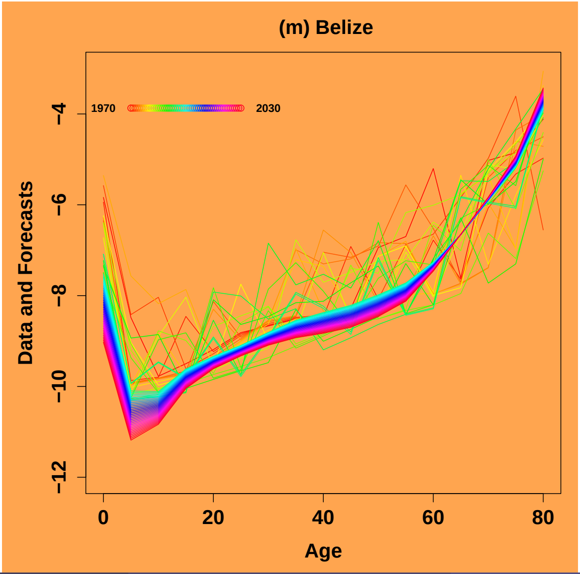
Mortality from Respiratory Infections, males, $\sigma = 0.87$

Smoothing over Age Groups



Mortality from Respiratory Infections, males, $\sigma = 0.66$

Smoothing over Age Groups



Mortality from Respiratory Infections, males, $\sigma = 0.50$

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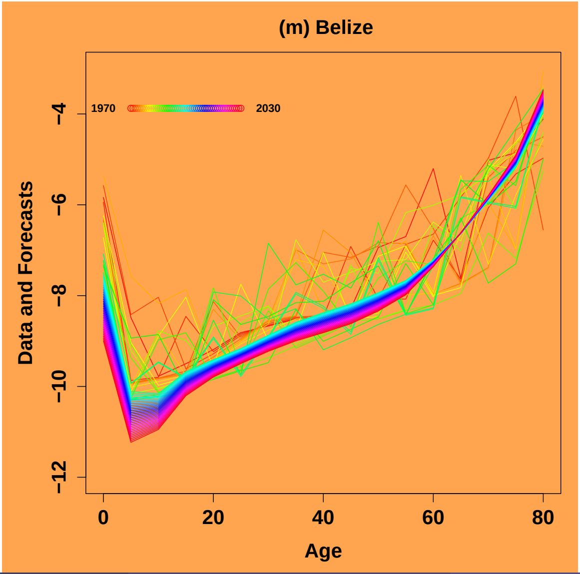
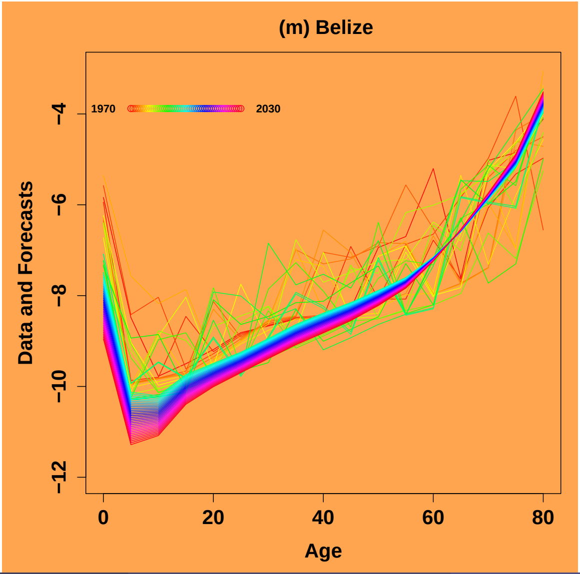


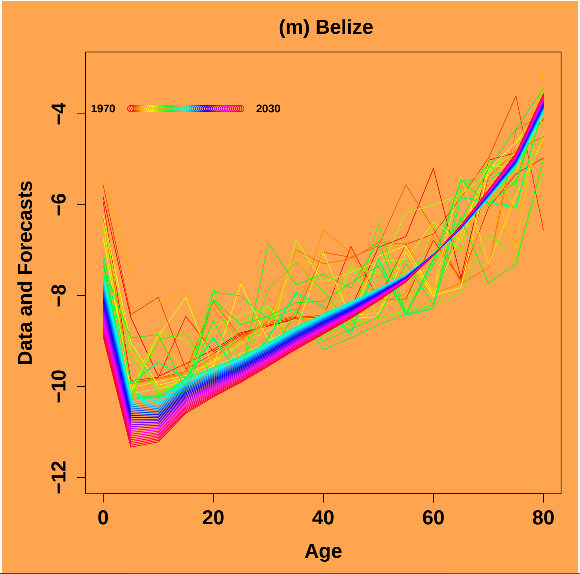
Mortality from Respiratory Infections, males, $\sigma = 0.38$

Smoothing over Age Groups



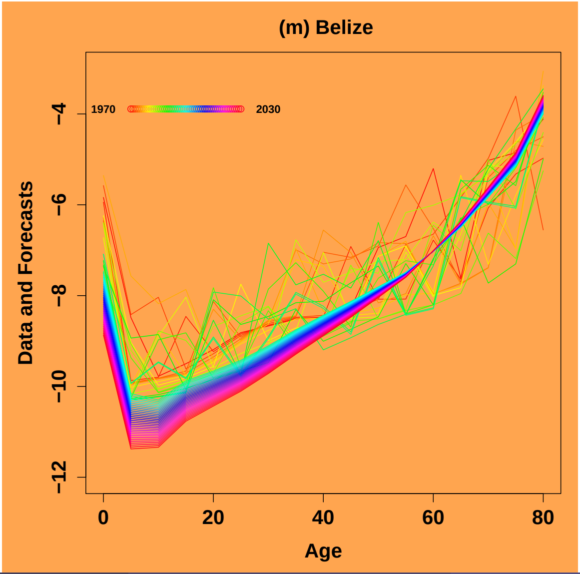
Mortality from Respiratory Infections, males, $\sigma = 0.28$

Smoothing over Age Groups



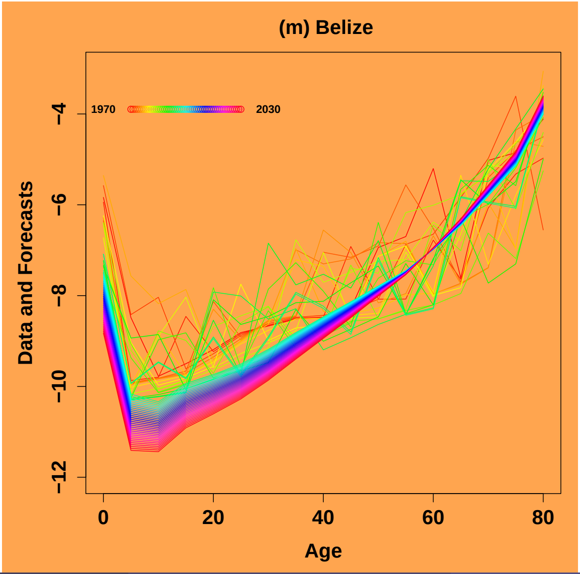
Mortality from Respiratory Infections, males, $\sigma = 0.21$

Smoothing over Age Groups



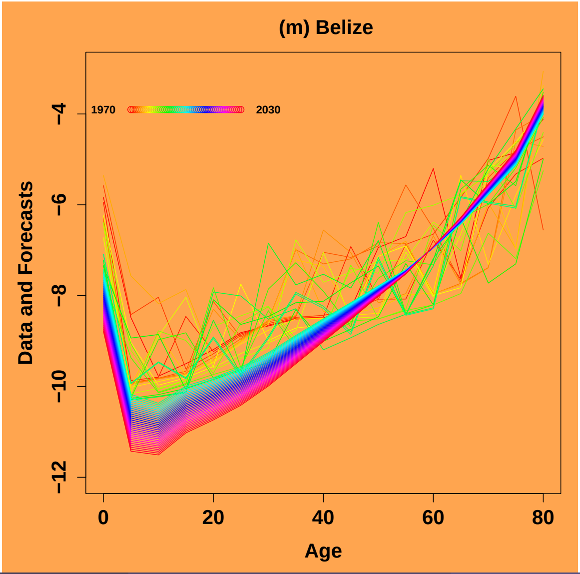
Mortality from Respiratory Infections, males, $\sigma = 0.16$

Smoothing over Age Groups



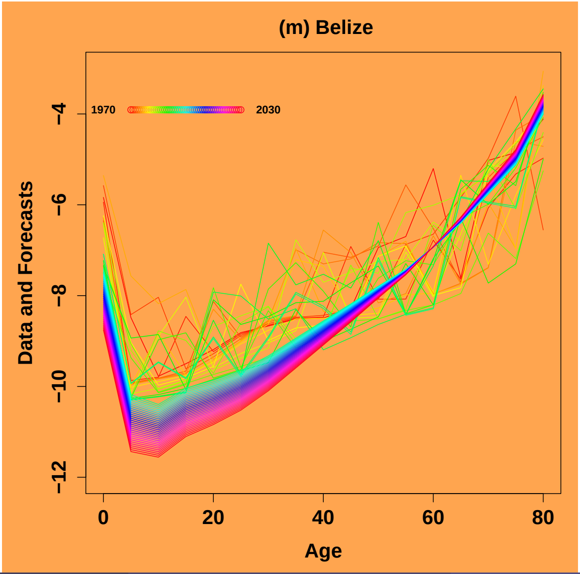
Mortality from Respiratory Infections, males, $\sigma = 0.12$

Smoothing over Age Groups



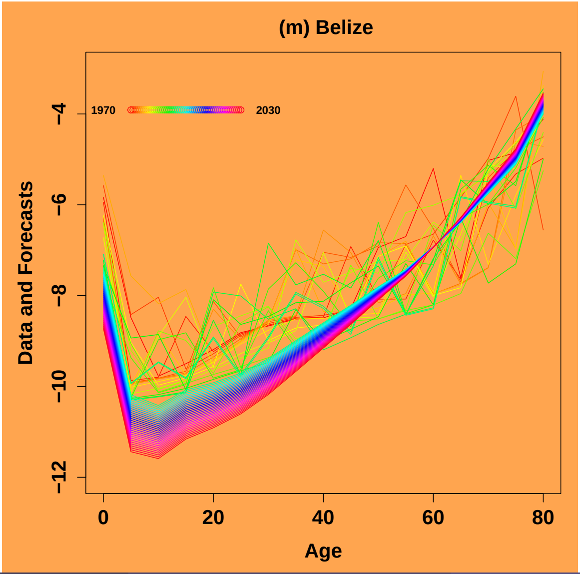
Mortality from Respiratory Infections, males, $\sigma = 0.09$

Smoothing over Age Groups



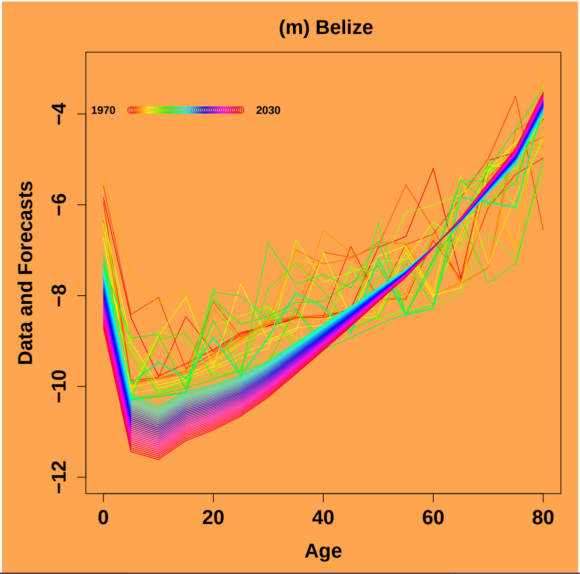
Mortality from Respiratory Infections, males, $\sigma = 0.07$

Smoothing over Age Groups



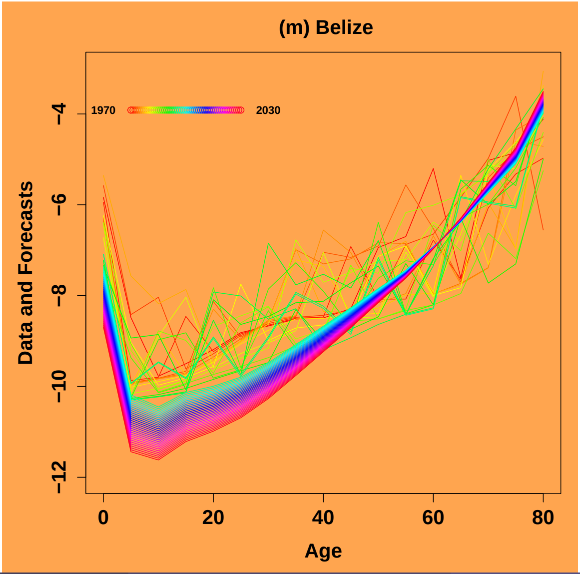
Mortality from Respiratory Infections, males, $\sigma = 0.05$

Smoothing over Age Groups



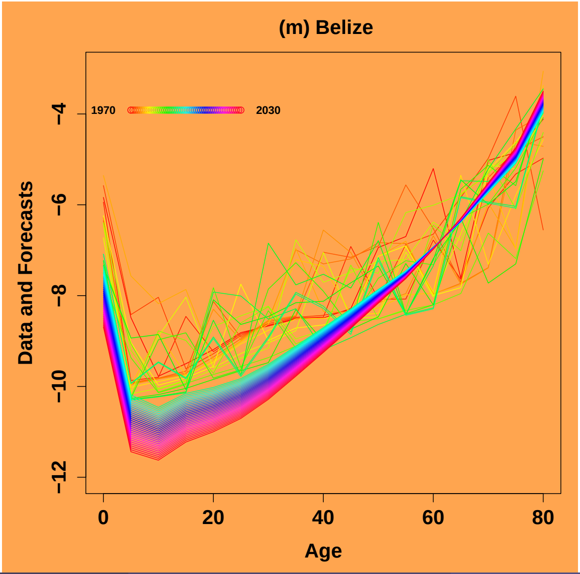
Mortality from Respiratory Infections, males, $\sigma = 0.04$

Smoothing over Age Groups



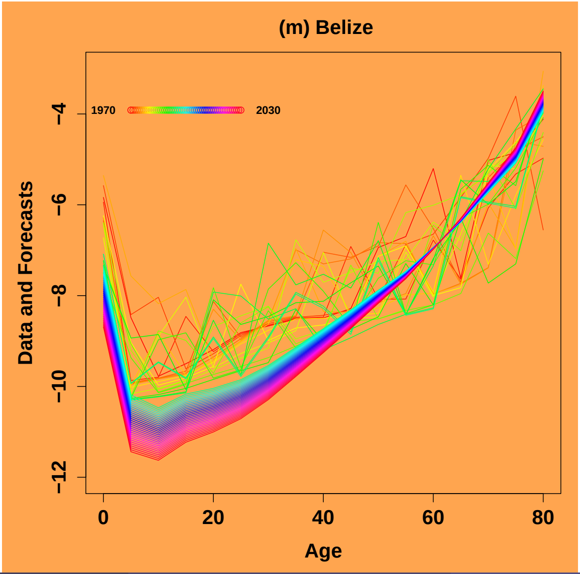
Mortality from Respiratory Infections, males, $\sigma = 0.03$

Smoothing over Age Groups



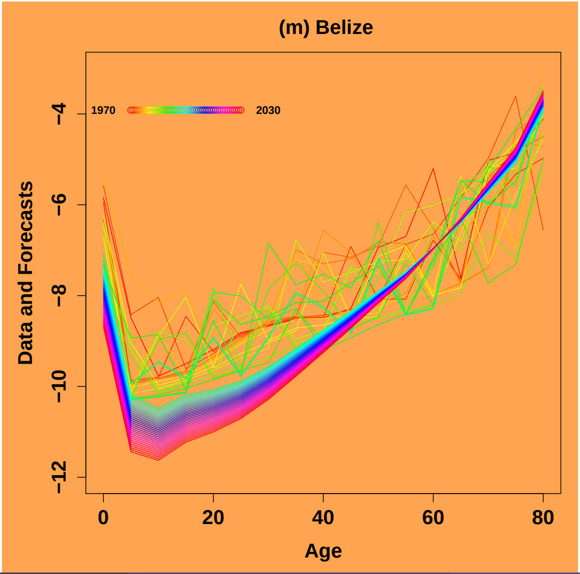
Mortality from Respiratory Infections, males, $\sigma = 0.02$

Smoothing over Age Groups



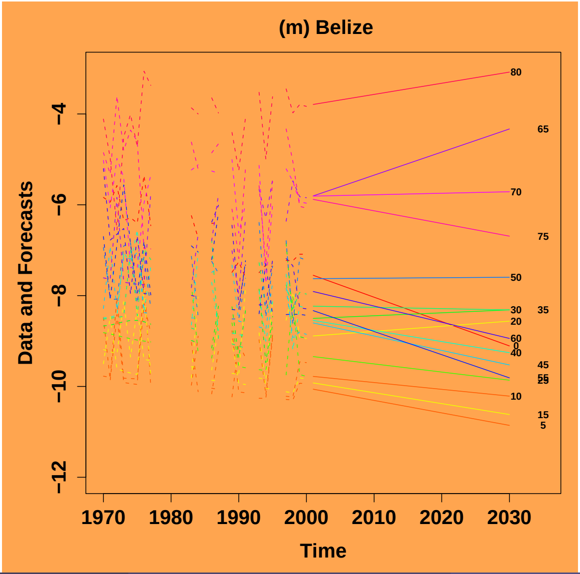
Mortality from Respiratory Infections, males, $\sigma = 0.01$

Smoothing over Age Groups



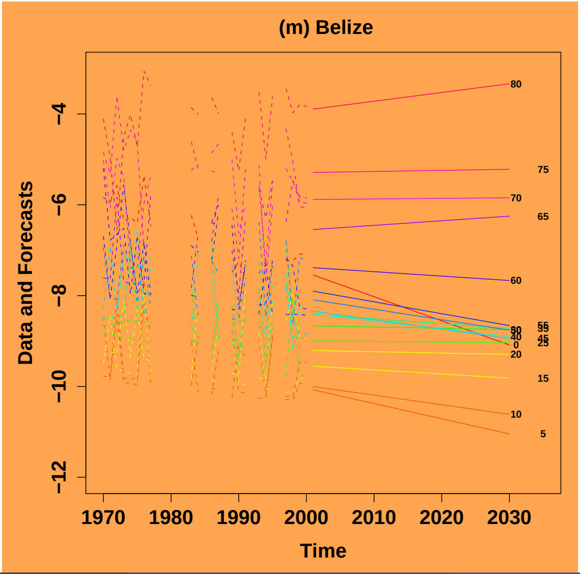
Mortality from Respiratory Infections, males

Least Squares



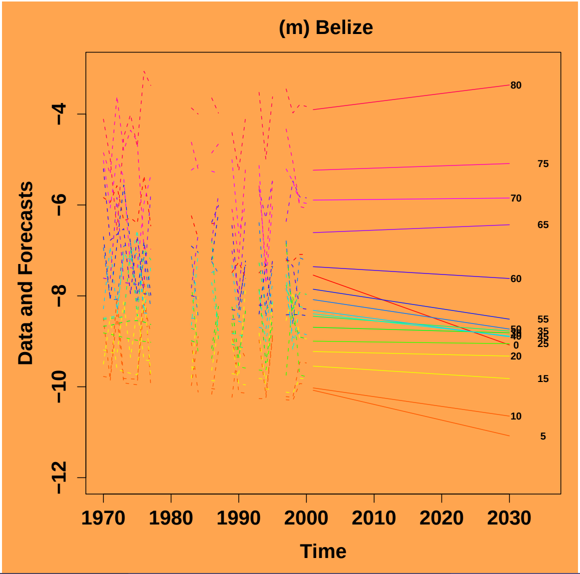
Mortality from Respiratory Infections, males, $\sigma = 2.00$

Smoothing over Age Groups



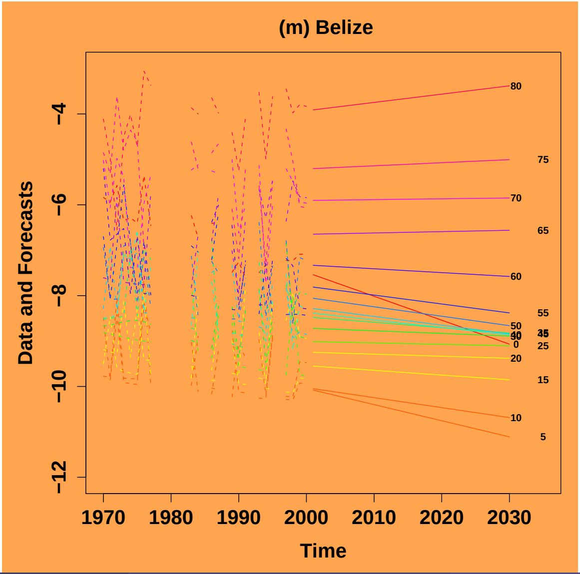
Mortality from Respiratory Infections, males, $\sigma = 1.51$

Smoothing over Age Groups



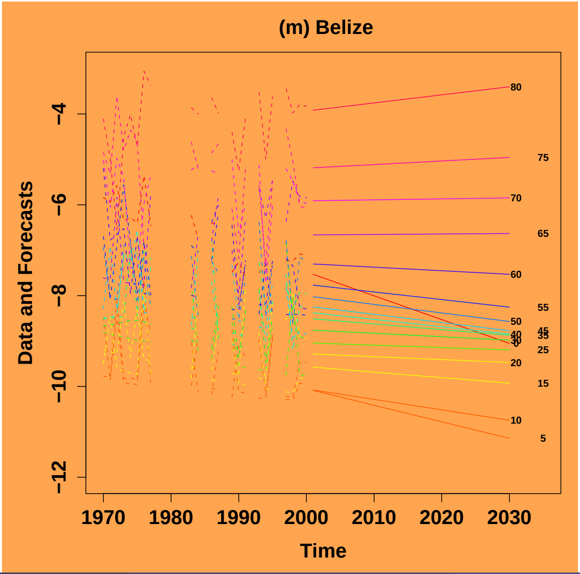
Mortality from Respiratory Infections, males, $\sigma = 1.15$

Smoothing over Age Groups



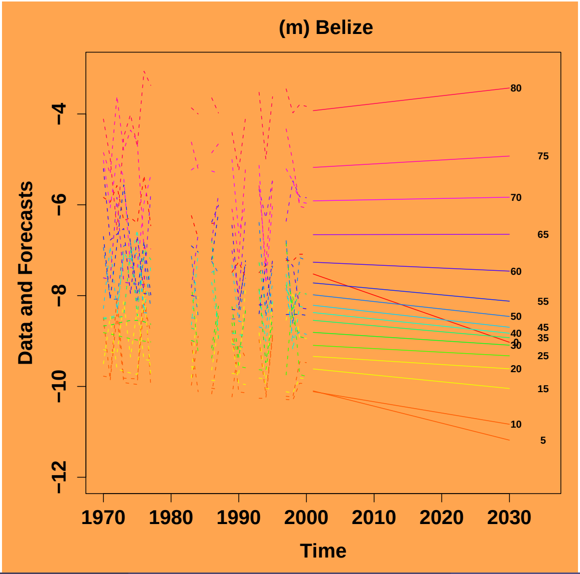
Mortality from Respiratory Infections, males, $\sigma = 0.87$

Smoothing over Age Groups



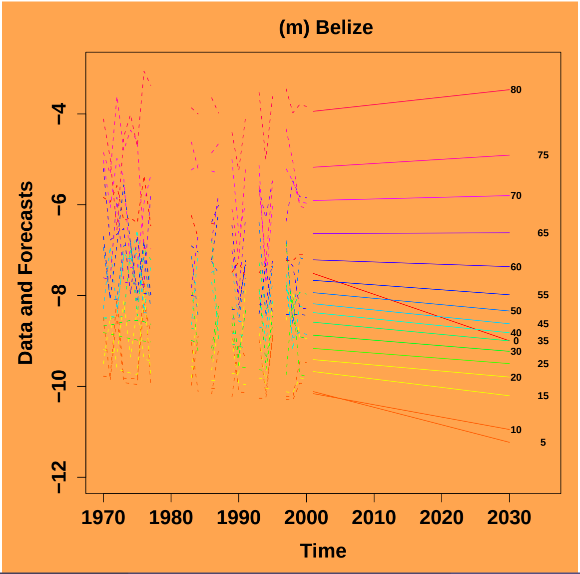
Mortality from Respiratory Infections, males, $\sigma = 0.66$

Smoothing over Age Groups



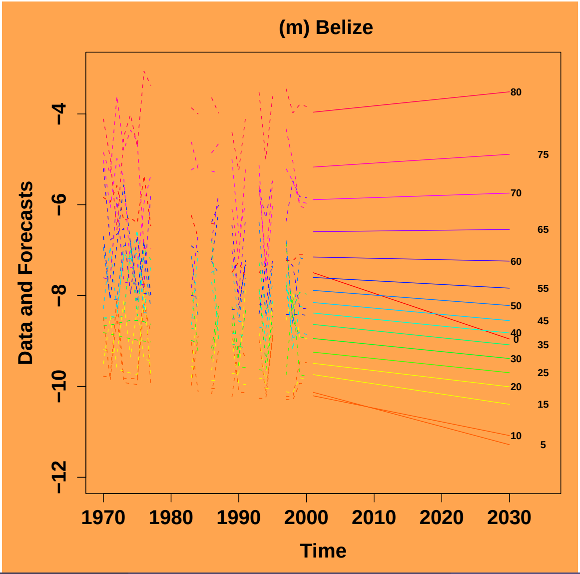
Mortality from Respiratory Infections, males, $\sigma = 0.50$

Smoothing over Age Groups



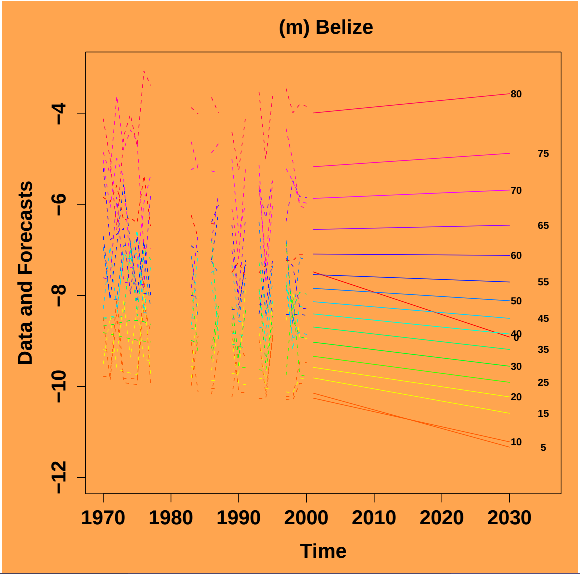
Mortality from Respiratory Infections, males, $\sigma = 0.38$

Smoothing over Age Groups



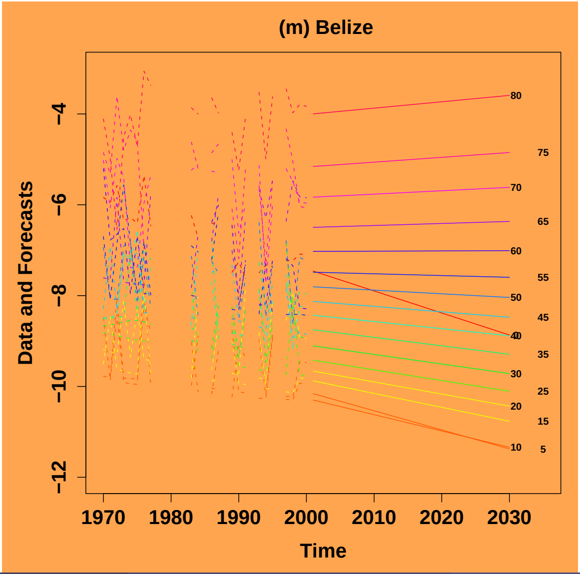
Mortality from Respiratory Infections, males, $\sigma = 0.28$

Smoothing over Age Groups



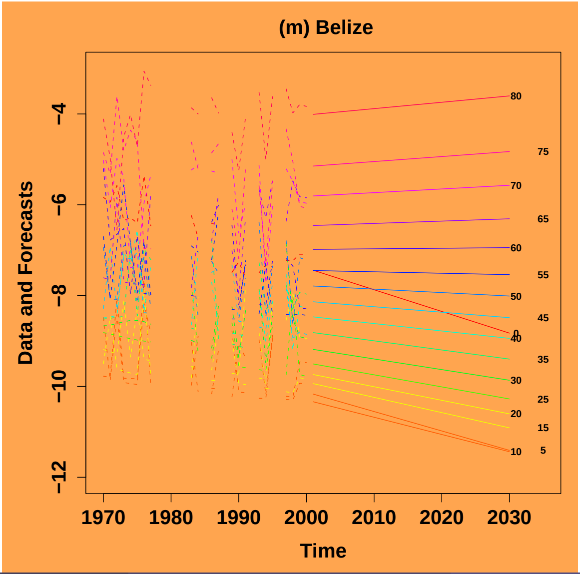
Mortality from Respiratory Infections, males, $\sigma = 0.21$

Smoothing over Age Groups



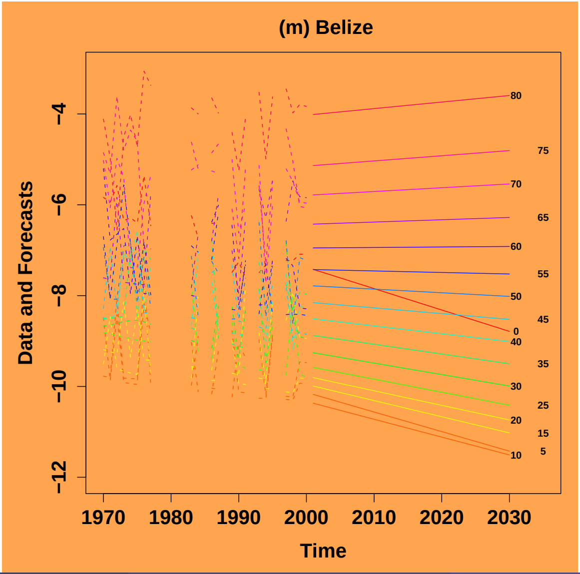
Mortality from Respiratory Infections, males, $\sigma = 0.16$

Smoothing over Age Groups



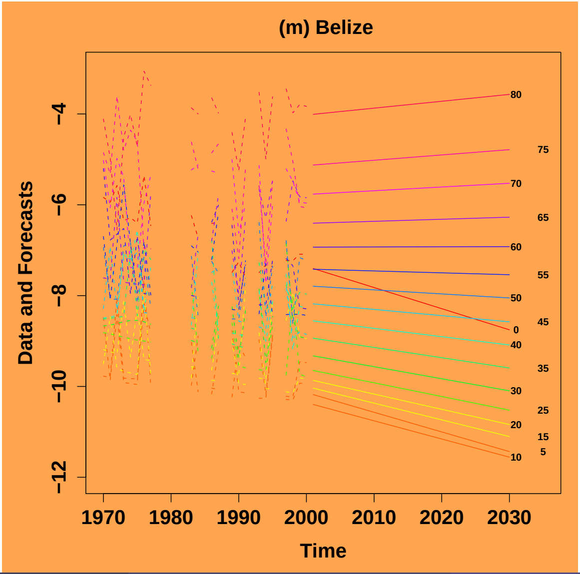
Mortality from Respiratory Infections, males, $\sigma = 0.12$

Smoothing over Age Groups



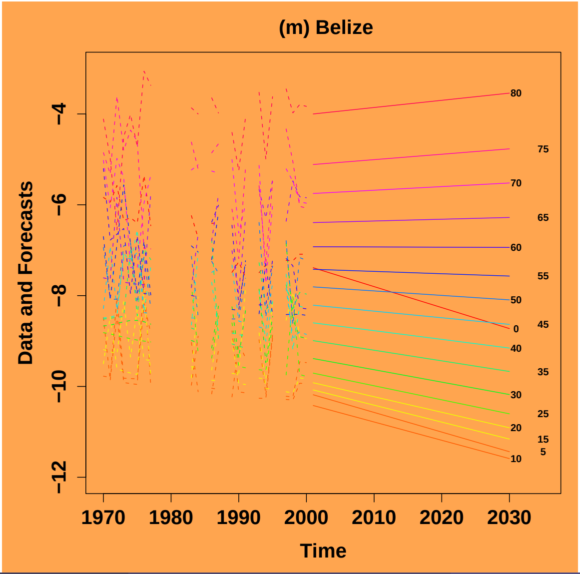
Mortality from Respiratory Infections, males, $\sigma = 0.09$

Smoothing over Age Groups



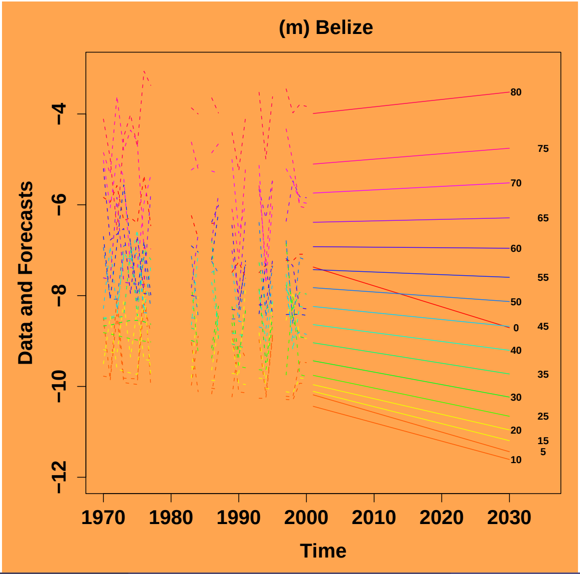
Mortality from Respiratory Infections, males, $\sigma = 0.07$

Smoothing over Age Groups



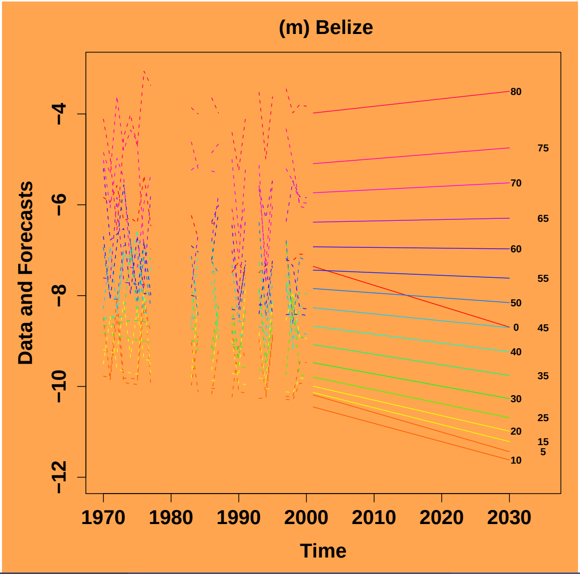
Mortality from Respiratory Infections, males, $\sigma = 0.05$

Smoothing over Age Groups



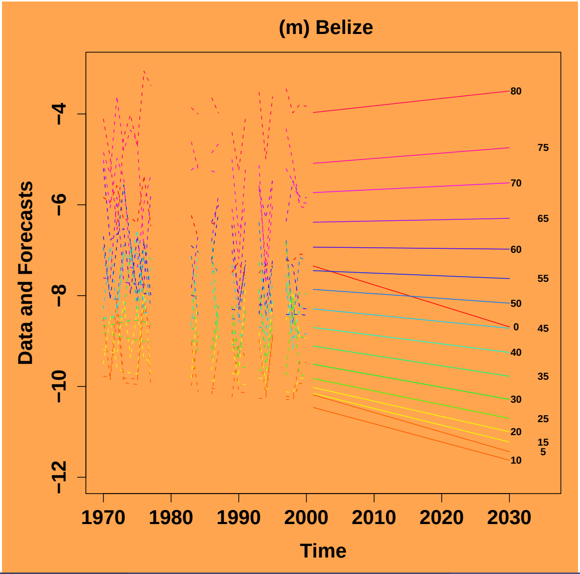
Mortality from Respiratory Infections, males, $\sigma = 0.04$

Smoothing over Age Groups



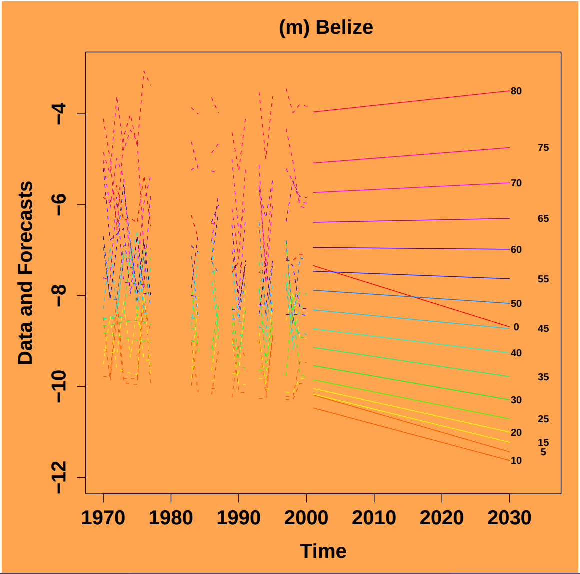
Mortality from Respiratory Infections, males, $\sigma = 0.03$

Smoothing over Age Groups



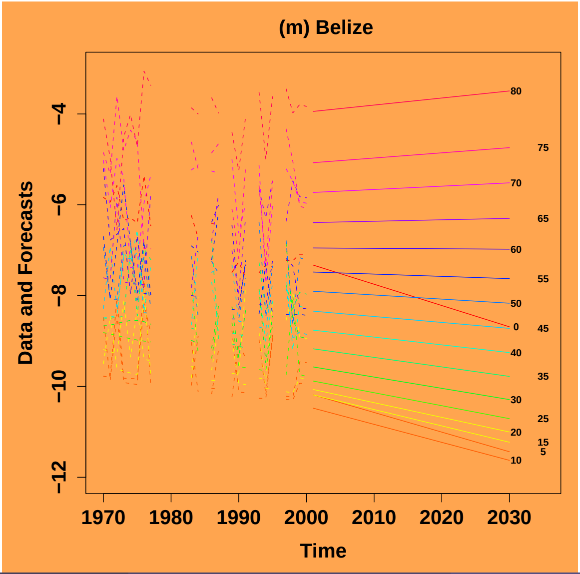
Mortality from Respiratory Infections, males, $\sigma = 0.02$

Smoothing over Age Groups



Mortality from Respiratory Infections, males, $\sigma = 0.01$

Smoothing over Age Groups



Smoothing Trends over Age Groups

Smoothing Trends over Age Groups

Log-mortality in Belize males from respiratory infections

Smoothing Trends over Age Groups

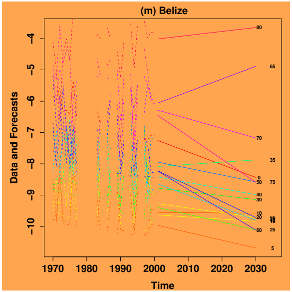
Log-mortality in Belize males from respiratory infections

Least Squares

Smoothing Trends over Age Groups

Log-mortality in Belize males from respiratory infections

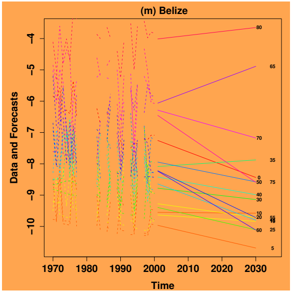
Least Squares

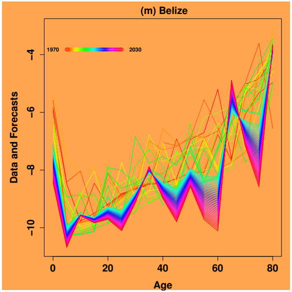


Smoothing Trends over Age Groups

Log-mortality in Belize males from respiratory infections

Least Squares





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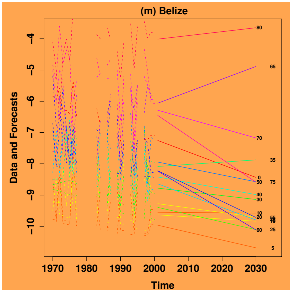
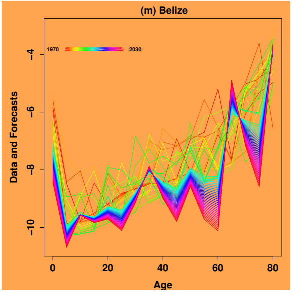


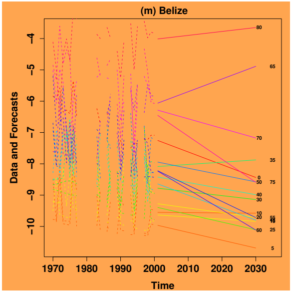


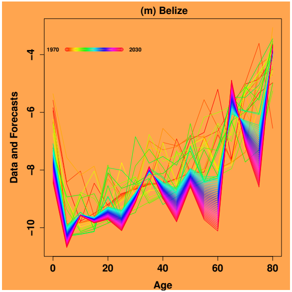
Smoothing
Age Groups

Smoothing Trends over Age Groups

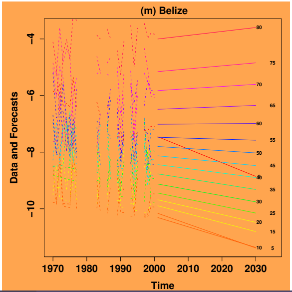
Log-mortality in Belize males from respiratory infections

Least Squares





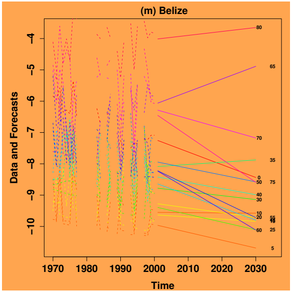
Smoothing Age Groups

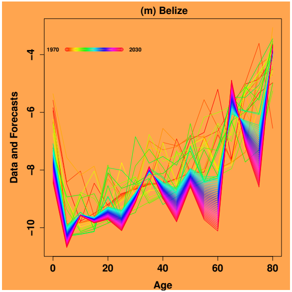


Smoothing Trends over Age Groups

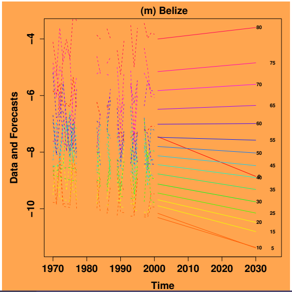
Log-mortality in Belize males from respiratory infections

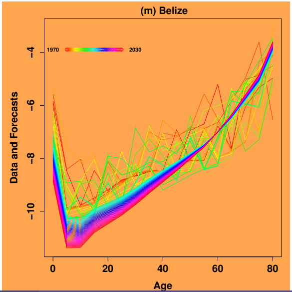
Least Squares





Smoothing
Age Groups





Smoothing Trends over Age Groups and Time

Smoothing Trends over Age Groups and Time

Log-Mortality in Bulgarian males from respiratory infections

Smoothing Trends over Age Groups and Time

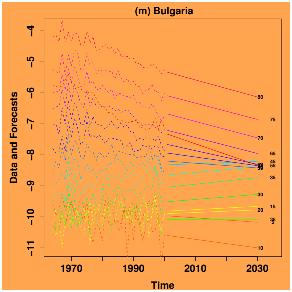
Log-Mortality in Bulgarian males from respiratory infections

Least Squares

Smoothing Trends over Age Groups and Time

Log-Mortality in Bulgarian males from respiratory infections

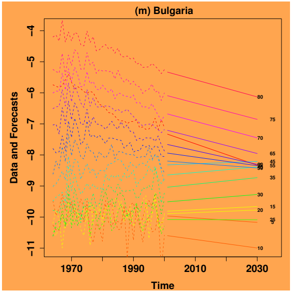
Least Squares

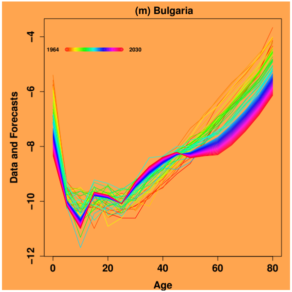


Smoothing Trends over Age Groups and Time

Log-Mortality in Bulgarian males from respiratory infections

Least Squares

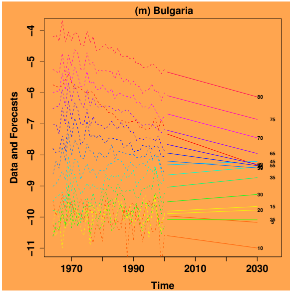


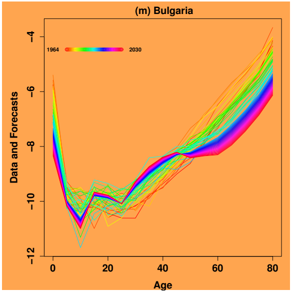


Smoothing Trends over Age Groups and Time

Log-Mortality in Bulgarian males from respiratory infections

Least Squares



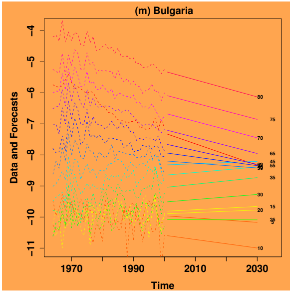


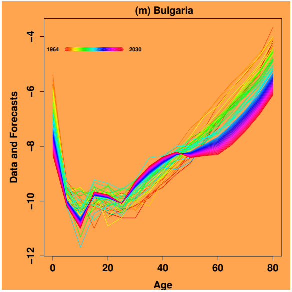
Smoothing
Age and Time

Smoothing Trends over Age Groups and Time

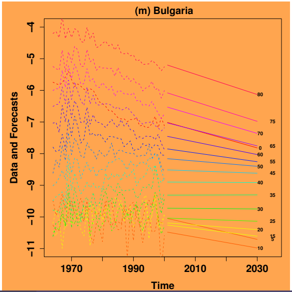
Log-Mortality in Bulgarian males from respiratory infections

Least Squares





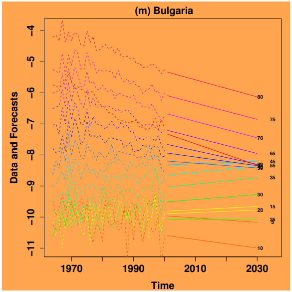
Smoothing
Age and Time

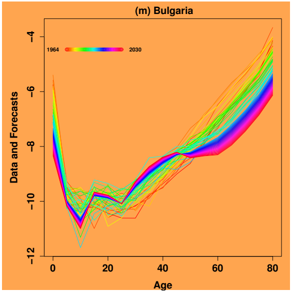


Smoothing Trends over Age Groups and Time

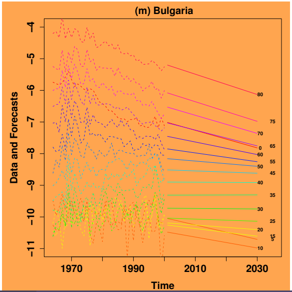
Log-Mortality in Bulgarian males from respiratory infections

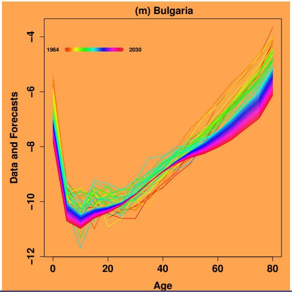
Least Squares





Smoothing
Age and Time





Using Covariates (GDP, tobacco, trend, log trend)

Using Covariates (GDP, tobacco, trend, log trend)

Lung cancer in Korean Males

Using Covariates (GDP, tobacco, trend, log trend)

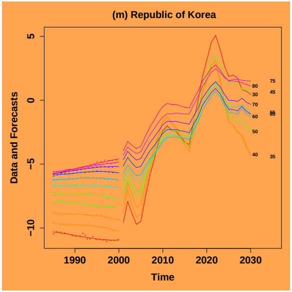
Lung cancer in Korean Males

Least Squares

Using Covariates (GDP, tobacco, trend, log trend)

Lung cancer in Korean Males

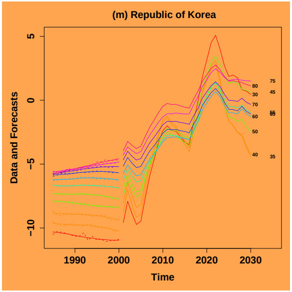
Least Squares

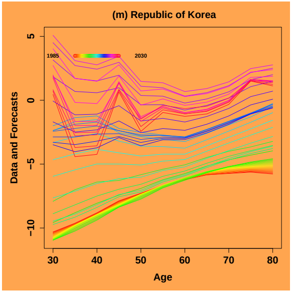


Using Covariates (GDP, tobacco, trend, log trend)

Lung cancer in Korean Males

Least Squares

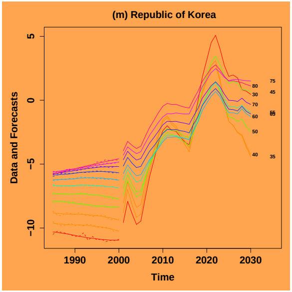


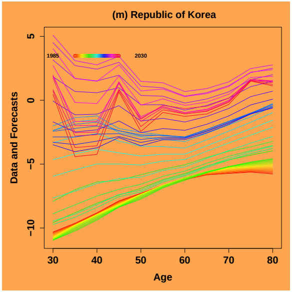


Using Covariates (GDP, tobacco, trend, log trend)

Lung cancer in Korean Males

Least Squares



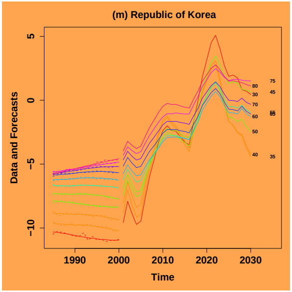


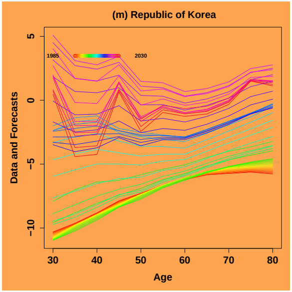
Smooth over age, time, age/time

Using Covariates (GDP, tobacco, trend, log trend)

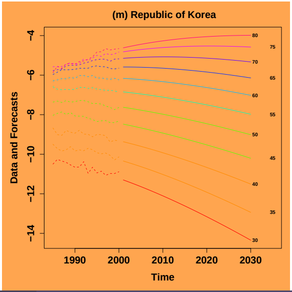
Lung cancer in Korean Males

Least Squares





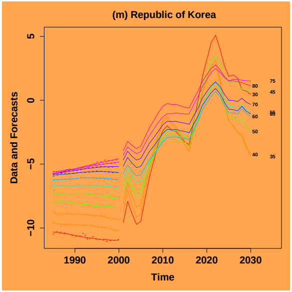
Smooth over age, time, age/time

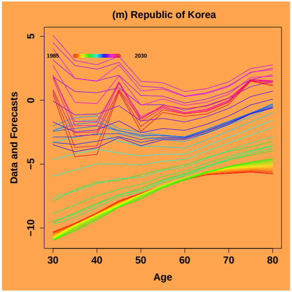


Using Covariates (GDP, tobacco, trend, log trend)

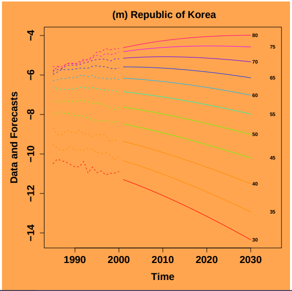
Lung cancer in Korean Males

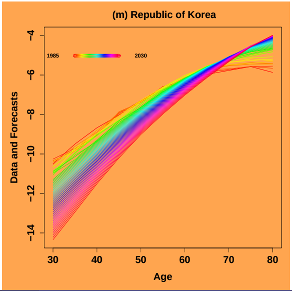
Least Squares





Smooth over age, time, age/time





Using Covariates (GDP, tobacco, trend, log trend)

Using Covariates (GDP, tobacco, trend, log trend)

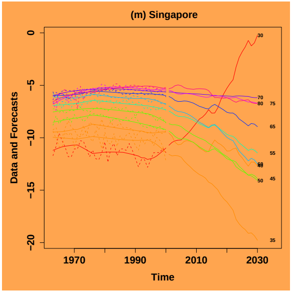
Lung cancer in Males, Singapore

Least Squares

Using Covariates (GDP, tobacco, trend, log trend)

Lung cancer in Males, Singapore

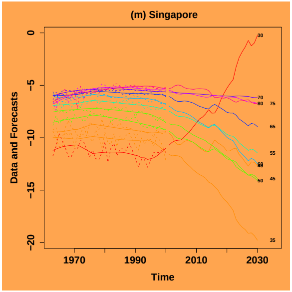
Least Squares

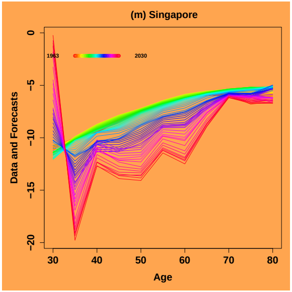


Using Covariates (GDP, tobacco, trend, log trend)

Lung cancer in Males, Singapore

Least Squares

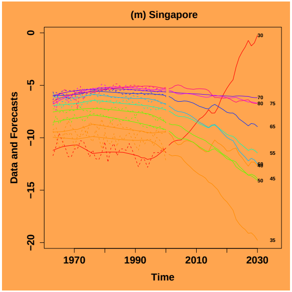


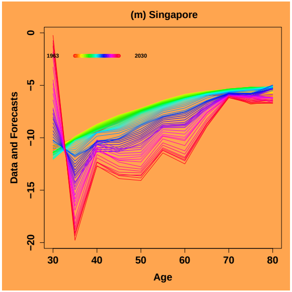


Using Covariates (GDP, tobacco, trend, log trend)

Lung cancer in Males, Singapore

Least Squares



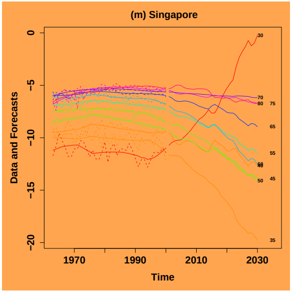


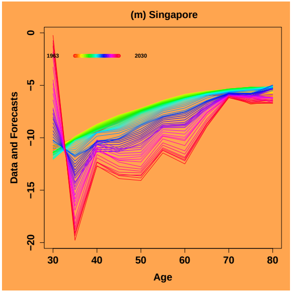
Smooth over age, time, age/time

Using Covariates (GDP, tobacco, trend, log trend)

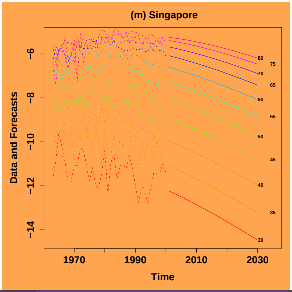
Lung cancer in Males, Singapore

Least Squares

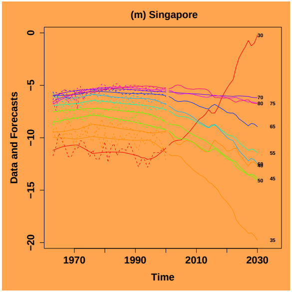


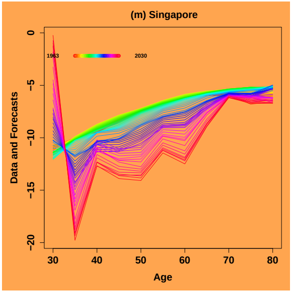


Smooth over age, time, age/time

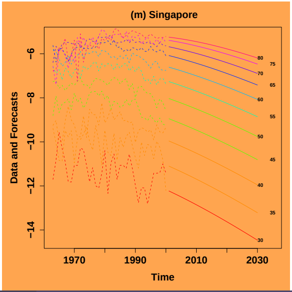


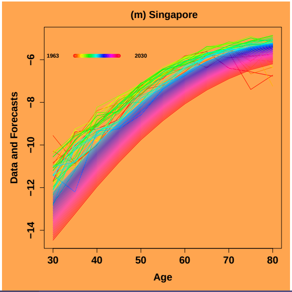
Least Squares



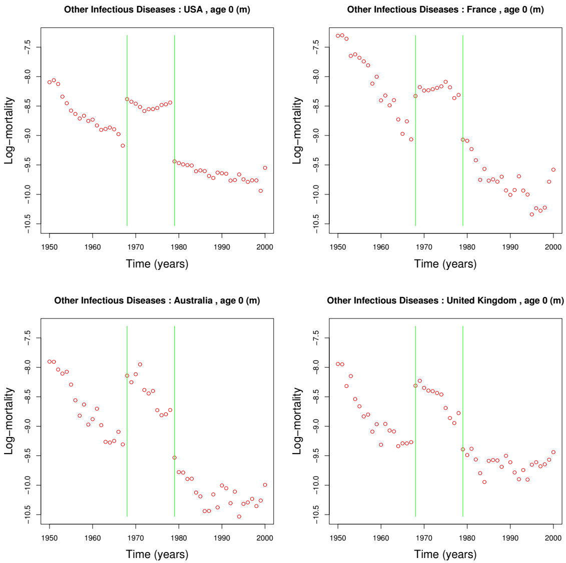


Smooth over age, time, age/time

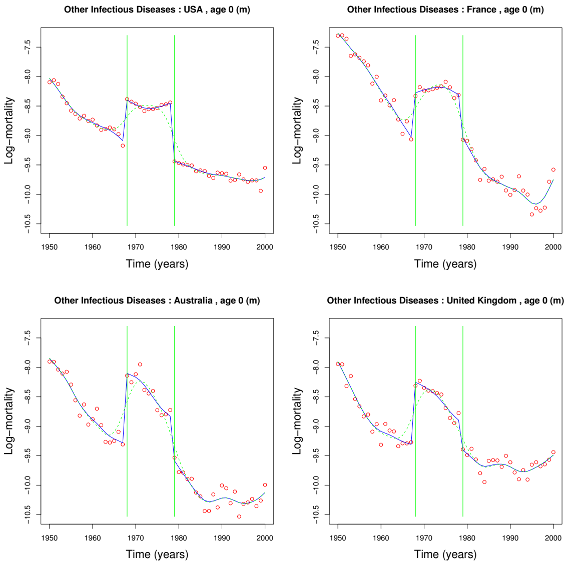




What about ICD Changes?



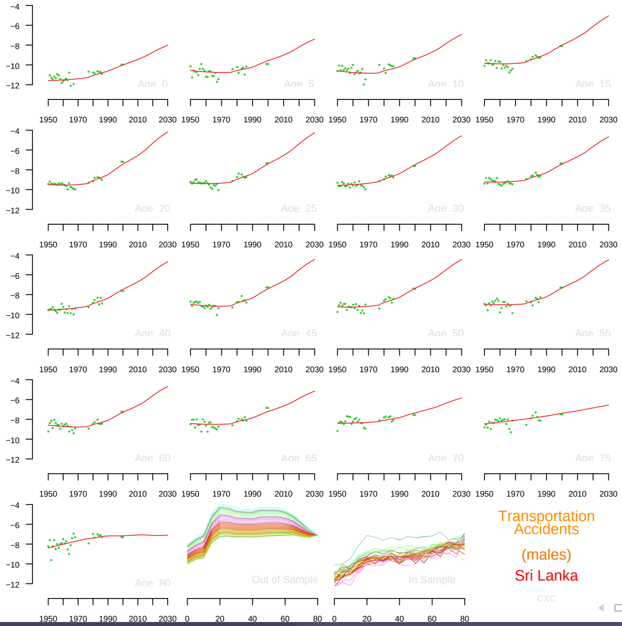
Fixing ICD Changes



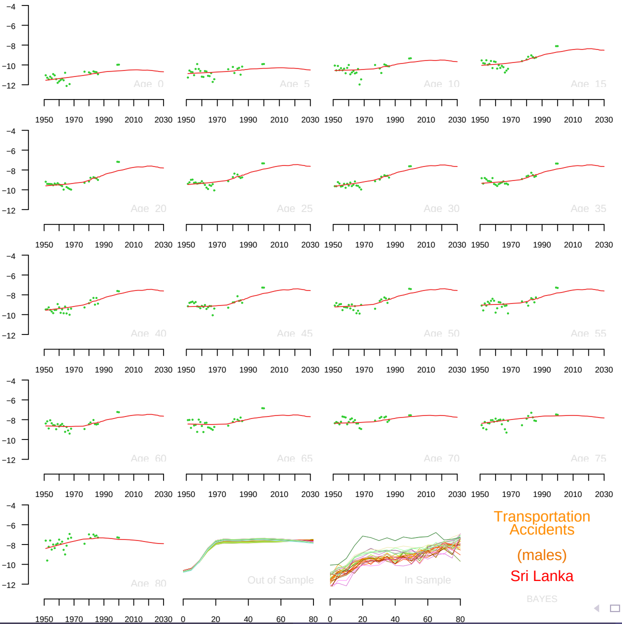
A book manuscript, YourCast software, etc.

http://GKing.Harvard.edu

Without Country Smoothing



With Country Smoothing



Formalizing Similarity

Formalizing Similarity

Standard Bayesian Approach

Formalizing Similarity

Standard Bayesian Approach

- Assume coefficients are similar

Formalizing Similarity

Standard Bayesian Approach

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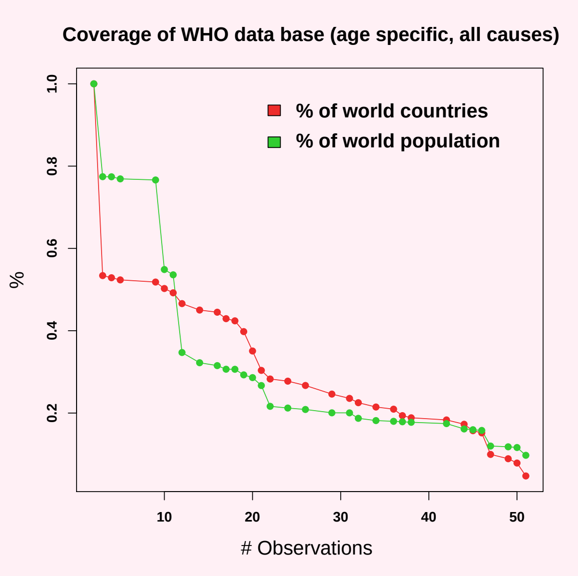
Many Short Time Series



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Mean Absolute Error in Males (over age and country)

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	Best Previous	Our Method	Best Conceivable	Over Best Previous	to Best Conceivable
Cardiovascular	0.34	0.27	0.19	22	49
Lung Cancer	0.36	0.27	0.17	24	47
Transportation	0.37	0.31	0.18	16	31
Respiratory Chronic	0.45	0.39	0.26	13	30
Other Infectious	0.55	0.48	0.32	12	30
Stomach Cancer	0.30	0.27	0.20	8	24
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- Does much better with better covariates